COMPLEMENTARY NETWORKING: ENRICHING UNDERSTANDING

Ferdinando Arzarello (*), Angelika Bikner-Ahsbahs (°), Cristina Sabena (*)

(*) Dipartimento di Matematica, Università di Torino (Italia),
(°) Fachbereich Mathematik und Informatik, Universität Bremen (Deutschland)

Our analysis of data about one learning situation from two theoretical perspectives yields results that on the surface seem to be in conflict. Through networking of two theories we produce a fresh combined analysis tool, which deepens our understanding of the data in an integrated way. We elaborate this example to make explicit our two theoretical approaches and our networking strategies and methods.

INTRODUCTION

The goal of the paper is to show how networking different theories can help researches in entering more deeply into their research questions. More precisely, we will illustrate the limits of two theoretical approaches when used alone to analyse a classroom teaching situation, and the benefits of networking. As a result, data analysis and learning processes understanding is strongly enriched.

The main question faced in our research concerns how mathematical knowledge about the growth of the exponential function is achieved in a specific socially supported learning processes. This requires properly defining the objects of our research, the method and the tools for observation (Prediger et al., 2008). As to the objects, we distinguish two deeply linked components: the social interaction among the subjects, and the epistemic issues in such learning processes.

Our networking strategy is worked out through analyses of empirical data. The same teacher-student-interaction is analysed from two theoretical perspectives that on the surface seem to be in conflict: the *interest-dense situation* and the *semiotic bundle* analysis. Using the former, it appears that the thought process of a student is disturbed by the social interaction with the teacher. However, no disturbances appear using the latter. We will show that through adding an epistemological perspective this conflict can be cleared away since the results can be integrated into a common view deepening our insight from both theoretical perspectives. This experience will be a starting point for a case of local integration of the two theoretical perspectives and some methodological reflection concerning networking strategies and methods.

ADOPTING TWO DIFFERENT PERSPECTIVES

Interest-dense situations and its epistemic process

So called interest-dense situations (Bikner-Ahsbahs, 2003) are those in which a maths class shows interest in the mathematical topic or activity, they occasionally occur within discursive processes in everyday maths lessons. In these situations the students become deeply involved in the mathematical activity, deepen their mathematical insight constructing further reaching mathematical meanings and begin to appreciate the mathematics they learn. To achieve some mathematical knowledge the students activate epistemic actions (actions that are executed in order to come to know more). Through social interactions the class collectively coordinates the epistemic process. In this way collective epistemic actions are constituted by social interaction. In contrast to non interest-dense situations, all interest-dense situations lead to the epistemic action of structure seeing (perceiving a mathematical pattern or rule referring to an unlimited number of examples).

The genesis of interest-dense situations is supported by a special kind of social interactions: The students are driven by their own way of thinking. They follow their own questions and ideas about the mathematical object that they want to know more about. In this case the students' actions are independent of the teacher's expectations. In interest-dense situations the teacher's expectations do not control the situation. Rather the teacher focuses on supporting the students' thinking. If the teacher's behaviour is controlled by his own expectations the emergence of an interest-dense situation is interrupted, and the learning process is disturbed (Bikner-Ahsbahs, 2003).

The ways in which the teacher and students socially interact can be analysed on the three levels (Davis, 1980; Beck & Meyer, 1994). Speaking, a person expresses something on three different levels. On the locutionary level he/she says something, on the illocutionary level, he/she tells something through the way of saying something. The perlocutionary level is concerned with effects: "a speaker saying something produces an effect on feelings, thoughts, or actions of the audience, other persons, or himself" (Davis (referring to Austin and Searle), 1980, p. 38). In our example, G locutionarily says: "for a very big variable a, when the exponential function ($f(x) = a^x$) and this straight line (which he assumes), meet each other, it (meaning the straight line) approximates the function very well because..." being interrupted by the teacher's request: "what straight line, sorry?". By using broken language, G tells the teacher that (illocutionarily) he is working out his train of thought while speaking. Starting the sentence with "because", he indicates on the illocutionary level that his train of thought is not yet finished. On the perlocutionary level we observe an effect; the teacher's request. In order to comprehend how the epistemic process in a discursive learning situation is socially supported or hindered; the analysis of social interactions is done on these different levels and is complemented by an analysis of the epistemic process. The term "non-locutionary level" will embrace the illocutionary and perlocutionary level.

