

TEACHER'S DIDACTICAL VARIABILITY AND ITS ROLE IN MATHEMATICS EDUCATION

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We are looking for the explanation of the differences in learners' flexibility when using the learned knowledge in new contexts. The main aim of our contribution is to combine various theoretical perspectives of investigating teachers' variability and students' flexibility when applying the learned knowledge. We consider the interpersonal differences as an effect of the teacher's didactical variability. Sarrazy (2002) claims that the question of the use of algorithms and taught theorems by students is more an anthropological than psychological problem. The contribution relates to the question B2: Do different frameworks make us look at different aspects of the learning process, that is, at different research questions and different data, or at different interpretations of the same data about the learning process?

1 INTRODUCTION

Learning mathematics is successful only when the learner is able to identify conditions for the use of knowledge in new situations. These conditions, however, are not present in the algorithms itself and cannot be carried over by teachers to their learners. This is one of the didactical contract paradoxes: "The more the teacher gives in to her demands and reveals whatever the student wants, and the more she tells her precisely what she must do, the more she risks losing her chance of obtaining the learning which she is in fact aiming for." (Brousseau, 1997, p. 41).

In (Novotna, Sarrazy, 2005) we presented two studies originally carried out as independent entities both dealing with the same topic: problem solving. One of them belonged more to the psychological perspective while the second one examined the effects of variability in the formulation of problem assignments on students' flexibility when using taught algorithms in new situations; the research was developed in the framework of the theory of didactical situations. These two studies proved themselves to be perfectly complementary. The first one allowed the detection of a set of phenomena, whereas the second gave them precision through an action model of the problem focusing on the variability in word problems. Connecting these two approaches allowed opening interesting perspectives for a better understanding of the role of problem solving in teaching and learning mathematics by giving precision to certain conditions of their use.

Why is it worth to combine the two approaches? Novotná (2003) showed that the analysis of models created by students enables the teacher to help them in case that

their effort to solve the problem correctly is not successful (mainly in determining the type of obstacles the student has faced). The individual differences in the form of graphical models could be explained by the internal student's cognitive processes (Novotná, 1999). However, this approach did not enable us to explain the striking difference “spontaneity versus copying” in the student groups. The psychological perspective did not offer any explanation of the fact observed. It was to be searched for outside the psychological approach. A suitable tool for the explanation was found in the frame of the Theory of didactical situations by Brousseau (1997), namely in the of variability of teachers introduced by Sarrazy (see Part 3).

Sarrazy (2002) presents a model based on the following idea: The more versions of realisations a particular form includes, the more uncertainty is attached to this form. To satisfy the teacher's expectations, the student must ‘examine’ the domain of validity of his/her knowledge much deeper than a student who is exposed to strongly ritualised (repetitive) teaching and therefore considerably reduced variability.

2 INTERPRETATION OF EFFECTS OF VARIABILITY

We are investigating effects of variability of teachers on learners' flexibility in applying algorithms from three perspectives (for more details see Novotná, Sarrazy, to be published):

a1 – *Psychological interpretation*: Variability gives priority to the change of learners' operational register by diversifying their relationship to the object of teaching or to their action (Richelle, 1986; Dré villon, 1980). In fact, the diversity of modes of relationship to the object of teaching, which is typical for didactical environments with strong variability, brings in an alternation between the phases of knowledge integration and differentiation in their usage. Dré villon (1980, p. 336) states that learners would possess a plurality in their access to objects that would be efficient to help “not only to proceed to the operational formal stage but to construct a repertoire of cognitive registers. This repertoire enables, if asked or needed, to examine a problem and solve it at the functional level, i.e., practical and objective, or to extract the operational quintessence and thus to construct a more general activity model”¹.

According to Piaget (1975, 1981), it is also possible to consider variability as one of the sources of perturbations resulting from variations of didactical environments; this variability enables to provoke cognitive adaptations (accommodations) and thus to increase the student's cognitive register in relation to a conceptual field – e.g., additive and multiplicative structures studied by Vergnaud (1979, 1982, 1994).

