CERME 6 – WORKING GROUP 8

QUESTIONS AND THOUGHTS FOR RESEARCHING CULTURAL DIVERSITY AND MATHEMATICS EDUCATION

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INTRODUCTION

QUESTIONS AND THOUGHTS FOR RESEARCHING CULTURAL DIVERSITY AND MATHEMATICS EDUCATION

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CERME 6, in Lyon 2009, was the 4th meeting of the working group "Cultural diversity and mathematics education" (in previous meetings it was WG10 and it had slightly different titles). The group is particularly interested in understanding learning and teaching mathematics in culturally diverse schools, classrooms and other educational settings. It also acknowledges the relevance of studies on culture and cognition in outside school settings linked with mathematics and, in particular, with ethno mathematics. We constitute a multi-disciplinary group that includes researchers from a variety of disciplines, such as mathematics, education, socio-cultural and developmental psychology, philosophy, anthropology, linguistics, sociology, political sciences, etc. We are in ourselves a multinational community that in Lyon included contributors from Belgium, Brazil, Canada, Cyprus, Denmark, Italy, Portugal, Rwanda, Spain, Sweden United Kingdom and USA.

QUESTIONS RAISED DURING WG8 MEETINGS

The areas covered by the presentations during our meetings were different theoretical and methodological approaches as well as different research domains. Teaching, the relationship between home-family and school, out-of-school practices, particular cultural and linguistic groups were some of the domains discussed. The perspectives that all of us brought to the discussion led, in particular, to interrogating how culture links to diversity, practices and institutions.

Conceptual clarification

The discussion of several papers claimed for clarification of different notions, such as 'culture', 'diversity' and 'cultural diversity'. This was considered important both in relation to theoretical papers and to empirical papers. Broad conceptualisations meant that there were issues at stake for data collection. There was agreement that culture is something dynamic but it is also something which is re-interpreted for meaning. In other words, there was interest in the socio-cultural as co-constituted in the psychological. Furthermore, whilst new concepts are introduced into theoretical research others continue to be discussed over time.

Culture in practice

Whilst discussions on the conceptualisation of culture were useful to the group, many felt they needed to make sense of how this shapes and is shaped by practices in the classroom. Questions were raised such as – how can we teach mathematics whilst respecting cultural diversity? How do teachers/parents of other cultural backgrounds explain mathematical problems? Can culture help us understand identities development in mathematical practices within and outside school?

Culture and institutions

The tensions between the school as a normalising institution and the diversity of students in society were raised. It was questioned what the dangers of bringing culture to a normalising institution may be? When one thinks of school as an institution whose goal it is to transmit culture, one has to think "whose" culture is being referred to. In other words, in which ways do educational institutions reproduce inequalities? It was suggested that this 'tension' or 'gap' between cultural diversity and the institution is as symbolic as the notion of 'normal'. The normalised institution, an idea developed and reproduced by school, is also symbolic and can be perceived as exotic and outside the lives of most pupils. Furthermore, institutions are culturally composed by people and these people may influence the institution.

SHARED INTERESTS WITH OTHER GROUPS

During reporting sessions, it was made apparent that there are different overlaps between WG8 and papers presented in other working groups. This was mainly expressed through an interest in a socio-cultural perspective when applied to a specific domain which was covered by another group. This perspective is felt to be more relevant since, nowadays, our schools are recognized to be more and more culturally diverse, and inequity in education has become under socio-political scrutiny.

For some groups, the intersection is wide and obvious. This would be the case with the working group dealing with mathematics and language, since culture is inextricably linked to language. It seems also clear to us that there is an intersection with the group working on Early Years Mathematics, since nowadays it is becoming clearer, especially for this age group, that learning is situated on its context.

For some other groups, one has to go deeper to see the overlapping. However, one of the participants in the Applications and Modelling group explicitly contributed to the reporting session by affirming that *"modelling in mathematics can also benefit if the cultural backgrounds of learners is taken into account while modelling learning situations"*. It did not surprise us either that people that had attended the Algebraic Thinking or Geometrical Thinking groups told that the curricular issues that they have addressed could benefit from a socio-cultural perspective.

AFTERTHOUGHS

To finish this introduction, we would like to share with the readers how we explain the overlapping with other research groups and the dilemmas that it poses to us as coordinators of the group.

The engagement of participants in WG8, ***Cultural Diversity and Mathematics Education,*** comes from our shared interest in and commitment to a particular empirical domain, that of multicultural settings. Other CERME working groups are organized either around the study of theoretical perspectives or the content domain of the research –language issues, teacher education, theoretical perspectives, algebraic thinking or modelling, just to name some of them. It is clear that any of the above mentioned focuses could be researched in a multicultural setting. And it is this last point where both our strengths and our weaknesses come from.

Our interest in addressing non-prototypical situations requires that we try to broaden both our theoretical perspectives and our methodological approaches. Both theories and methodologies could be of use to other researchers in mathematics education.

However, each of us as participants to WG8, has once asked him/herself questions such as: Do I want the focus of my presentation to be the fact that I am dealing with a culturally diverse situation? Do I want to stress that I am using a theoretical perspective that is new to mathematics educators? Or do I want to suggest a discussion on curricular issues or content matters? This is where our dilemmas arise. If we keep within our group, the research done in culturally diverse situations becomes closed, making it difficult for others to come to know about our developments. However, if we go to other groups, then we risk losing our primary focus and then a new question arises: who is going to foster research in culturally diverse situations and other neglected empirical domains? What we as a group, and the larger community, will loose or gain if we move from a title of WG8 that has to do with our empirical domain into a title that has to do with a theoretical perspective? How things would change if next meeting WG8 was renamed “Socio-cultural perspectives on mathematics education”?

A SURVEY OF RESEARCH ON THE MATHEMATICS TEACHING AND LEARNING OF IMMIGRANT STUDENTS [1]

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This paper presents key themes that emerged from a review of the literature and from solicited contributions from researchers around the world on the teaching and learning of mathematics of immigrant students. Researchers strongly suggest the need for schools to look at the different kinds of mathematics that immigrant students bring with them and to use this knowledge as a resource for learning. There is a clear need for teachers to gain a better understanding of their immigrant students' and their families' knowledge and experiences. The emphasis on language as "the problem" promotes approaches that segregate immigrant students and raise issues of equity in the mathematics education they are receiving. Little research documents experiences that center on diversity and multiculturalism as a resource for learning.

This paper presents the key themes that emerged from a review of the literature on the topic of the mathematics teaching and learning of immigrant students. This topic was one of the four areas that ICME 11 Survey Team 5 addressed as part of our task to examine the research topic of mathematics education in multicultural and multilingual environments since ICME 10 in 2004. One of my main sources of information for my part of the survey team was the work of researchers actively involved in CERME's working group on Cultural Diversity and Mathematics Education.

The purpose of this paper is to highlight the main findings, advances, challenges, and indicate topics for further research in the area of mathematics teaching and learning of immigrant students. Much of this work is actually centered on research in Europe, hence the role of CERME papers. I also draw on the different contributions received from researchers across the world in response to our survey team's call for contributions. Finally, I also looked at aspects of research in the mathematics education of Latino/a students in the U.S. These three sections (proceedings, contributions, and research with Latino/a students) are discussed at length elsewhere (Civil, 2008b). For reasons of space, in this paper I am only highlighting some of the main ideas with special attention to those that relate to CERME research, as a way to encourage further discussion of this topic, the teaching and learning of mathematics of immigrant students, during the working group sessions.

Different forms of mathematics

Several studies address issues related to everyday mathematics, critical mathematics, community mathematics, school mathematics, and so on. Researchers in Greece have been looking at Gypsy / Romany students' use of mathematics in everyday contexts, in particular computation grounded on children's experiences with their involvement in their families' business (Chronaki, 2005; Stathopoulou & Kalabasis, 2007). These researchers note that schools and teachers seem to show little interest in what knowledge minority students (in this case Gypsy) bring with them and thus, in how to build on this knowledge for classroom teaching. It may be little interest on the part of the teachers, or it may be due to an unawareness on how to build on this knowledge. Elsewhere I have argued for the complexity of the pedagogical transformation of community knowledge into modules for the classroom setting (Civil, 2007).

In Civil (1996) I raised two questions that still seem relevant today: “Can we develop learning experiences that tap on students’ areas of expertise and at the same time help them advance in their learning of mathematics?” and “What are the implications of critical pedagogy for the mathematics education of ‘minority’ and poor students?” More recently Powell and Brantlinger (2008) discuss some of the tensions around their own work with Critical Mathematics (CM) and write, “CM educators should not be satisfied with engaging historically marginalized students in politicized investigations of injustices (e.g., wage distributions) if they do not have access to academic mathematics” (p. 432). As we consider different forms of mathematics and whose mathematics to bring to the foreground, issues of power and valorization of knowledge become prominent. Abreu has written extensively on the concept of valorization of knowledge (Abreu & Cline, 2007).

Teacher education

Much of the research I reviewed for this topic addressed teachers’ attitudes and knowledge of immigrant students. This body of research presents a rather grim picture and thus opens the door to several possibilities for further research. Reports on an European project that is looking at the teaching of mathematics in multicultural contexts in three countries, Italy, Portugal and Spain, point out that teachers feel unprepared to work with immigrant students. César and Favilli (2005) report that teachers in this study underscore the issue of language as being a problem and do not seem to recognize the potential for richer learning grounded in different problem solving approaches and experiences that immigrant students may bring with them. They also note that teachers seem to have different perceptions on immigrant students based on their country of origin. Overall, these reports point to a deficit view by teachers of their immigrant students.

Abreu (2005) reports that most teachers in the studies she examined tended to “play down cultural differences” arguing for general notions of ability and equity, as in treating everybody the same. Gorgorió (personal communication, April 28, 2008) writes, “teachers tended to make invisible the cultural conflict that would arise in their classrooms as a result of the discontinuities between different school cultures and different classroom cultures.” Abreu points out the need for teacher preparation programs to pay more attention to the cultural nature of learning.

Gorgorió and Planas (2005) discuss the role of social representations in teachers’ images and expectations towards different students. In particular, they write, “unfortunately, too often, ‘students’ individual possibilities’ do not refer to a cognitive reality but to a social construction. Teachers construct each student's possibilities on the basis of certain social representations established by the macro-context" (p. 1180). Researchers are critical of the public discourse that frames immigration as being a source of problems rather than a resource for learning since this discourse is counter-productive to the education of immigrant children (Alrø, Skovsmose, & Valero, 2005). Unfortunately, as Gorgorió and Planas (2005) point out, some teachers use this public perception as their orientation to assess immigrant students in their classrooms, rather than a direct knowledge and understanding of their individual students and families.