The semiotic bundle perspective

The semiotic bundle perspective lies on two basic assumptions:

- the teaching-learning process inherently involves *resources of different kinds*, in a deep integrated way: words (orally or in written form); extra-linguistic modes of expression (gestures, glances, …); different types of inscriptions (drawings, sketches, graphs, ...); different instruments (from the pencil to the most sophisticated ICT devices), and so on (for some examples see Arzarello, 2006);

- such resources may play the role of signs (according to Peirce's definition[1]) and therefore can be considered as *semiotic resources*.

Differently from other semiotic approaches, the semiotic bundle construct allows us to theoretically frame gestures and more generally all the bodily means of expression, as semiotic resources in learning processes, and to look at their relationship with the traditionally studied semiotic systems (e.g. written mathematical symbolism):

> "A semiotic bundle is a *system of signs* — with Peirce's comprehensive notion of sign — that is produced by one or more interacting subjects and that evolves in time. Typically, a semiotic bundle is made of the signs that are produced by a student or by a group of students while solving a problem and/or discussing a mathematical question. Possibly the teacher too participates to this production and so the semiotic bundle may include also the signs produced by the teacher" (Arzarello et al., in print).

In teaching-learning contexts the different semiotic resources are used with great flexibility: the same subject can exploit simultaneously many of them, and sometimes they are shared by the students and by the teacher. All such resources, with the actions and productions they support, are important for grasping mathematical ideas, because they help to bridge the gap between the worldly experience and the time-less and context-less sentences of mathematics. An interesting phenomenon that has been identified within such an approach is the so called *semiotic game* (Arzarello, 2006; Arzarello et al., in print). A semiotic game happens in the teacher-students interaction when the teacher tunes with the students' semiotic resources and uses them to guide the evolution of mathematical meanings. We have analysed various examples in which the teacher repeats a student's gesture, and correlates it with a new term or with the correct explication given using natural language and mathematical symbolism (ibid.). Semiotic games constitute therefore an important strategy in the process of appropriation of the culturally shared meaning of signs.

An example analysed from the two perspectives

In this example, students (grade 10 of a scientific oriented high school) are working in pair on an exploratory activity on the exponential function. They are using a dynamic geometry software to explore the graphs of $y = a^x$ and of its tangent line[2] (a is a parameter whose value can be changed in a sliding bar). At a certain point the teacher has asked the students the following question: what happens to the exponential function for very big x? We propose a short excerpt from the interaction between the teacher and one pair of students (G and C) about this question.

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[1] As *sign* or *semiotic resource*, we consider anything that "stands to somebody for something in some respect or capacity" (Peirce, 1931-1958, vol. 2, paragraph 228).

[2] The line is actually a secant line; the secant points are so near that the line appears on the screen as tangent to the graph. This issue has been discussed in the classroom in a previous lesson.

1 [00:00] G: but always for *a* very big this straight line (pointing at the screen), when they meet each others, there it is again…that is it approximates the, the function very well, because…

2 T: what straight line, sorry?

3 G: this …(pointing at the screen) this, for *x* very, very

(00:14) big

00:14 G: the hand goes upwards

4 T (00:16): will they meet each other (00:17)? [suggestive connotation in the sense of "do you really think so?"]

00:16 T: pointing two forefingers

00:17 T: crossing the two pointed forefingers

00:19 G: two forefingers touching each other

5 G: that is [cioè][3], yes, yes they meet each other (00:19)

6 T: but after their meeting, what happens?

7 G: eh..eh, eh no, it make so (00:24)

00:24 G crosses the left hand over the right one; T is keeping the previous gesture

8 T: ah, ok, this then continues (00:27), this, the vertical straight line (00:28), has a well fixed *x*, hasn't it? The exponential function later goes on increasing the *x*, doesn't it (00:31)? Do you agree? Or not?

00:27 T moving rightwards his left hand

00:28 T: right hand vertically raised

00:31 T: moving rightwards his right hand

[3] The expression "cioè" in Italian means literally "that is". Over-used by teenagers, it introduces a reformulation of what just said. As it is likely in this case, it can have the connotation of "I am sorry but".