This first aspect can be précised didactically by changing the frameworks as proposed by Douady (1986) in the theory of “dialectic ‘tool-object’ (outil-objet)”: “A student possesses mathematics knowledge if he/she is able to provoke its functioning as explicit tools in problems he/she must solve [...] if he/she is able to adapt it when the normal conditions of its use are not exactly satisfying for interpreting problems or for posing questions with regards to it”² (Douady, 1986, p. 11).

a2 – *Anthropological interpretation*: When interpreting variability effects in relationship to what could be called “school culture” of the class, variability creates a characteristic of the environment in which learners develop and learn mathematics. In case of weak variability, a repetitive teaching, poorly varying in its forms of organisation and in the content, leads the learners to a hyper-adaptation to proposed situations. In order to adapt themselves to the usual teacher’s demands, the learners develop strategies of *coping* (Woods, 1990) with the criteria usually used. They can easily detect indicators allowing them to adapt their decisions and their behaviour to their teacher’s didactical requests. In that case, learners can very well apply suitable behaviour without exactly understanding the sense of the lesson or of the problem they were assigned. In case of strong variability, the learners cannot rely solely on the “rituals” because they can neither anticipate nor manage the succession of sequences or behaviours expected by the teacher. The learners’ engagement in the situation is much more probable.

It is well known that a particular teacher’s attitudes create educational environment, let us call it climate. Flanders (1966) showed the influence of teachers’ ways of functioning on the class climate. This climate was defined as “common attitudes that learners have, in spite of their individual differences, with respect to the teacher and the class”. In individual cases, this climate can support or block learners’ future successful development of their relation towards learning. Certain works in the domain of didactics of mathematics, e.g., Perrin-Glorian (1993) or Noirfalise (1986) support the previous interpretation.

The authors observe that some teachers focus their teaching more on the content to be taught while others on their learners privileging the relationship with the student. The first mainly look for progress in the subject matter and gaining new knowledge, they appreciate all attitudes with which the learners manifest their interest in what they are taught; the latter prefer production of ideas and communication among students. Achievements obtained by students differ significantly according to the considered domains: focus on the content favours success in algebra while focus on the students leads to better results in geometry and to making mathematics more attractive for the student.

a3 – *Didactical interpretation*: As mentioned in a1, Douady’s results (1986) allow clarifying the processes enabling to report on the effects of variability. This research is done in two frameworks: Theory of conceptual fields by Vergnaud (1990) and Theory of didactical situations by Brousseau (1997). For Douady, teaching a mathematical concept requires a transformation, a completion to see even the rejection of learners’ previous knowledge. The proposed problems must be perceived in such a way that the learners have an opportunity to engage at least one basic solving strategy but this strategy is insufficient: the taught knowledge (*object*) must correspond to the *tool* best adapted to the problem.

Douady distinguishes 6 different phases constituting the process of the “dialectic tool-object”:

Phase a – Mobilisation of “former”: Corresponds to the phase of the problem adaptation by the student.

Phase b – “Research”: Corresponds to the phase of action of the Theory of didactical situations (Brousseau, 1997). During this phase, students encounter difficulties caused by the insufficiency of their previous knowledge and consequently look for new, better adapted instruments.

Phase c – “Local explication and institutionalisation”: The teacher points out the elements that played an important role in the initial phase and formulates them in terms of the object with the condition of their use at the given moment.

Phase d – “Institutionalisation” (in the sense of the Theory of didactical situation by Brousseau, 1997): The teacher gives a cultural (mathematical) status to the new knowledge and he/she requests memorization of current conventions. He/she structures the definitions, theorems, proofs, pointing out what is fundamental and what is secondary.

Phase e – “Familiarisation - reinvestment”: It concerns the maintenance of what was learned and institutionalised in the various exercises.

Phase f – “Complexification of the task or a new problem”: The aim of this last phase is to allow the students to make use of the new knowledge in order to allow new objects to occupy their position in the students’ previous knowledge repertoire.

According to Douady, the aim is to exploit the fact that most mathematical concepts operate in several frameworks – in fact in diverse types of problems. For example, a numerical function can be presented at least in three frameworks: numerical, algebraic, and geometrical. These changes of frameworks (“game of frameworks”) allow varying the significances (supports of significations) for the same concept and allow avoiding that the learners make them function in a partial or in over-contextualised ways. The interactions among diverse frameworks allow, according to Douady, to make the knowledge progress and to keep all the conceptual potential of the taught object.