There is a clear need for teachers to understand other ways of doing and representing mathematics (Abreu & Gorgorió, 2007; Moreira, 2007). As Abreu and Gorgorió (2007) write in relation to a teacher's reaction to differences between representations of division in Ecuador and in Spain, "the relevant question is not whether there are any differences in the representation of the algorithm of the division, but how teachers react to the differences" (p. 1564). Related to the need for teachers to know about others' ways of doing mathematics, is a need for an expanded view of what mathematics is. Teachers tend to view mathematics knowledge as culture-free and universal (Abreu & Gorgorió, 2007; César & Favilli, 2005). This relates directly to the previous section on different forms of mathematics. Teacher education programs should address this view of mathematics as being culture-free. Moreira (2007) brings up the need for teacher education programs to prepare teachers to research this locality of mathematics (e.g. everyday uses of mathematics).

Issues related to educational policy

Researchers from different countries are critical of educational policies that push towards assimilation of immigrant students. These policies convey a deficit view on immigrants' language and culture, instead of promoting diversity as a resource for learning (Alrø, Skovsmose, & Valero, 2007). Anastasiadou (2008) writes,

> The de facto multiculturalism (…) which now describes the Greek society, … [which] continues to function with the logic of assimilation (…). In the field of education the adoption of the policy of assimilation means that it continues to have a monolingual and monocultural approach in order that every pupil is helped to acquire competence in the dominant language and the dominant culture. (p. 2)

The work of Alrø et al. (2005) is particularly relevant here as these authors take a socio-political approach to the discussion of the teaching and learning of mathematics with immigrant students. They write about the influence of public discourse and in particular of the view of immigration as a problem rather than a resource:

> In Denmark, the sameness discourse has spread into a variety of discourses, which highlight that diversity causes problems – it is not seen as a resource for learning. And this idea brings about a well-defined strategy: Diversity has to be eliminated. (p. 1147)

Then, as researchers in other parts of the world have noted, these authors point to the emphasis in educational policy on students' acquisition of the Danish language as the priority. The idea that mathematics education is political is particularly true when studying the mathematics education of immigrant students.

Language, mathematics, and immigrant students

Many of the contributions I received from across the world were on this theme. Here I can only give snippets of some of those. Most of them point to a clear concern among researchers for restrictive language policies that limit the use of home languages in the teaching of mathematics. For example, Clarkson (personal communication, May 25, 2008) writes,

> Mathematics teaching, like all the teaching that occurs in a school, normally is mandated to be carried out in the dominant language of the society. The use of other languages is normally proscribed. For immigrant children this may be an important matter. If they are from homes that speak a language different to the dominant societal language, then much of their formative early learning undertaken before schooling has begun will be encoded in their home language. Hence for schools to take no or little notice of these extra hurdles that such students have to leap is to simply not be realistic.

Staats (personal communication, June 8, 2008) brings another language-related issue emerging from her work with Somali immigrant students in the U.S. She wonders what happens when students do not really know their home language. She writes,

> With the educational history of Somalis they do not know their math vocabulary. It is a point of sadness, in fact, for many young people that they feel they do not know any language well, they might know parts of Somali, Swahili, Arabic, Italian, or English but feel insecure speaking any of these.

Elbers provided thought-provoking comments on the situation of mathematics education in the Netherlands. His comments relate to both the prior section on issues related to educational policy and this section on language:

> Realistic Mathematics was also criticized as being not real math (also by leading mathematicians in the Netherlands), and being based more on semantics and interpretation of assignments than on math knowledge and skills. They claim that the Dutch good achievement in math in the PISA studies is because the PISA studies do not test real math. Many plead for a return to transmission of knowledge in classrooms. The bad results of minority children in schools, in the recent debate, was partly explained with a reference to educational methods such as students learning by collaboration and investigation. These methods, the argument runs, depend on students' skills in Dutch and therefore these students, because of their language gap, can never be successful in math. (E. Elbers, personal communication, May 14, 2008)

As we can see, once again, language is singled out as the obstacle to immigrants' learning of mathematics. Elbers' comment is even more pointed as it is focusing on a critique of discussion-rich approaches to teaching mathematics that could be problematic for students for whom Dutch is not their first language. Moschkovich

(2007) addresses this topic in her research with English Language Learners in the U.S. She writes,

> The increased emphasis on mathematical communication in reform classrooms could result in several scenarios. On the one hand, this emphasis could create additional obstacles for bilingual learners. On the other hand, it might provide additional opportunities for bilingual learners to flourish (p. 90).

As we have seen, in the eyes of education policy-makers and many teachers, not knowing the language of instruction is seen as a major (and in most cases the main) obstacle to the teaching and learning of mathematics of immigrant students. Hence, the push is for these students to learn the language(s) of instruction as quickly as possible. As Alrø et al. (2005) point out, the emphasis on learning the language of the receiving country may occur at the expense of these students' learning of mathematics. Gorgorió and Planas (2001) have documented a similar situation in Catalonia. In my local context there is long history of changes in language policy for education, with some states now having banned or severely limited bilingual education. In Civil (2008c) I present the case of one student who was Spanish-dominant and had a good command of mathematics (she had already learned much of what she was being currently taught in Mexico), but was in a context in which English was the language of instruction. I raise questions about equity and the opportunities for participation and further learning of mathematics for this student.

What about immigrant parents' views on issues of language policy and mathematics education? This is a less researched topic, but one that is quite prominent in our Center CEMELA (Center for the Mathematics Education of Latinos/as)[2]. For example, in Acosta-Iriqui, Civil, Díez-Palomar, Marshall, & Quintos-Alonso (2008), we look at two CEMELA sites (Arizona and New Mexico) that have different language policies (in Arizona, bilingual education is extremely restricted, while in New Mexico it is endorsed in their state constitution). This allows us to contrast the effect of such different language policies on parents' participation in their children's mathematics education. An interesting theme emerging from our research with immigrant parents is that for many of them the language also seems to be the main obstacle to their children's learning of mathematics (this parallels what teachers think as we have illustrated earlier). This is the case in our research with mostly Mexican parents in the U.S. (Civil, 2008a) but is also the case with immigrant parents in Barcelona (Civil, Planas, & Quintos, 2005). As immigrant parents focus on the language as being the main obstacle, I wonder whether they are aware of the actual mathematics education that their children are receiving. In particular, I am referring to issues of placement: are the students placed in the appropriate mathematics classroom (based on their knowledge and understanding of the subject) or are schools basing their placement on their level of proficiency in the language of instruction? I wonder about the thinking behind these placement policies. Not only are parents not aware of the implications of this policy on their children's learning (or not) of mathematics, but also teachers often are not either (Anhalt, Ondrus, & Horak, 2007).

Research with immigrant parents

Most of the research I found on immigrant parents and their views of mathematics education was done by Abreu and her colleagues in the U.K. (Abreu & Cline, 2005; O'Toole & Abreu, 2005) and by Civil and her colleagues in the U.S. (Civil & Bernier, 2006; Quintos, Bratton, & Civil, 2005). Civil, Planas, and Quintos (2005) look at immigrant parents' perceptions about the teaching and learning of mathematics in two different geographic contexts, Barcelona, Spain, and Tucson, U.S. Besides these studies in U.K., U.S., and the one study with immigrant parents in Barcelona and in Tucson, I found one study with immigrant parents in Germany by Hawighorst (2005).

There are three related themes that emerged and that cut across all immigrant parents in these studies. Overall, immigrant parents in the four geographic contexts shared a concern for a lack of emphasis on the "basics" (e.g., learning of the multiplication facts) in the receiving country, a perception that the level of mathematics teaching was higher in their country of origin, and a feeling that schools are less strict in their "new" country. Abreu and colleagues as well as Civil and colleagues have looked at these themes in some depth, thus providing an analysis related to issues of differences in approaches, issues of valorization of knowledge, and potential conflict as children are caught between their parents' way and the school's way.

The research with immigrant parents on their perceptions of the teaching and learning of mathematics underscores the need for schools to establish deeper and more meaningful communication with parents. Parents tend to bring with them different ways to do mathematics that are often not acknowledged by the schools, and vice versa, parents do not always see the point in some of the school approaches to teaching mathematics. Although this may be the case with all parents (e.g., in the case of reform vs. traditional mathematics), the situation seems more complex when those involved are immigrant parents and their children. As the research of Civil and colleagues shows (Civil, 2008a; Civil, Díez-Palomar, Menéndez-Gómez, Acosta-Iriqui, 2008) differences in schooling (different approaches to doing mathematics) and in language influence parents' perceptions of and reaction to practices related to their children's mathematics education.

Implications for further research

My hope is that this paper will serve as a starting point to hear from other researchers who are working in mathematics education and with immigrant students. There are several implications that this review points to and that I want to briefly mention here. Abreu, César, Gorgorió, and Valero (2005) raise two important questions that should frame, I think, further research in this field. They write, "Why research on teaching and learning in multiethnic classrooms is not a bigger priority? Why issues of teaching in multicultural settings are not central in teacher training?" (p. 1128)

Based on the research reviewed, there seems to be a clear need for action-research projects with teachers of immigrant students engaging as researchers of their own practice to counteract what appears to be a well-engrained deficit view of these students and their families. Through a deeper understanding of their students' communities and families (e.g., their funds of knowledge), maybe teachers can work towards using different forms of doing mathematics as resources for learning instead of the current trend that seems to view diversity as an obstacle to learning (there are of course exceptions to this view and I address those in Civil, 2008b). Related to this idea of understanding immigrant students' communities, there is very little research looking at the sending communities. That is, what do we know about the teaching and learning of mathematics in the countries / communities that these immigrant students come from? We have recently started one such project in CEMELA, in which we look at the mathematical experiences of the students who are recent immigrants to the U.S. by studying the teaching and learning of mathematics in some sending communities. Specifically, we are looking at mathematics instruction at one school in Mexico across the border from Arizona to gain a better understanding of Mexican teachers' conceptions about the teaching and learning of mathematics. I argue that there is a need for more research along these lines to gain a better understanding of the background experiences of immigrant students.