9 G: yes […]

10 T (addressing C): He [G] was saying that this vertical straight line (pointing at the screen) approximates very well (00:43) the exponential function

11 G: that is, but for very big *x* (00:46)

12 T: and for how big *x*? 100 billions? (00:51) x = 100 billions?

00:43 T raises both hands

00:46 G moved his left hand high wards

[00:51 T: raised his hand and keeps it still]

13 G: that is, at a certain point…that is if the function (00:57) increases more and more, more and more (00:59) then it also becomes almost a vertical straight line (1:03)

00:57 G raises his left hand

00:59 G moves his hand upwards

01:03 G's hand is vertical

14 T: eh, this is what seems to you by looking at; but you have here x = 100 billions (01:08), is this barrier overcome sooner or later, or not?

01:08 T: keeps his right hand in the vertical position

15 G: yes

16 T: in the moment it is overcome (01:12), this *x* 100 billions (01:13), how many *x* do you have at disposal, after 100 billions? (01:14)

01:12 T crosses left fore-finger over right hand

01:13 T raises his right hand

01:14 T moves right hand rightwards, repeatedly

17 G: infinite

18 T: infinite… and how much can you go ahead after 100 billion?

19 G: infinite points

20 T: then the exponential function goes ahead for his own business, doesn't it? [01:26]

The analysis from the perspective of interest-dense situations

How is the emergence of an interest-dense situation supported or hindered? In line 1 G begins to construct mathematical meanings about the growth of the exponential function in broken language as described above. In this moment the teacher interrupts him: Apologising, the teacher illocutionarily indicates that he normally would not interrupt the student, but in this case an interruption is necessary. The teacher perlocutionarily might want G to feel accepted, however, saying sorry indicates also that there is something wrong with the "straight line". Locutionarily the teacher says: 'tell me what straight line you mean'. However, G does not react on the locutionary level; he describes the condition for his explanation in line 1: "for very big *x*"; just as he was asked to do in the task. The teacher's question "They will meet each other?" is (illocutionarily) posed in a suggestive way. Perlocutionarily, the teacher wants to get the answer: 'no, they don't meet'. However, G withstands the teacher's demand and answers *that they meet* (5). This is supported through adopting the teacher's finger crossing gesture (6, 7). On the locutionary level, we would see only the question and the answer. On the non-locutionary levels there is negotiation underneath. Looking only at the lines 1 to 5, an interest-dense situation is about to emerge. From the theory of interest dense-situation we could predict how the teacher could support or hinder the emergence of interest-density. Focussing on the student's ideas he would support it, acting according to his own thinking process or his expectations he would interrupt the emergence of it.

In the sentence that follows, the teacher starts to build up an argumentation as a proof of contradiction following his own train of thought and not that of the student. In line 8, he constitutes his base of argumentation. In order to include G into the process, his rhetorical questions "do you agree? Or not?" demands G's agreement. Summarising G's statement from line 1 grammatically more precise (10), the teacher establishes the statement that he wants to prove being false. G's modification "but for very big *x*" locutionarily looks like a complementary argument, but illocutionarily he corrects the teacher. G only partially agrees, because his description was based on 'very big *x*' (11). Again, G indicates that his train of thought is a bit different. Perlocutionarily G succeeds at this moment because the teacher changes his focus; locutionarily taking up the student's idea in the question: "for how big *x*?" (12). G seems to feel encouraged to explain: "that is, normally does not arrive at a certain point, the function increases more and always more, then still it becomes almost a vertical straight line …". Again, an interest-dense situation is about to begin. Then, on the non-locutionary level, the teacher expresses understanding G's view (14). However, through saying that, he also says that the student's way of arguing is false. He proves this by a proof of contradiction which he closes by the rhetorical question: "or not?" After the proof, G gives up to follow his own train of thought. The emergence of interest-density dries up.