3 EXAMPLE: SARRAZY’S MODEL OF TEACHERS’ VARIABILITY

For the characterisation of teachers’ modes of didactical activity, typology of modes and examination whether these modes enabled awareness of the differences in the sensitivity to didactical contract in groups of students, Sarrazy (1996) introduced a model that allows describing the modes of teachers’ actions. This model is sensitive in learners’ treating of problem types. It uses the following three dimensions, the six variables being defined in order to measure variability in organisation and management of the teacher’s work during and between lessons:

- i) *Didactical structure of the lesson* (what the teacher really does from the perspective of the knowledge to be taught);
 - v₁. What is the type of didactical dependence? Does the teacher proceed from simple to more complex tasks or the other way round?
 - v₂. Place of institutionalisation: At which moment does the teacher present a solving model? Closer to the beginning or to the end of the lesson? Or only at the beginning or at the end?
 - v₃. Types of validation: How are the students informed about validity of their answers? Does the teacher always use the same type of evaluation and assessment (by the milieu, by direct evaluation, by the Topaze effect³, by peers ...)?
- ii) *Forms of social organisation* (this domain corresponds to the teacher's activities regarding class management)
 - v₄. Interaction modes: teacher-student(s), student(s)-student(s)
 - v₅. Management with regard to the students' groupings: the whole class, small groups, individual work
- iii) *Variability of arithmetical problem assignment*
 - v₆. The variable is related to editing the problem assignment. It is given by an indicator which measures the teacher's "capacity" to consider diverse modalities of the same didactical variable in the assignment.

This model makes it possible to describe the teacher's teaching practices from a triple perspective: presentation of the content (i), desired forms of teaching (ii) and variety of the proposed situations (iii). It is not an isolated variable that affects the students' learning (mainly defined by the notion of sensibility – i.e. their ability to use the taught algorithms in various contexts). On the contrary, it is an effect linked to a set of variables (that may be called a profile of the didactical action); this profile enables a characterization of one way of letting the students do mathematics. This is why we proceeded to a hierarchical classification in order to show similarities by clustering of variables.

Using the above variables, teachers' different profiles were hierarchically classified (Sarrazy, Novotná, 2005, where the experimental disposition, that allowed characterising teacher's variability and thence to show the influence on the way how the students do mathematics, is presented; the crucial role of didactical contract and the sensitivity to it is documented).

Let us recall here the general idea: Submission of students to a teaching style poorly varied (and strongly repetitive in the forms of organisation in the presentation of the content) will decrease the possibilities of opening the didactical contract; vice versa, more variable the teaching is, the more the students will be confronted with new situations and the more flexible their use of the taught algorithms will be. Let us con-

sider a simple (and therefore caricaturing) example which serves as an illustration of the theoretical position:

The mother spent 13 EUR at the market. Now, she has 19 EUR. How much had she had when she went to the market?

This problem, although simple, presents several difficulties to the students. These difficulties are based on the fact that the problem evokes the framework of subtraction but the numerical operation to be executed is addition. Here is an example of the variety: the more the student will be confronted with the situations that involve diverse contexts of the use of additive structures, the higher the probability that his/her answer will be guided by conceptualising the relations in play; vice versa, the less diverse the situations are, the more the students will be lead to rely on the apparent characteristics of the tasks when producing their answer (e.g.: every time seeing the verb “spent” they will subtract, “anybody” divide etc.).

Using the above variables we defined three teaching styles of the school culture that are in strong contrast:

“*Devolving*”: This style corresponds to what, in the first approximation, could be called “active pedagogy” in which the students need to be “active”. This style is characterised by strong variability in the organisation and management of situations: the teachers regularly use group work although they by no means restrict only to this form of student work; generally speaking, the problems are complex; classroom work is very interactive (students interact spontaneously, “choral” answers are not rare, ...); in the lesson, institutionalisation is diverse. These are the main features of the first style.