There is also a need to analyze the learning conditions in schools with large numbers of immigrant students. What Nasir, Hand, and Taylor (2008) write in reference to African American and Latino and poor students is likely to be the case with immigrant students in many countries:

> African American and Latino students and poor students, consistently have less access to a wide range of resources for learning mathematics, including qualified teachers, advanced courses, safe and functional schools, textbooks and materials, and a curriculum that reflects their experiences and communities. (p. 205)

Issues of valorization of knowledge and different forms of mathematics need to continue to be explored, as there are still many open questions. Related to this is the idea of non-immigrant students' views of immigrant students. This topic has received little attention (a notable exception is Planas, 2007), yet it seems important to understand how all the students see and understand the experience of being in a multicultural classroom (Alrø et al. (2007) address this topic to a certain extent).

Another area that needs further research is that of immigrant parents' perceptions about the teaching and learning of mathematics. Furthermore, an important and under-researched area is that of interactions between immigrant parents and teachers and perceptions of each other's in terms of the children's mathematics education. Civil and Bernier (2006) address this to a certain extent, but much more work is needed in this area.

Language is a prominent theme in the research with immigrant students and mathematics education. More research is needed that focuses on multiple languages as resources for the teaching and learning of mathematics, once again to counteract the deficit perspective, particularly in the public discourse that sees the presence of other languages and not knowing the language of instruction as obstacles to the mathematics education of immigrant children. Issues of placement based on language proficiency and the impact that these decisions have on students' learning of mathematics also need to be studied further.

Finally, a clear implication from the research reviewed on this topic is the need for interdisciplinary teams with expertise in different areas including mathematics education, immigration policy, linguistics, socio-cultural theories, anthropology, just to name a few. There is a need for this interdisciplinary expertise, as well as for the development (or refinement) of theoretical and methodological approaches. I find Valero's (2008) comment on this (in the context of mathematics education in situations of poverty and conflict, which are often the norm in immigrant contexts) very insightful:

> The theories that have been used to study mathematics learning build on a fundamental assumption of continuity and of progression in the flow of interactions and thinking leading to learning. (…) When [these theories] are simply applied without further examination the result has often been the creation of deficit discourses on the learners or the teachers. (…) The question then becomes how can (mathematics) "learning" be redefined as to provide a better language to grasp the conditions and characteristics of thinking in situations where continuity and progression cannot be assumed. (p. 161)

I leave the reader with the challenge Valero raises in the last sentence.

Notes

[1] This paper is adapted from a longer paper (Civil, 2008b) prepared for ICME Survey Team 5: Mathematics Education in Multicultural and Multilingual Environments, Monterrey, Mexico, July 2008.

[2] CEMELA is a Center for Learning and Teaching (CLT) funded by the National Science Foundation under grant ESI-0424983. The views expressed here are those of the author and do not necessarily reflect the views of the funding agency.

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PARENTAL RESOURCES FOR UNDERSTANDING MATHEMATICAL ACHIEVEMENT IN MULTIETHNIC SETTINGS

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This paper examines parental understandings about their child's mathematical achievement and the resources they use to go about gaining information in culturally diverse learning settings. This examination takes place within a critical-developmental framework and draws on the notion of cultural models to explicate how resources are used. Three parental resources of mathematics achievement are scrutinised: (i) the teacher, (ii) exam test results and (iii) constructions of child development. The interviews with twenty-two parents revealed that some resources were concrete, such as examination results. Other resources were symbolic, like the representation of child development, and were less likely to be shared with the school community. Either way, these resources were open to parental interpretations and influenced by parents' own experiences and cultural representations.

Key words: parents, resources, cultural models, achievement, ethnic minority

INTRODUCTION

Within the English school system, like many Western/English-speaking countries, there is a strong emphasis on testing and measurable outcomes for success at school. The introduction of the National Numeracy Strategy and nationwide testing in the primary sector led to a greater pressure for parents' involvement in their children's school education (Bryans, 1989). While many could see problems with using parents as teachers in the home, the problems of engaging parents specifically from culturally diverse backgrounds remained largely uncontested.

The education of ethnic minority children has been given some attention, although less seems to be said about mathematics learning in particular in the UK context. The pitting of one ethnic group over another has tended to overshadow the sociocultural composites of school practices or the "gaps" in cultural understandings of what counts as mathematics learning. The current UK government position is to play down cultural influences on home learning even though the precise form in which home learning is delivered depends on the parents' understanding of the individual child and their development (Goodnow, 1988) as well as judgements of value and cultural practices, often filtered by community experience and past experience (O'Toole & Abreu, 2005).

This paper examines parental understandings about their child's mathematical achievement and the resources they use to go about gaining information in culturally diverse learning settings. Resource is a concept which refers to the way in which the individual is simultaneously a seeker and provider of information which is open to resistance, interpretation and multiple representations. This examination takes place within a framework which suggests that institutional systems like school reflects a dominating and particular way of looking at children's learning where singular pathways to development, often age-related, are considered "appropriate" or "correct" (Burman, 1994). These conceptualisations influence what we think children should learn and what achievement outcomes are necessary by certain stages of development. As such, expectations for children's achievement are "normed" against particular developmental milestones (Fleer, 2006). The "colonization" of the home by school practices does not attempt to reflect or value family practice but marginalises practices which are not represented by White, middle-class groups (Edwards & Warin, 1999). Equally, parents are privy to limited amounts of information about their child's school life, including their child's mathematics learning and therefore seek other avenues for constructing meaning from an environment from which they are largely excluded.

It is also suggested that when parents utilise and incorporate the resources available to them they do so within the boundaries of particular cultural models (Gallimore & Goldenberg, 2001). Cultural models can be understood in terms of a shared understanding of how the individual perceives the way the world works, or should work. A cultural model is described as:

> Encoded shared environmental and event interpretations, what is valued and ideal, what settings should be enacted and avoided, who should participate, the rules of interaction, and the purpose of the interactions (p.47).

Cultural models are often hidden and unrecognisable to the individual and quite often assumed to be shared by others around them. As such, mathematical learning also comes with a knowledge structure which is a reflection of the family or community practices (Abreu, 2008). Parents draw on their own understandings of mathematics learning to make sense of how their child is achieving. The resources they use to do so may have concrete or tangible aspects to them such as discussions with the class teacher or examination results. Others err more towards a cultural model that is representational or symbolic. Both are susceptible to miscommunication and interpretation.

A STUDY OF PARENTAL RESOURCES FOR UNDERSTANDING THEIR CHILD'S MATHEMATICS ACHIEVEMENT IN SCHOOL

The twenty-two parents participating in this study had children in primary schools (ages 5-11 years) situated in a town in the South East of England. Eleven of the twenty-two parents were from ethnic minority backgrounds and the remaining participants were White and British born. The children are characterised as being either high or low achievers in mathematics and were placed as such by their teachers. Data collection took place in three multiethnic schools that are known as school A (mainly White), school B (ethnically mixed) and school C (mainly South Asian). Data from parents was collected using the episodic interview (Flick, 2000), a method which assumes a shared common knowledge on behalf of the participants about the subject under study. It specifically facilitates the exploration of meanings, representations and experiences. The procedure for analysing the interviews was borrowed from Flick (2000) and based upon the analysis of themes.

Although the study was specifically about mathematics, parents within the sample used this opportunity to talk about their child's education as a whole and therefore the data is highly inclusive of other educational issues. For parents, constructing meaning in relation to their children's mathematics education is like fixing together the pieces of a puzzle and this is managed in a holistic way. In their accounts, parents utilised a varied number of resources to help them construct an understanding of their child's "achievement." The three dominating resources were: (a) the teacher, (b) exam test results and (c) constructions of child development.

Using the teacher as a resource for understanding the child's achievement

The teacher was cited most often as the resource of information about mathematics achievement for the parents in the current study. Of interesting note, is that parents of high achieving children mentioned using the teacher as a resource more than parents of low achieving children (19, 11[1]). Furthermore, White British parents mentioned using the teacher for this role more than the ethnic minority parents (17, 13). There are a number of potential explanations for why this might be the case. The parents of high achieving children may not have to worry so much about what will be discussed during consultations, therefore there is less at stake in discussing their child's progress with the teacher. Parents of high achieving, and indeed White British parents are more likely to share cultural models of education, teaching and learning with the school. The discrepancies and conflicts in value positions between home and school for those who do not share cultural models with the school have been well documented by Hedegaard (2005).

On the whole parents' communication with teachers tended to centre around the parent-teacher consultation evening on a twice-yearly basis. Communication between parents and teachers surrounding achievement is complex, and teachers couch many of their descriptions of the child to parents using "teacher talk" whereby descriptions could connote two different meanings. For example, if a child is described as having "leadership qualities" this can also be interpreted as "the child is bossy." "Teacher talk" can produce a discrepancy between the teacher's discussion of the child's mathematics achievement and the parent's understanding of that achievement. For instance, Rajesh's mother asked the teacher in the parents' consultation, "how's he getting on, will he be alright?" and Rajesh's mother recalled that the teacher said:

[1] The figures used in this paper are based on the number of times a resource is mentioned, therefore there are times when one parent mentioned a resource more than once.

> Rajesh's mother: "he'll be fine, no point to worry or anything…if he just carries on the way he's doing he's fine" (Indian mother: yr 2, LA)

However, the teacher described Rajesh to me as a low achieving child and his family were categorised as having a low level parental involvement. However, this parent has taken at face-value the message. There are opposing cultural models of Rajesh's learning held by home and school here. Rajesh still struggled to undertake calculations with number below ten, whilst curriculum guidelines stipulate that children of his age should be capable of working with numbers up to 20. This parent has assumed that the teacher would offer the most concrete information around her son's mathematical achievement. Another parent, Fazain's mother, reported a similar conversation she had with a teacher at her son's school:

> Fazain's mother: Mr. Headworth, he was saying that he is really good in maths because he comes home and you know, because I improve my maths, you know, a lot. So I teach him, and he's coming really good, he's top in his class (Pakistani mother: yr 6, LA)

Age-related views of mathematics learning are representative of generalised and dominant forms of knowledge which places children outside of these brackets of being an achiever. Fazain was by no means top of his class and was described to me by his classroom teacher a low achieving child. Fazain's mother has attempted to align her own models of mathematics with the schools by improving her own learning, but using the teacher as a resource of information still creates discrepancies.