Semiotic-bundle analysis

We see both student and teacher enacting a semiotic bundle composed by words, gestures, and inscriptions on the screen of the laptop. The basic point of discussion regards the behaviour of the exponential function for big base *a* and big values *x*. G thinks that in this case, the function can be approximated by a vertical line (#1-3). Such a conjecture is fostered by the image from the dynamic geometry software the students are using (see Figure 1): the tangent line appears in fact as almost vertical, and the exponential function comes to be perceptually confused in it. The teacher wants to clarify whether the student is thinking to a vertical asymptote (#4-6). Asking about an hypothetic meeting of the function with the straight line, he is representing the graphs by means of his iconic gesture (00:17): his right forefinger stands for a vertical line, and his left forefinger is inclined to represent the exponential function graph. G (#5-7, 00:19 and 00:24) is tuning with the teacher's semiotic resources, both speech and gesture. With his hand, he represents the graph of the exponential crossing the vertical line (00:24): he is answering the teacher's question by means of the gesture. The teacher (#8) accepts such an answer and endeavours in making explicit the idea that the domain of the exponential function is not limited, and therefore its graph intersects any vertical line. To do so, he uses both speech and gestures (see #8-20, and the related pictures). Let us enter into the dynamics of the semiotic bundle. In order to include C in the discussion, the teacher reports G's observation. By repeating G's words (#10) he is tuning with the student's semiotic resource (speech). But through gestures (00:43, 01:12, 01:13), he is making explicit the behaviour of the exponential function, i.e. the fact that it crosses any vertical line. The teacher is showing what we call a semiotic game, in that he is tuning with the student's semiotic resource, and is using another resource to make meanings evolve towards mathematical ones. The gesture appears a powerful resource, since it allows him to refer to what cannot be seen in the representation on the screen, and that is still difficult for the students to be conveyed in speech. In particular, gesture seems a suitable means to refer to very big values and to evoke their infinite quantity (01:14). If we now turn to G, we see that he does not appear to have profited from the teacher's semiotic game. Let us focus on lines 11-13 and related pictures. In his words we can see that he is still insisting on the idea that the function will become "almost a vertical straight line", but above all his gestures appear very different from the teacher's ones. In fact, whereas the teacher's gestures link big values of *x* with the right location in space (hand moving rightwards: 00:31, 00:51 and 01:14), the student's ones link big values of *x* to top location in space (hand moving upwards (00:46, 00:57, 00:59 and 01:03). From a cognitive point of view, they are

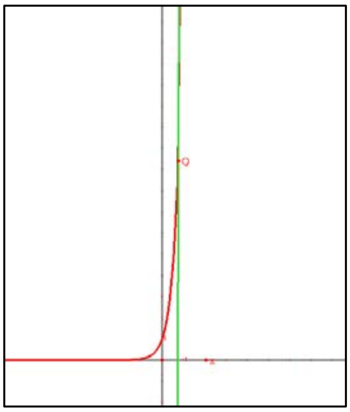

Figure 1

adopting different metaphorical references and only the teacher's one is consistent with mathematical signs (i.e. the Cartesian plane).

AN EMPIRICALLY BASED INTEGRATION

Based on the theoretical account and the empirical analysis, we can consider the two theories as *complementary*: they shed light on different aspects of the teacher-students interaction. However, by using the two theoretical lenses separately it appears that there is something important missing in each case. The strength of the interest-dense situations perspective is the possibility to predict their emergence according to the type of social interactions that hinder or foster it. In fact it includes the analysis of the locutionary and non-locutionary levels of speech and shows negotiations underneath the content. This approach is able to describe how the epistemic process proceeds and provides deeper insights into the social interaction process that foster or hinder the emergence of interest-dense situations, including structure seeing. However, the student and the teacher are not able to merge their argumentations although there is a lot of negotiation about whose train of thought will be followed. Neither the teacher nor the student is able to engage with the other’s perspective. The analysis shows a gap that cannot be overcome, but is unable to give the tool to find out why this is so. By looking at a wide range of signs (in Peirce's sense), the semiotic bundle analysis identifies the semiotic game between teacher and student, and allows the game to be properly described. However the theory is not able to fully explain the reason why the student does not gain much from such semiotic game. In most other cases we had observed that the students succeeded to learn through semiotic games (e.g. see Arzarello et al., in print). One difference that can be identified within the theoretical frame is that this time the semiotic game applies the gesture-speech resources in reverse way with respect to semiotic games analysed as "successful". In this case, in fact, the teacher tunes with students' speech and uses gesture to foster meaning development; in other cases (see Arzarello et al., in print) it was the other way round: tuning with gestures and fostering meanings through words. We could conjecture that the characteristics of gestures as semiotic resource are not apt to this kind of didactical support, and indeed this can be a research problem to investigate. But within the semiotic bundle theory we are not able to say why such semiotic game did not work. The discussion so far leads us to argue that the simple juxtaposition of the two perspectives is not enough to deeply understand what's going wrong in the analysed episode. To go a step further, we start from the example to combine and locally integrate the two theories. The *combination* provides a tool to investigate how each sign of the semiotic bundle may contribute to the locutionary or non-locutionary aspects of the interaction. For instance, a gesture can support locutionary as well as non-locutionary features that play important roles in the interaction (see Figure 2). In the episode, gestures illustrated in pictures 00:19 and 00:24 at the locutionary level show the behaviour of the graph in iconic way, and at the non-locutionary they show that the student is trying to agree with the teacher's perspective. The hands in fact are used in the same configuration as the teacher (observe the teacher in the same pictures); in the entire episode this is the only case in which it happens. In all the other cases, G's gestures have very different configurations. Concerning the words, a similar situation is constituted; at the locutionary level G's words affiliate to the teacher's perspective. But at the non-locutionary levels the teacher and G do not fully agree with each other using words.