The other extreme is the “*institutionalising*” style. This climate is characterised by a weak introduction and a weak variety of situations presented to students; we could call it ‘classic teaching’ in which the scheme “show–remember–apply” seems to be the rule. These teachers institutionalise one solving model very quickly and then present students with exercises of growing complexity. First, the exercises are corrected locally – the teacher passes through the rows and corrects them individually. Then the teacher gives the complete correction on the blackboard; here he/she gives details of the solution and, depending on the time he/she has, occasionally invites some students to the board either to make sure that they are paying attention, or to recall certain knowledge. Now, the interactive climate is quantitatively as well as qualitatively very different from the interactive climate of the preceding style: Students’ spontaneous interactions or “choral” answers hardly ever occur.

The third style is the “*intermediary*” style. As its name indicates, this style is closer to the institutionalising style, even if the teachers ‘open’ the situations more and more frequently. In any case, here the students have more chances than students of “institutionalising” teachers to encounter research situations, and debate, but markedly less than those exposed to the “devolving” style.

As we expected, we observed strong internal coherence of each of the styles (climates) confirmed by the stability of the results acquired using various methods of data analysis (implicative analysis, dynamic clusters, hierarchical classification, and so on). It seems to provide evidence in favour of the existence of an organising principle for the practices. This organising principle could at the same time be linked with didactical conditions (meant in relation to the knowledge dealt with) and with anthropological conditions (independent of knowledge but linked with teachers' pedagogical or political convictions, with influences of fashionable constructivist, cognitive, and other psychological models).

4 CONCLUDING REMARKS

There are two concluding topics to be discussed: the consequences of the presented results for teacher training and the theoretical positions of the studies about variability.

The presented results are of great interest for improving the teaching of mathematics by focusing on the flexibility in the use of the taught algorithms. But is it possible to foster an increase in the variability of the teachers? It seems to be difficult to directly influence the conditions allowing increasing the variability of teachers. Even if we find it important to present teachers with models of the analysis of problem assignments (e.g. those of Vergnaud concerning additive and multiplicative structures), there are good reasons to believe that mere presentation is not sufficient. In fact, on the one hand these models when only presented to teacher trainees to have a look at them do not affect their variability directly (Sarrazy, 2002); on the other hand, we could observe that variability is the dimension of the teacher's activities that is statistically linked with other dimensions of his/her didactical activities (e.g. the use of group work, the volume of didactical interactions, his/her pedagogical philosophy). Variability should be understood as one of the elements of the teacher's system of didactical activities that interacts with other components.

This last aspect bids for discussion of its theoretical status. We do not pretend to submit here a new theoretical concept of a teacher's didactical activity but more modestly, we situate this approach as an extension of the Theory of Didactical Situations by Brousseau (1997). During the "ordinary" teaching situations that we observed, we found rarely those where the "milieu" contained an a-didactical component, i.e. those where the situation allowed to delegate to students the retroaction to their actions. We believe that a developed variability when the a-didactical "inside" of the situation is absent, would allow the students to establish a quasi a-didactical relation only. As we indicated, it is the consequence of the fact that they cannot go upon the formal aspects of the proposed assignments.

An important question arising from our research is: What kind of training is likely to increase the variability of teachers? Although it is certainly an important question, we find solving it premature as long as the problem of conditions favouring the variabil-

ity has not been clarified. This problem, first opened in anthropo-didactical approach in DAESL about fifteen years ago, needs to be explored in further research in the area where didactics and pedagogy meet.

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¹ Translation from French by J. Novotná. Original text: « non pas seulement à passer au stade opératoire formel mais à construire un clavier de registres cognitifs. Ce clavier permet à la demande, et en cas de besoin, d'examiner un problème et de le résoudre au niveau fonctionnel, c'est-à-dire pratique et objectif, ou d'en extraire la quintessence opératoire et de construire ainsi un modèle plus général de l'activité. »

² Original text: « Un élève a des connaissances en mathématiques s'il est capable d'en provoquer le fonctionnement comme outils explicites dans des problèmes qu'il doit résoudre [...] s'il est capable de les adapter lorsque les conditions habituelles d'emploi ne sont pas exactement satisfaites pour interpréter des problèmes ou poser des questions à leurs propos ».

³ *Topaze effect*. When the teacher wants the pupils to be active (find themselves an answer) and when they can't, then the teacher suggests/disguises the expected answer or performance by different behaviours or attitudes without providing it directly. Example: Teacher: 6×7 ? Pupils: 56. Teacher: Are you sure?