This next quote from Michael's mother shows what can happen if the interaction with the teacher creates a dissonant cultural model of achievement from the one held by the parent. Michael's mother describes a negative parent-teacher consultation she had experienced. In his first two years schooling, Michael's parents had always been told that he was achieving well. At the most recent parent-teacher consultation, Michael's parents were surprised to be told that he was not doing as well as the others. This change in the representation of her son's achievement by the mother, as a consequence of the teacher consultation, prompted her to questions the teacher's judgement:

> Michael's mother: As I say, this consultation with Mrs. Edwards didn't even sound like Michael…I thought, she doesn't know this child at all, doesn't even sound like him…and I remember being so cross…and I said to [the head teacher] "what does this child have to do to get any praise?" because I thought it was so unfair. Because he was working hard and yet there wasn't a single thing said that was positive. (White British: yr 2, HA)

Although the teacher was an important resource of information for all the parents as a means of understanding their child's achievement, parents may challenge their opinion if it runs counter to well established models of understanding.

On the whole, parents placed a great deal of emphasis and importance on the teacher's judgement of their child's achievement without always realising that teachers' discourse can be framed to connote multiple meanings. One might speculate that these discrepancies are even more problematic for the more marginalised parent (such as ethnic minority parents, working class parents, or parents of low achievers), like the mothers' of Rajesh and Fazain, who may have been socialised to understand a more literal educational discourse. For example, these parents took at face-value the "no need to worry" teacher talk. This is unsurprising when models of success are more desirable and the teacher is considered the key authority. Using the teacher as a resource means that conversations take place in a setting which is rigidly framed by a White middle-class institutional structure (Rogoff, 2003) and as such, teachers are in a powerful position. Michael's mother has fewer qualms about challenging achievement representations of the teacher. As such she has the resources to challenge the institutional perspective. It was suggested earlier that using the teacher as a resources of information was tangible or concrete and yet "teacher talk" creates models of achievement which are not necessarily congruent with normative age-graded levels, or parents constructions of their child's achievement.

Using examination assessment results as a means of understanding achievement

Examination results from the Standard Assessment Tests (SATs) conducted in year 2 and year 6 were also resources used by some of the parents. Parents of high achieving children were most likely to speak of examination results in relation to achievement (13, 9), although there was no difference between the White British and ethnic minority parents. In principle, parents should be able to use examinations as a concrete means of understanding achievement. Yet how parents come to understand or use these tests for assessing their child's achievement and construct subsequent cultural models is open to considerable interpretation.

For a start, many of the parents failed to understand how the tests are scored (tests are scored using levels rather than A-G classifications which parents are familiar with). Once again though, parents in this sample of high achieving children had a clearer idea of the scoring system used for the SATs tests. Why this should be the case is uncertain, since the scoring is new for all parents of children currently in the school system. It is likely that these parents are confident in accessing resources like the teacher, websites and shop-bought information books.

The majority of the parents who knew that the SATs examinations were taking place had negative feelings about the tests. Some thought the children were too young and therefore ran counter to their cultural models of appropriate child development practices. Others felt that the SATs examinations were for the schools benefit, and not for the children since results are published publicly and are used to measure the school's success. Rajesh's mother was unique in her opinion about testing and its usefulness in understanding achievement. This may have been because she may have been naïve about how the schools use the test results:

> Rajesh's mother: I reckon tests are good because it will show him what he needs to go further on and what he needs to learn…I think he's going to have tests his whole life so

> he might as well start now…they're not going to judge the kid, if he's bad or anything it just means he needs more help which is good in a way (Indian mother: yr 2, LA)

Rajesh's mother also held the belief that there would be some kind of positive feedback from the tests, which would help her son realise his mistakes and improve. However, once the final examinations had been finished, none of the schools in this sample revisited the papers and other parents had a stronger insight into institutional motives for testing mathematical achievement. Dale's father shared this low opinion on the value of examinations as a resource for understanding his son's mathematical achievement:

> Dale's father: I find going into school reinforces my idea that they put you in a pigeonhole at the earliest opportunity; that's the line, you're this side of the line, you'll always be the worst. Well, all right, he's a couple of digits down on a maths test, it's not the end of the world but to listen to them talk sometimes; is that because of the concern for Dale or is it because they're concerned the school is going to get a bad report because the Stats *[sic]* are down…and I sometimes wonder exactly what it's for, this sort of test thing (White British: yr 6, LA)

Parents described how, in their view, SATs examinations have little value as a tool for helping the child, but are instead used as a form of classification. As such institutional practices are at odds with parental cultural models of what counts as a useful learning experience. Also, the parents look at the SATs exercise with justifiable scepticism. Perhaps these parents know better than Rajesh's mother, that the papers will not be re-visited or used as a learning tool.

With two exceptions the parents of low achieving children had more negative feelings towards the examinations than parents of high achieving children. Parents here were concerned about seeing their children fail, something that is more likely to happen to the low achieving children. Parents' difficulties in interpreting the SATs mathematics examination results revealed that even as a concrete resource of information about the child's achievement, examination results can have their own interpretive problems.

Resources of child development for understanding achievement

One other piece of the educational puzzle, perhaps built upon the most symbolic of all the resources for understanding achievement, was the use of models of child development. Juxtapositioned against the need to understand mathematics achievement was the belief that the children were very much in the early stages of their own development. Parents maintained a cultural model of their children as still being very young which are not necessarily shared by teachers or school as an institution. As a consequence of these dissonant models of child development, tensions were created between home and school. The next quote from Rajesh's mother reveals the conflict between her own model of child development and her desire for her child to be successful early in life:

> Rajesh's mother: But then I'm thinking like, his education is important at the moment but it's still a bit of a laugh for him so I don't really want to burden, like I don't want to be like a fussy parent saying I'm pushing him or something…but at the moment you think he's only seven, you don't really want to push him too much, cos you're stuck in the middle. Then you think if he has a good start now then he'll have a good start, you know. I don't know, it's a bit difficult (Indian: yr 2, LA)

Her conflicting model of appropriate parenting and educational expectations for achievement are both tied in with her identity as a good parent. Contained within the quote are three messages which are no doubt conflicting but lead back to her model of child development as the resources of understanding. She does value education and considers it important, but for a boy of 7 years old it should be fun. She is also worried about being perceived as "pushy" if she broke away from her own cultural model of child development. However, Rajesh's mother is unaware that it is her own cultural model of child development which is marginalised by against expectations of the school.

Even when parents have a keen awareness of the cultural models held by the school, these may still be challenged by parents own models of child development. Simon's mother drew on her own experiences as a school child to understand the anomalies between her own cultural models of child development to what her son was experiencing:

> Simon's mother: I just think that he's seven, he's in the infants and if I related to when I was in the infants, we never brought homework home until; I think we just had reading. And part of me thinks they're just children, let them be children, you know, if they're happy they'll be learning and I don't want too much pressure on him really (White British: yr 2, HA)

Past educational experiences are embedded in cultural models and linked to the settings where practices take place (O'Toole & Abreu, 2005). Based on these past experiences, Simon's mother has a strong model that school is for learning and home for playing/recreation. Once again, she draws on child development as a resource of knowledge for her cultural model.

A recurrent idea running through parents' models of child development was that of learning as a progressive activity. Learning was viewed by many of the parents as a building block, which develops with the child. The stage-theory representation of child development established through developmental psychology is widespread in these parents' accounts. Learning is described as progressive and based primarily in the childhood years. The crux of the problem is that parents' stage-related views on child development are more varied than one might expect. The variations in parents' models of learning and development are strongly influenced by their own values and experiences, which were culturally situated. However, school as an institution in England relies heavily on constructs established by stage-related theories. Moreover, they are not necessarily congruent with the models held by the teacher. One of the teachers, Richard, in School B told me:

> I still think some parents haven't quite caught onto the idea that they're seven so we should be expecting quite a lot of them. Their expectations of what a child can do isn't as high as our expectations…(yr 2, mixed achievement class)

CONCLUSIONS

When parents talk about their children's mathematics learning they draw on much more than just isolated accounts of mathematics as a subject. Parents try to make sense of their child's mathematics experience by using both concrete and symbolic resources. While some resources, like the teacher and examination results might be considered fairly concrete forms of information for parents, they carry their own problems of interpretation and expectation. For example, whilst "teacher talk" may be a kindness to the parents and child, not all parents have the resources to reinterpret the double meaning. In culturally diverse situations there remains the possibility for discrepancy between the cultural models of learning and achievement between home and school through literal educational discourse. It is noteworthy that the two resources most used, the teacher and examination results, come from the most powerful setting where the knowledge is unidirectional; from home to school. Parents with strong cultural models about their child's achievement can challenge the school. Marginalised parents, or those that sit outside White middle-class institutional confines, tend not to have the resources to either challenge the school or recognise incongruent pieces of information. The least tangible cultural model, child development, resides mostly in the home and is born out of values, expectations, practices and past experiences. This is a resource which is least likely to be shared with the school but is still a pervasive influence in the home.

Furthermore, cultural models and knowledge about achievement have a reciprocal influence on each. A question was raised about whether the cultural model is established before the representation of achievement or whether images of achievement precede the model. The use of cultural models and representations of achievement are seen as constituted from each other, in that they have the power to be transformed, reconstructed and rejected based on the resources that are utilised. In other words, new information about achievement (perhaps resourced from test examination results) may change a cultural model. On the other hand, a steadfast cultural model (perhaps resourced from representations of child development) might be resisted or rejected in light of discussions with the teacher about what a child should be able to achieve by seven years of age.

Whilst institutional practices continue to be dominated by universal/western notions of development which are characterised by White, middle-class value-positions then some homes and their cultural practices will be marginalised. Furthermore, these homes and their families will be positioned as incompetent or lacking knowledge.

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DISCUSSING A CASE STUDY OF FAMILY TRAINING IN TERMS OF COMMUNITIES OF PRACTICE AND ADULT EDUCATION

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SUMMARY

This paper focuses on adult mathematics learners working on their children's algebra problems in high school. These "adult learners" have their own characteristics and dynamics as a group. Therefore we define them as a socio-cultural group. In addition we assume that to reach an identity as a member of a group is something good in terms of learning. For different reasons we have chosen Wenger's idea of "community of practice" to look at this socio-cultural group. However we are not looking at this group of parents as a community of practice, but the process of how this group of people becomes it. To understand how a group of people becomes a community of practice may provide some hints to improve our teaching and learning strategies.

KEY-WORDS: Adult Learners, Family Training, Mathematics Education and Communities of Practice.