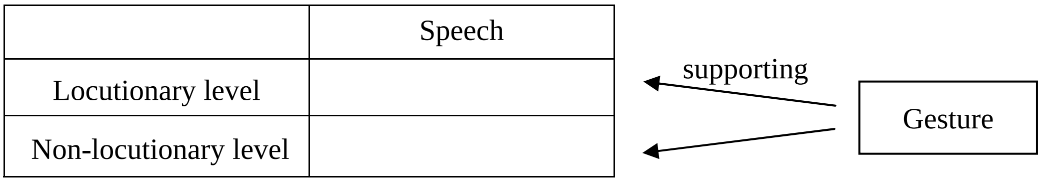

Figure 2: Two-level-analysis of semiotic resources

With the aim to answer the question what exactly did not work in the student-teacher interaction of the episode, we propose an integration of the two combined theories adding an *epistemological* dimension to the analysis above; that means to carefully consider the epistemological points of view of the teacher and of the students. By *epistemological points of view* we mean the background of the piece of knowledge that a subject thinks can give sense to a specific situation. The epistemological point of view is not always explicit: it appears not only from the locutionary dimension of the semiotic resources used by a subject but also from the non-locutionary ones. Moreover, it can be partially revealed by the epistemic actions produced by the subject. Of course the epistemological point of view with respect to a situation can vary with the subjects. For example, that of a student can be different from that of the teacher or of another student. But this difference might not be apparent although the dynamics of a didactic situation in the classroom might be deeply influenced by it, especially when the teacher is not aware of it or does not take into account the epistemological points of view of his students. This is exactly what happened in the episode analysed above. We observe a semiotic game articulated in a tuning in words and a dissonance in gestures: the teacher is repeating G's words (#11-12), but he is performing completely different gestures (see, that in 00:46 G's hand is moving upwards, to indicate big values, whereas in 00:51 the teacher's hand is moving rightwards). The dissonance in gesture is a signal that the teacher and the student are showing different points of view: the teacher relies on a formal theory (Weierstrass definition of limit) using potential infinite; the student relies on his perception imagining what happens "for very big *x*" (#11). It is not so clear what the student means: possibly he has been influenced by perceptive facts (see the discussion above) and perhaps he is thinking within an "actual infinite" perspective, even if this point is not so explicit here. The analysis of the semiotic game including the epistemological dimension allows us therefore to say that there is an *epistemological gap* between teacher and student, and to hypothesise that this gap prevents the teacher from suitably coaching the student's knowledge evolution and the student from profiting by the interaction with the teacher. Therefore the emergence of an interest-dense situation was not successful.

CONCLUSIONS

Presenting an empirical case of networking of theories, we showed that through a local integration two theoretical approaches can be enriched (Prediger et al., 2008). This was possible because the theories provided two complementary observation tools: one at the level of discourse analysis describes social interactions and their epistemic processes; the other at the level of gesture analysis describes learning from a semiotic perspective. The starting point of the theoretical integration was based on the empirical data analysis whose meaning was not clarified by any of the two theories. This stall was overcome by suitably combining the two approaches: adding an epistemological dimension made possible to locally integrate the two theories, so uncovering blind spots in both.

The results of our analysis could have important didactical consequences: in fact from them it seems possible to design a fresh role for the teacher in supporting students' learning processes. According to the combined analysis of the semiotic and linguistic features, integrated with the epistemological dimension, the teacher could develop suitable interventions, taking care both of the social interaction and of the epistemological issues with the help of semiotic resources.

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