INTRODUCTION

People who work in the field of education know that classrooms work better, and students achieve better scores, when they identify as members of a community. Many teachers look for strategies to build these complicities at the beginning of the school year, thus students could become a group[1] of people working together to learn. Much research draws on this image by providing supporting evidence to demonstrate that grouping is better in terms of learning strategies (Lou, et. al. 1996). Drawing on the prior research, some relevant questions implicit in the process of building a group of people may include issues such as how the group works, what type of elements provides unity to the group, what are the main characteristics of the "culture" of the group, and so forth. The processes of support, as well as the solidarity between students, stresses the uniqueness of a milieu that encourages inclusion and learning for all the members of the group. The positive interactions held between the different members of the group promotes a working environment that positively strengthens each member. The result in terms of learning is usually better than the one obtained when this group identity is not present (or when it is a group of people with no cohesion).

The idea of "community of practice" is present in a number of articles and books on Mathematics Education (Cobb & Hodge, 2002, Lerman, 2001, Jaworski, 2006). Usually the "community of practice" is related to good practices, because as Renshaw

(2003) claims there is “kindness” in the word “community;” and this “kindness” makes this concept attractive. However, the concept proposed by Lave and Wenger (1991) and developed by Wenger (1998) is a notion precisely used by Wenger in a particular context (the business). It was not created as a tool to be used in the context of educational research. All the research reviewed in this paper use this notion in a finalistic meaning, presenting the group studied as a “community of practice” already established.

Data used in this paper come from a research project titled “Teacher training towards a Mathematics Education of parents in multicultural contexts” (ARIE/2007 program, number of reference 00026), funded by the General Office of Research and Universities (AGAUR) from Catalonia. In this exploratory case study the focus is on families and Mathematics Education. Our main aim is to use the concept of “community of practice” as tool of analysis, in order to understand if people involved in our study are (or not) a community of practice. We consider that the process of how a group of people become a community of practice is an interesting topic to be analyzed. On one hand this transition step is something that has not been studied in the scientific literature, on the other we think that this process may present key elements to understand how this ideal situation of “community of practice” appears, and what aspects play an important role on it. We are not looking at a “community of practice” already built but discuss a process. Data collected suggests that there is some kind of correspondence between the examples found in our study and what Wenger calls a “community of practice” (1998). We look at these situations because previous research suggests that groups working as communities of practice achieve better results than groups where there is not a sense of cohesion. Our research work was held in a classroom with adult people, and as such is a set of people different from other educative targets.

ADULT EDUCATION: TOWARDS AN UNDERSTANDING OF SOME KEY ELEMENTS AROUND ADULTS’ LEARNING AS A CULTURAL GROUP

In this paper we use Woods’s (1990) and Geertz’s (1973) notions of culture to define the adult learners as subjects of our study. The notion of “culture” has been used broadly with many different meanings. The aim of this paper is not to explore the scope of this idea and its definition but we do want to highlight how we use the term “culture” in our research.

Geertz (1973) define culture as a notion that:

> “Denotes an historically transmitted pattern of meanings embodied in symbols, a system of inherited conceptions expressed in symbolic forms by means of which men communicate, perpetuate, and develop their knowledge about and attitudes toward life” (p. 89).

According to his definition “culture” is defined in this paper as a characteristic of individuals related not only to the ethnicity, language, country of origin, or social background, but also all the small groups to which these individuals belong to. In this sense, we define “adult people” as a particular cultural group, with their own characteristics and dynamics, impacting and determining how the educational process works inside the classroom of mathematics. As Woods (1990) claimed, every single group of people has their own culture, thus we need to analyze it in order to understand the practices carried out by the members of this group.

Drawing on phenomenology, Rogers (1969) showed that all persons exist in a world of experience, which is always changing. This “world of experience” becomes the filter through which we perceive all of what is around us. Talking about how adults learn, Rogers (as well as Piaget) argued that there is a cognitive process of adjustment: when somebody finds that some kind of information coming from the outside (the real world) does not accord to his/her previous [cognitive] schemes. This person then assimilates the new information by accommodating it into his/her mental schemes. From this point of view, to learn is a “learner centred” process where the individual tries to solve the incongruence between what s/he perceives and what would represent (according to his/her previous schemas). This argument may explain why many adults have a common set of values and schemes (because their common background), which distinguish them from other social groups.

Other researchers offered key contributions to the learning theory in Adult Education, such as Knowles (1984) and Mezirow’s (1997) who both differentiate adult individuals as a particular cultural group in terms of their own learning. Knowles (1984) claims that adults are individuals who learn by drawing on their own experience and their “self-concept” (that is: the capacity to move from one being a dependent personality toward one of being a self-directed individual). Mezirow (1997) adds that this learning process in grounded in a dialogue. Before Mezirow (1997) was working on these ideas, Freire (1977) discovered the importance of dialogic action. The Brazilian professor had already demonstrated the power of the word (“la palabra”) as a tool to read the world critically. Drawing on this idea, Freire proposed what he called “Dialogical Method of Teaching.”

Drawing on the ideas of Freire and Habermas, among others, Flecha (2000) proposed what he calls “Dialogic Learning Theory.” The most important concept embedded in this learning theory is the egalitarian dialogue: learning is the result of an intersubjective process of interaction that occurs when learners use the egalitarian dialogue in order to share their prior knowledge with others. Thus the learning process is not unidirectional between teacher and students, but the result of a dialogue. Arguments always are discussed grounded on validity claims, not power claims. Flecha (2000) explains this approach using seven principles (egalitarian dialogue, cultural intelligence, solidarity, transformation, creation of meaning, instrumental learning, and equality of differences), which are the central axe of the “Dialogic Learning Theory.” Learning is a powerful experience for adult people; it really transforms their lives. In addition, learning is reached when it makes sense for them. This is a particular difference with children since adult people already have experiences to build upon new knowledge. Drawing on these principles we can affirm that adult learners are a particular group, with their own ways of thinking and functioning.

THE NOTION OF COMMUNITY OF PRACTICE AS A METHODOLOGICAL TOOL

Wenger (1998) introduces a learning theory grounded on the notion of Community of Practice in his book *Communities of Practice: Learning, Meaning, and Identity*. This concept has the "three dimensions of the relation by which practice is the source of coherence of a community" (Wenger, 1998 p.72) as a key idea. These three dimensions are: mutual engagement, joint enterprise and shared repertoire.

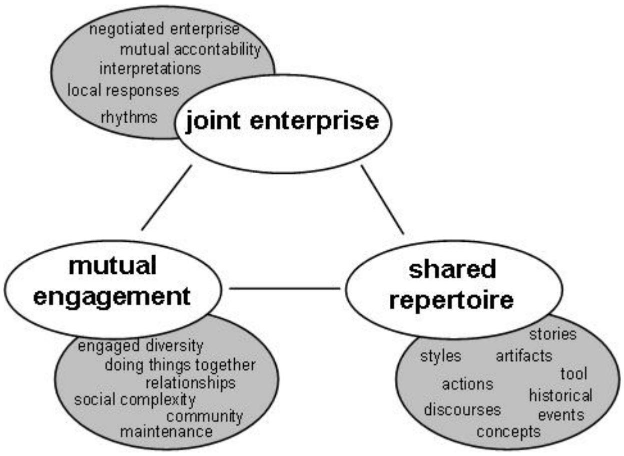

Figure 1. "Dimensions of practice as the property of a community" (Wenger, 1998 p.73)

The concept "community of practice" was created to define a group that acts as an "alive-curriculum" for the learner. For this reason the "community of practice" is a type of community present everywhere, and this is not linked necessarily to a formal system of learning.

The notion of community of practice is more than a group of people with similar (or common) interests, involved in a regular activity. This is not a synonym of group, team, or network. This does not mean (only) to be affiliate to some kind of organization, or to connect with other people (close in terms of geography or social class). This is a dynamic concept, including all members of the community of practice (not just the own participants in the practice which is studied).

Wenger's (1998) concept of community of practice has been used as tool of analysis more than the theory embedded in it. However this "operationalization" of the theoretical concept cannot be made without taking into account several considerations to avoid doing an incorrect use from the methodological standpoint. [2]

In this paper we use the concept of "community of practice" as tool of analysis, in order to analyze if parents involved in the study became a community of practice (or not). At the same time, we also analyze how this process impacts on teaching and learning practices. Thus the research question is: what type of (social and cultural) processes happen while a group of people became (or not) a Community of Practice?

In order to answer this question, our start points are the 14 “indicators that a community of practice has formed” (Wenger, 1998, p 125). These 14 indicators are specific descriptors of the 3 dimensions quoted before (mutual engagement, joint enterprise and shared repertoire).

These 14 indicators are:

“1) Sustained mutual relationships – harmonious or conflictual

2) Shared ways of engaging in doing things together

3) The rapid flow of information and propagation of innovation

4) Absence of introductory preambles, as if conversations and interactions were merely the continuation of an ongoing process

5) Very quick setup of a problem to be discussed

6) Substantial overlap in participants’ descriptions of who belongs

7) Knowing what others know, what they can do, and how they can contribute to an enterprise

8) Mutually defining identities

9) The ability to assess the appropriateness of actions and products

10) Specific tools, representations, and other artefacts

11) Local lore, shared stories, inside jokes, knowing laughter

12) Jargon and shortcuts to communication as well as the ease of producing new ones

13) Certain styles recognized as displaying membership

14) A shared discourse reflecting a certain perspective on the world” (Wenger, 1998 pp.125-126).

In this paper a series of classroom sessions of mathematics are discussed. Data was collected using videotape. The dynamics generated by the parents involved in the study are analyzed according to Wenger’s 14 indicators. A father and 19 mothers were part of the group. Almost everybody was from Catalonia, although at the beginning of the school year there were also two Latina women. Their children were freshmen in the high school (12-13 years old).

ANALYSING AN ADULT LEARNING GROUP

The group of adult learners took place in a high school classroom in Barcelona city. The learners were a group of parents come together to work on algebra problems. It is a group of people that have deliberately joined together in order to learn mathematics, although some of them knew each other before because they usually came to the high school in order to collaborate in other activities organized by the centre. The group was open to everybody (immigrant and native people, parents of low and high achieving pupils, etc.). Wenger’s (1998) community of practice concept asserts that we can neither build this type of groups as a result of a mandate, nor establish them from the outside. We cannot generate or design these communities either. According to this viewpoint, "communities of practice are groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly" (Wenger, 2007). That means that a group of people may become a community of practice over the time (if they follow the 14 criteria pointed out by Wenger).

Data discussed in this paper comes from the fourth session of the workshop. People involved in this group had been working together for four successive weeks doing mathematics in this classroom. Videotapes show how they were becoming (functional) as a "group" over these four sessions. The identity of every single person of the group became more defined little by little. Analyzing our videotapes in terms of Wenger's (1998) notion of community, several clips suggest that some of the 14 indicators are achieved (or they are in the way to be achieved), such as indicators 1, 2 and 8 ("sustained mutual relationships – harmonious or conflictual," "shared ways of engaging in doing things together," and "mutually defining identities"). A longitudinal analysis of the videotapes indicates that people define their identity collectively (indicator 8). This process produces a number of sustained mutual relationships (indicator 1), and at the same time shared ways of engaging in doing things together appears (indicator 2). The first quote is an example of this type of dynamics. The adult learners are in a classroom placed in a high school and are taking part in an activity of translation: from natural to algebraic language. They are working with first grade equations with one unknown. The facilitator had asked how they solved the problem. Pere is the only man of a group of 20 people (all of them are involved voluntarily in the group). Some of them participate actively in the class. Pere intervenes:

> Pere: Me too. Two times x, and then plus two times x.
>
> Facilitator: You wrote two times x, and then?
>
> Pere: One, plus two times x (a noise from the chalk when writing on the chalk board is heard, when the facilitator write on the chalkboard what Pere is saying).

It is interesting to highlight that Pere (who usually is not the protagonist, in the sense that he is not the person who has the highest index of interventions) usually intervenes before the mothers to answer the questions proposed by the facilitator (almost always). This practice always occurs when some kind of explanation or validation is required from the learners. According to this interpretation the role played by Pere is "a person who already has a prior knowledge in mathematics, and who is able to make connections between his ideas and what the facilitator explains, as well as to consolidate this knowledge in the group."

Another aspect emerging from the data analysis is the definition of learners' identities as members of the group (indicator 8) in opposition to their children's identity.

People from the group identify themselves as such because all of them are parents (indicator 6). The variable "generation" becomes a common characteristic of their identity as a group, because it is also connected to their motivation to participate in this workshop of mathematics (and consequently, to consolidate themselves as a group and, perhaps, as a community of practice in the future). This aspect of their identity also helps us to understand the conflict emerging between these people and their children, in terms of teaching and learning mathematics. All these parents have children in the high school, and all the children have difficulties with mathematics. This situation produces a plethora of common experiences shared by all the members of the group. They, as parents, have a different "way to see the world" than their children. This fact, and especially how they have faced this situation as "people who engage in a process of collective learning in a shared domain of human endeavour" (Wenger, 1998), suggests that this group has some characteristics similar to what Wenger defines as a community of practice (1998).

We have observed several clips suggesting that the "parents' group" and the "children' group" (implicit in parents' discourse) have characteristics that may be defined as a culturally different, in terms of Woods (1990). The values shared by parents, as well as the cognitive referents linked to mathematics (ways to act and solve problems), are really different from those used by their children. This difference may explain the "generational" conflict between parents and children, because the culture of each group is not the same. In this next quote the adult learners are once again in a classroom in the high school. The parents are working with a first grade topic "how to solve an equation." The facilitator solves the problem using one method, and one mother claim that her daughter uses another way to do it. At this point the facilitator explains the method used by the daughter. She has divided the chalkboard into two columns: on the left there is the method used by the facilitator – which is the one known by the mother; on the right the facilitator wrote the daughter's method – which is the one used by teachers and children in the school):

> Facilitator: How it is going? Good?
>
> Mothers: yes... very good (the mom who asked the question is the one who speaks louder).
>
> Mother: We didn't understand it at home.
>
> Facilitator: eh?
>
> Mother: I didn't understand it like this at home; this that you have explained to us my daughter used to say "mom, we wrote this here," and I say "where do you put this?" because I know it in the other w... in the old way (a noise in the background is heard, like admitting she is right) and I was not able to understand it because there is no explanation on the text book.
>
> Facilitator: But, now did you get it?

Mother: (Some mothers agreeing on the background are heard) Kind of, but what happens is that here is so easy... but to me... (She starts to laugh and makes gestures with her hands to say that sometimes the activities are difficult).

Facilitator: ... well... this is the same... but you have to go to....

Mother: (At the same time) now you're getting it, because, because...

Facilitator: (At the same time) to everybody.

Mother: she explains that she does it that way, but I don't know how to explain it....

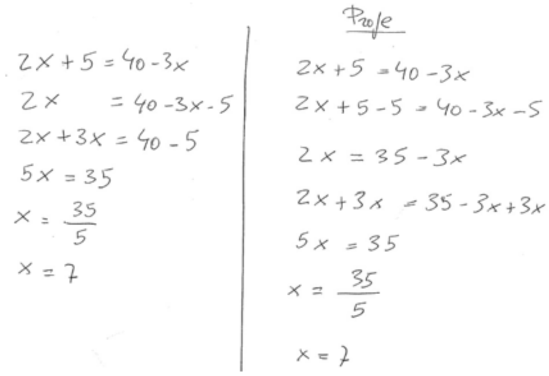

Figure 2. Detail of the chalkboard grounded on the field notes.

The problem described in the above quote is common for many families as they experience difficulties in helping their children to solve home mathematics. Those difficulties are sometimes related to mathematics itself and how much mathematics the parents understand themselves. However, other times the problem is the difference between the methods used by parents and the ones used by children (and teachers). One possible reason may be the reforms in mathematics that have changed the procedures used in the classroom to teach mathematics. Figure 2 illustrates the difference between the way used by the mother to solve the equation, and the procedure used by the teacher (of her daughter) to do the same thing. In this figure we can see that while the mother puts all the unknowns[4] together in one side of the equation, and the numbers in the other side of the equal sign, what the teacher does is simplify the expression eliminating the same numbers in both sides of the equation. Both results are the same, but the procedure reasoning implicit is different.

The lack of more opportunities (such as the workshops of mathematics for parents) to connect school and family results in parents having less opportunities to learn what teachers explain in the classroom. Consequently there is no possibility to create a unique discourse about how to teach mathematics. Parents solve the mathematical problems using different strategies grounded on their own methods. But they do not know the methods used by their children (or they just have forgotten them). Then the conflict between them and their children (and more broadly the school) arises. This conflict makes it more difficult for them to get involved in their children' education.

SOME CONCLUSIONS

As a concluding remark, this preliminary data provides evidences that the process of became a Community of Practice are not an easy process, neither lineal. It involves definition of roles, interactions, identities, etc. Some indicators appear at different moments, and not according to a prefixed order. In this process some conflicts between actors arise as well. Data shows that there is some kind of generational gap between parents and children (working from a parent involvement approach to the learning of mathematics).

FURTHER RESEARCH

The analysis suggests that when a group is new, every member plays a particular role that becomes part of his/her identity. One question arising from this situation is what is the impact of the role-identity definition process in terms of individual confidence to do and solve mathematical problems? Prior research highlights that self-image (in terms of ability to do/solve mathematics) has a key impact on the self-confidence that everyone has as a mathematics solver/doer. Taking this into account, it is important to analyze the effect that may have the construction of the identity in the process of building a group (being or not a community of practice). Could somebody who is not confident about him/herself feel able to learn mathematics? What is the role of gender in this process? Can the guarantee that everyone has an opportunity to participate ensure that everyone would learn mathematics?

On other hand, in the analysis we have also observed that families and their conflicts with their children doing mathematics may open further analysis to find the elements that affect the relationship between parents and children. The community of practice offers us methodological tools (indicators) to analyze how aspects that define one group could be different for other groups, thus conflicts may be explained because of these differences (contradictions). Consequently a strategy to improve mathematics performances should take into account all the elements that may be defined as "culture" of a particular group (such as prior experience, mathematical knowledge, procedures, etc.) in order to find ways to solve the contradictions (Woods, 1990). In this sense learning approaches such as Dialogic Learning Theory (Flecha, 2000) may be a way forward for further analysis and exploration. However, before that, more in-depth analysis of culture (defined in terms of everyday life) may be needed in order to find hints to bridge the functioning of the different groups. Finally, one more question to be further analyzed is our assumption regarding the impact of "generation" conflict.

NOTES

1. We use the term "group" referring to the people involved in the study because the aim of this study is to elucidate if this "group of people" are (or not) a Community of Practice. For this reason we only use the term "community" when referring to the theoretical concept / definition.

2. "However, it is not clear how to make these learning theories operational from a methodological point of view." (Gómez, p. 283).

3. All names are pseudonyms.

4. Unknown is "an unknown quantity of variable" (Pearsall, J. (Ed.). (1999). *Concise Oxford Dictionary* (10th Edition). New York: Oxford University Press.

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UNDERSTANDING ETHNOMATHEMATICS FROM ITS CRITICISMS AND CONTRADICTIONS[2]

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We considered articles from six researchers on the field of mathematics education, in which we identified two categories of criticisms to ethnomathematics: epistemological, related with the way ethnomathematics positioned itself in terms of mathematical knowledge; and pedagogical, related to the way ethnomathematical ideas are implicated in formal education. From this analysis we conclude firstly that it is not easy to criticize a research field so diverse and internationalized as ethnomathematics. Those difficulties are related with the different contexts on which ethnomathematics is pedagogically implicated. Secondly ethnomathematics itself as a research field rejects any dogmatic position, and is aware of contradictions implicated in their pedagogical aims.

Key-words: ethnomathematics, criticisms, contradictions, school, education

THE RADICALITY OF ETHNOMATHEMATICS

To associate the prefix 'ethno' to something so well defined, exact and consensual as mathematics can cause strangeness. The idea of a science that is human-proof, as mathematics is in a platonist perspective, is splintered when we associate it with the prefix 'ethno'. 'Ethno' shifts mathematics from the places where it has been erected and glorified (university and schools), and spread it to the world of people, in their diverse cultures and everyday activities. Ethnomathematics as an approach sullies mathematics with the human factor. Not an abstract human, but a human situated in a space and a time that implies different knowledge and different practices to live. Ethnomathematics as a research program is less a complement to mathematics, than a critique to the knowledge that is valorised as being mathematical knowledge.

Ethnomathematics does not restrict its research to the mathematical knowledge of culturally distinct people, or people in their daily activities. The focus could be academic mathematics, through a social, historical, political and economical analysis of how mathematics has become what it is today. As mentioned by Greer (2006), it is

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part of ethnomathematical research to understand the historical development of mathematics as a scientific discipline, the understanding of that development as the intersection between knowledge from different cultures, and the way the validation of what is considered to be true mathematical knowledge is less related with issues of rationality, than with the social and political contexts.

According to D'Ambrosio (2002)[3] academic mathematics is the basis of our modern world, upon which rests our faith in science and enlightenment ideas. So, if ethnomathematics aspired to be more than just the study of different mathematical ideas, but also the critical study of the social, political and anthropological aspects of academic mathematics, it assumes itself a critical stance on how mathematics is involved in the maintenance of our modern world. Ethnomathematics wishes to be an epistemological and educational alternative but, above all and this is not always given, a social and political alternative to our modern world. [4]

Given the radicalism of the ethnomathematical program (at least as it is put by D'Ambrosio (2002)), it is not surprising that its emergence has been the target of strong criticism. In our days research on ethnomathematics is numerous and scattered around the world.[5] It's difficult to have an international perspective on how ethnomathematical research is being done. Hence, to criticize something with so different practices and discourses as ethnomathematical research could result in an unreal chimera, if we don't take into consideration the different contexts on which research is made. A way to surpass those difficulties requires criticizing ethnomathematics as a well defined research program, and by analysing the work of the most important ethnomathematical researchers. That was the path chosen by Rowlands and Carson (2002) and Horsthemke and Schäfer (2006), in the epistemological and educational critique made on ethnomathematics. This critique, we argue, although apparently pedagogical, is an epistemological critique that pretends to highlight academic mathematics as one of the biggest achievements of mankind. In what concerns the pedagogical critique made by the latest researchers, and also by Skovsmose and Vithal (1997), we will articulate the contradictions raised by ethnomathematical researchers. Even among these researchers there are contradictions in how they understand the pedagogical implications of ethnomathematics.

EPISTEMOLOGICAL CRITICISMS

In 2002 Rowlands and Carson wrote an article published in *Educational Studies in Mathematics*, where they make a critical review of ethnomathematics, by comparing the ethnomathematical program to the curriculum of school mathematics. This article was subsequently answered by Adam, Alangui and Barton (2003), which Rowlands

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[3] But also to the philosopher Heidegger (1977) considerer the most important of 20th century by Slavoj Žižek (2006).

[4] At least, as D'Ambrosio (2002, 2003) put it.

[5] All those references are present in the bigger version of the paper.

and Carson (2004) later responded to in turn. As raised above, this paper also draws on arguments by Horsthemke and Schäfer who wrote two articles presented at the International Congress on Ethnomathematics in 2006, where they follow most of the arguments presented by Rowland and Carson. Those two sources of criticism present themselves as an educational critique on ethnomathematics but, in the way we analysed the texts, they are above all an epistemological critique, especially the articles from Horsthemke and Schäfer.

Against a nominalist posture assumed by ethnomathematics, Rowlands & Carson (2002, 2004) and Horsthemke & Schäfer (2006) advocate an essentialist position, based on the idea that although knowledge is constructed by humans, remains beyond. This is to say, there is some kind of invariant (an essence) that is repeated in all mathematical knowledge, despite this knowledge being developed in a Mongolian tribe or in a European university, the mathematics involved is the same:

> Mathematics is universal because, although aspects of culture do influence mathematics, nevertheless these cultural aspects do not determine the truth content of mathematics (Rowlands & Carson, 2002, p. 98).

The authors positioned themselves against the politicization of science: "mathematics is a science, and its laws, principles, functions and axioms have little to do with issues of social justice" (Horsthemke & Schäfer, 2006, p. 9). Or, as mentioned by Rowlands and Carson (2002) "rationality may be the preserve of an oppressive cultural system but that does not necessarily mean that rationality is in itself oppressive" (p. 82). Represented very strongly in this sentence is the idea that rationality exists *per se*, that is, as something disconnected from the social and political environment. In that sense, mathematics is taken by the authors as a piece of truth and neutral knowledge that could be used to the good and the evil, although mathematics itself is free from judgement: "the odious use of something does not make that something odious" (p. 98).

These authors embraced academic mathematics as a universal human good, shared by all people and considered to be one of the biggest achievements of mankind. This universal knowledge is presented as being the climax of a human evolution, and clearly more precious than others:

> The reason we are attempting to 'privilege' modern, abstract, formalized mathematics is precisely because it is an unusual, stunning advance over the mathematical systems characteristic of any of our ancient traditional cultures. (Rowlands & Carson, 2004, p. 331)

Finally, the authors adopted an epistemological position in which the genesis and consolidation of knowledge must be understood by analysing the internal logic of that knowledge and its pragmatic value, suggesting that social and political aspects have no influence in that genesis.[6]

> modern conventions of mainstream mathematics have become ‘privileged’ (i.e. accepted by the world’s mathematical community and numerous secular societies) for reasons that have little if anything to do with the politics of nations or ethnic groups, but have much to do with their pragmatic value. (Rowlands & Carson, 2004, p. 339)

EDUCATIONAL CRITICISMS

The tone for the educational critique developed by Horsthemke and Schäfer is the way the application of ethnomathematical ideas into South African schools contributed not to the inclusion, but to the exclusion of children. Ten years before, Skovsmose and Vithal (1997) had developed the same critique, although in a more constructive way. They called our attention to the way ethnomathematical ideas are implicated in schools of countries suffering from ethnic and racial tensions. In the case of South Africa, we can see how those ideas contributed to the creation of a lighter mathematical curriculum (based on students’ backgrounds) to those students considered being ‘ethno’[7]. As a consequence of that politics, those students were systematically excluded from access to academic mathematics then aimed at the white student: “in South Africa bringing students’ background into the classroom could come to mean reproducing those inequalities on the classroom” (p. 146).

This critique on the way ethnomathematical ideas in school could overshadow the access to academic mathematics is also made by Rowlands and Carson. These authors emphasise the dangers involved in not considering formal mathematics as an important part of all students’ education. According to the authors, it is formal mathematics that gives access to a privileged world, and that all students should know how to appreciate that knowledge:

> There is every danger that mathematics as an academic discipline will become accessible only to the most privileged in society and the rest learn multicultural arithmetic within problem solving as a life skill or merely venture into geometric aesthetics. (2002, p. 99)

In this sense, the authors defend a clear distinction between the local culture of a student, and the scientific and school culture:

> To preserve American Indian cultures, African tribal cultures, traditional cultures of Asia and elsewhere, their uniqueness must be recognised, not collapsed into a dreary and illusory sameness with scientific culture. (2002, p. 91)

Rowlands and Carson are against the use of ethnomathematical knowledge in the classroom, arguing that there may be incommensurable ways of understanding and

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[6] As was done in mathematics during the so called crisis on the foundations of mathematics, where mathematicians like Frege, Hilbert, Russell tried without success to epistemologically understand mathematics by using mathematics. The Gödel results showed what a chimera such enterprise is.

[7] Black students in the context of apartheid regime.

perceiving mathematics. It is that incommensurability that could make an artificial endeavour in trying to articulate ethnomathematical knowledge with school knowledge. They argue that people can master more than one culture, and school should be the place where people have contact with the more universalized culture, this is, the occidental culture.

Finally, Rowlands and Carson consider mathematics to be a foreign language to all students before they go to school. Contrary to the ethnomathematical stance which argues that students already have non-formalized mathematical knowledge before they start school, these authors argue that *protomathematical* knowledge is not important for learning school mathematics, because all students are equally positioned to learn a new knowledge:

> We go to great lengths to point out that children of traditional cultural backgrounds are probably not at any significant disadvantage when it comes to learning mathematics, since it is a 'foreign language' to all novices, regardless their cultural background. (2004, p. 335)

Skovsmose & Vithal (1997) acknowledge the importance of ethnomathematical ideas on a critical mathematics education. They identified four trends in the ethnomathematical research, and stressed that it is in the confrontation with school mathematical curriculum that ethnomathematics finds its greatest challenge, and also the possibility of critique. Firstly, the authors stressed the fact that research in ethnomathematics does not usually specify much about the relation between culture and power. Secondly, they identified a problem with the definition of 'ethnomathematics', and make the question: how can someone educated in formal mathematics identify other mathematics? According to them, ethnomathematics only makes sense through the perspective of academic mathematics. Thirdly, the authors argue that ethnomathematics lacks a critique on how mathematics formatted reality (Skovsmose, 1994). Finally, as mentioned before, Skovsmose & Vithal (1997) think it necessary to *problematize* the idea of students' background, and think not just in terms of the actual culture of students, but also in the aspirations and desires that students have of emancipation, what they called the students' foreground:

> Foreground may be described as the set of opportunities that the learner's social context makes accessible to the learner to perceive as his or her possibilities for the future. (p. 147)

According to Skovsmose (1994) all the importance given to students' background could inhibit them from emancipation, and more attention should be paid to the opportunities that the social, cultural and political context could bring to students. By emancipation Skovsmose means the access and participation in a world where mathematical knowledge is central.

SOME COMENTS ON EPISTEMOLOGICAL CRITICISMS

Before entering into a discussion on the epistemological criticisms made to ethnomathematics, we take the position that the interpretation of ethnomathematics carried out by Rowlands, Carson, Horsthemke and Schäfer is misleading. These authors understand ethnomathematics as an ethnic or indigenous mathematics. In fact, there is a vast diversity of studies in ethnomathematics, and part of them assume that ethnomathematics research consists of understanding, with the tools of academic mathematics, the mathematical ideas of culturally distinct people[8]. In that sense, ethnomathematics is indeed the study of an ‘ethnic’ mathematics:

> the prefix ethno refers to ethnicity, this is, to a group of people belonging to a same culture, sharing the same language and rituals, in other words, cultural well delimitated characteristics so we can characterize it as a specific group. (Ferreira, 2006, p. 70)

In this sense, the educational implications of ethnomathematics are focused on “how to bring ethnic knowledge to the classroom to allow for a meaningful education? How to establish the bridge between ethnic and institutional knowledge?” (Ferreira, 2006, p. 75). But there are other ways of addressing ethnomathematics. For instance, D’Ambrosio (2004) clearly says that “my view of ethnomathematics try to avoid the confusing with ethnic mathematics, as understood by many” (p. 286). That’s why D’Ambrosio prefers to talk about “ethnomathematics program”, as something more than the study of the ideas and uses of non-academic mathematics. We understand this program as a radical one, in the sense that it endeavours is to criticize, not just mathematics and mathematics education, but social orders and ideologies that feed our current world. As mentioned by D’Ambrosio (2004), “the ethnomathematical program focuses on the adventure of human species” (p. 286). Others like Knijnik (2006) and Powell & Frankenstein (1997) also criticize the idea of ethnomathematics as an ethnic mathematics and have developed investigations where the thematics of power and politics is taken seriously.

The epistemological discussion carried out by Rowlands, Carson, Horsthemke and Schäfer is an echo of a bigger philosophical discussion about the nature of knowledge that was intensively debated in the last decades under the label of “science wars”. As with any philosophical question, there are different ways of analysing it, and everyone has the right to choose the one that better fits its interests. We will not enter in such a discussion here. We just want to call attention to two points. First, in a philosophical line where we can include Nietzsche, Marx, Foucault, Durkheim, Weber, Wittgenstein, Freud, Lacan, Kuhn, Lakatos, Bloor, Restivo, Deleuze, Althusser, Zizek among others, knowledge is perceived from a nominalist perspective, that is, as something which creation, maintenance, valorisation or disqualification has nothing to do with its intrinsic or essentialist value, but with the way knowledge is exercised, whether it is in a language game (Wittgenstein, 2002),

[8] See for instance the work of Sebastiani Ferreira, Paulus Gerdes and Marcia Ascher.

in the webs of discursive modalities involving power relations (Foucault, 2004), as an ideological discourse (Althusser, 1970), and so on. The meaning and the knowledge we have of something is always contingent, full of historicity, and involved on power relations. As mentioned by Amâncio (2006) the idea of knowledge as something universal, with an existence *per se*, is itself a very ideologically loaded position. Hence, the important aspect of this epistemological discussion is less a discussion on whether knowledge is itself universal or situated, but, as mentioned by Foucault (2004), what intentions, what politics, are behind the claiming that some knowledge (like academic mathematics) is universal?

Secondly, unlike Rowlands, Carson, Horsthemke and Schäfer, we don't think there is a lack of theoretical and philosophical basis for ethnomathematics. Although there is a very diverse and disperse field of research, and also a recent one, there are several studies where the focus is not the ethnomathematical knowledge of groups of people, but philosophy, sociology and political science. Most of those studies use the work of the philosophers mentioned above.[9]

The authors of the essentialist perspective positioned themselves as the guardians of academic mathematics that fuelled this modern world, seen as being superior to any existing society, "the beliefs and practices of other societies are epistemic and vertically inferior to our own" (Horsthemke & Schäfer, 2006, p. 12). From their perspective, we are living the climax of a human evolution, in which academic mathematics is the substrate of a society based on humanistic ideals. This universal society is however problematic. Part of the research on ethnomathematics has been concerned to understand how these universal images of society generate through history[10]. As mentioned by Fernández (2006), the idea of such a universal society was possible through "the development of a set of formalisms characteristic of a peculiar way that has a certain tribe, of European origin, to understand the world" (p. 126). That is, the universal society (capitalist society) based on universal knowledge (mathematics and science) suggested by Rowlands, Carson, Horsthemke and Schäfer is a very particular way of understanding time and space, of classifying and ordering the world, of understanding economical and social relations. In short, of conceiving what is possible and impossible to think and do.

CRITICISMS AND CONTRADICTIONS ON THE EDUCATIONAL IMPLICATIONS OF ETHNOMATHEMATICS

Ethnomathematics carries with it a critique on school.[11] D'Ambrosio (2003), for instance, compares current school with a factory, where people are components of big machinery that aims uniformity. In school, as mentioned by Rowlands and Carson

[9] All those references are present in the bigger version of the paper.

[10] See for instance the book edited by Powell & Frankenstein (1997), which collects a set of articles where these ideas are deconstructed.

[11] See for instance the work of Ubiratan D'Ambrosio, Gelsa Knijnik and Alexandrina Monteiro.

(2002, 2004), we are introduced to a certain society. And if we are delighted with our current society, as apparently is the case of Rowlands, Carson, Horsthemke and Schäfer, then we must prepare students the best we can to be full members of that society. But part of the studies in ethnomathematics does not share this optimistic view on current society.[12]

Society should be problematized, and not taken for granted, especially when we are aware of the economical politics based on market priorities, and all the ideologies that fuel our way of living (like the liberal view on mankind). What does it mean to educate people to be participative, active authors in a more and more merchandized society? Do we all want "schooling to serve the needs of industry and commerce?" (Rowlands & Carson, 2002, p. 85). Hence, a problematization of society, and the role of school in society is, in our opinion, a priority in a research program like ethnomathematics. But that is far from happening.

For instance, and to speak to one of the criticisms made by Rowlands, Carson, Horsthemke and Schäfer regarding the use of ethnomathematical knowledge in regular schools, we can identify a contradiction on how ethnomathematicians understand this pedagogical implications. On the one hand, as mentioned before, some researchers defend the idea of using students' ethnomathematical knowledge to construct a bridge for the learning of formal mathematics. But, on the other hand, researchers like Knijnik (2006) clearly said that:

> it's not a matter of establish connections between school mathematics and mathematics as it is used by social groups, with the purpose of achieving a better learning of school mathematics. (p. 228)

Behind these two postures, is the way researchers understand the role of mathematics and school in our society. The problem with the first one, characterized by the "bridge metaphor", is the reinforcement of the hegemony of school mathematics because the 'other' is valorised only as a way to achieve the true knowledge. Thus, it contradicts the critique that ethnomathematics makes to the hegemony of academic mathematics. The same problem identified by the critics regarding the valorisation of background instead of the foreground, is also raised by Knijnik (2006), Monteiro (2006) and Duarte (2006). These authors raise questions about the usually folkloric way ethnomathematical ideas appear in the curriculum. According to them, the use of local knowledge as a curiosity to start the learning of school mathematics could be the cause of social inequalities, as is mentioned by the critics.

But to truly contemplate ethnomathematical ideas in the curriculum is no less problematic. If we focus on a regular school, and take into account its role preparing students to a market orientated society, with all the pressure to learn the mathematics

[12] In Powell & Frankenstein (1997) we can find a set of articles that articulate a critique on mathematics with a critique on society. See also the most recent writings of Ubiratan D'Ambrosio where he developed a social critique, based on the idea of peace.

of the standard curriculum that will be essential to students' approval in the high stakes tests, we can ask ourselves if there is a place for ethnomathematical knowledge (or other local, non scholar knowledge)? Our opinion, according to our review on ethnomathematical research in Brazil, is that those educational implications of ethnomathematics (in a regular school) ended up being phagocytised by a school that, as Rowlands, Carson, Horsthemke and Schäfer would agree, is worried with the uniformization of knowledge. In that sense, we agree with them and also with Skovsmose and Vithal when they say that focussing the learning of mathematics in students' local knowledge could be a factor for social exclusion. But the problem is not just in ethnomathematics, but in school itself. Monteiro (2006), a very well renowned ethnomathematicians makes the definitive question: "Is it possible to developing ethnomathematical work in the current school model?" (p. 437).

Hence, it is not just the valorisation of students' background that should be dealt with care, but also the valorisation of students' foreground. Although we realise the importance of students having the opportunity for emancipation, and for full participation in a technological world (that is also a capitalist world based on a liberal idea of economy that stress the individual above the social), we should criticize naïve and ideologically loaded ideas about society. Preparing students to become participants in a society is also preparing them to assume critical points of view about society, different ways of thinking, acting and doing mathematics. Using the words of D'Ambrosio, we need to emancipate students by learning academic mathematics, but also by reinforcing its roots. If we analyse the role of school in modern societies, this is obviously a paradox.

Critical mathematics education and ethnomathematics, as mentioned by Skovsmose & Vithal (1997), have common concerns. Both developed a critique of the way mathematics is usually understood as one of the biggest achievements of mankind, and the intrinsic resonance (seen as something inherently good) that feeds its education. But in the struggle for a better mathematics education, they should take care when suggesting pedagogical proposals to be implemented in a problematic school. Taking school for granted is the best way to trivializing critical and ethnomathematical ideas.

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USING MATHEMATICS AS A TOOL IN RWANDAN WORKPLACE SETTINGS: THE CASE OF TAXI DRIVERS

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The present study is part of an ongoing study of which the aims are twofold; to provide knowledge about why and how mathematics is involved in specific workplace settings, and to provide student teachers with culturally relevant examples to contextualise school mathematics for secondary school students. Observations and semi-structured interviews were conducted in the workplaces of two taxi drivers, one house constructor and one restaurant manager. The focus here is on taxi-drivers. The analyses draw on ideas from socio-cultural theory and the anthropological theory of didactics. A common main concern was economic profit and risk of loss; level of justification, mathematical problems to solve and techniques used differed. Among the taxi drivers, silent and taken-for-granted cultural knowledge were used.

INTRODUCTION

After the 1994-genocide, the Rwandan society was destroyed and disorganised in all sectors. In order to cater for capacity building, the Government of Rwanda has undertaken several measures in all economic sectors through its Vision 2020 for developing Rwanda into a middle-income country (Republic of Rwanda: Ministry of Finance and Economic Planning, 2000). For instance, in the educational sector, the Ministry of Education (MINEDUC) has embarked on prioritising the teaching and learning of science and technology (including mathematics) to provide human resources useful for socio-economic development through the education system. MINEDUC recommends that learning should be context-bound. This means that in order to serve the local society, teachers and researchers are encouraged to bring material to the students that are taken from national contexts. For instance, exploring mathematics via tasks from workplaces may support students to learn in ways that are personally meaningful (Taylor, 1998). Contextualising mathematics allows students both to understand the role of mathematics in solving different workplace problems and see ways in which mathematics is used out of academic institutions. They can also realize that such activities can be translated into mathematical language that is taught in different institutions.

However, before we embed mathematics in workplace settings, we should have a clear picture of the use of mathematics in such contexts. This is of crucial importance especially in Rwanda where this kind of research is relatively new and where mathematics is mostly seen as an abstract and hidden science which does not provide visible applications in workplaces (Niss, 1994; Williams & Wake, 2007).

In this study the use of mathematics as a mediating tool (Vygotsky, 1978) supports workers to solve problems related to the earning of their income, using culturally relevant concepts and experiences (Cole, 1996; Abreu, 1999) when seeking survival means is investigated. Therefore, the current study will provide knowledge about why and how mathematics is involved in three workplace settings: daily taxi driving, house construction, and restaurant management. Although the workplace settings are quite different and subject to change over time, the choice was made with the intention to understand mathematics in use in workplace settings where the actors perform differently but aim to achieve the same goal – to earn a good living. Within this study, the present paper will focus on the taxi driving context.

STUDIES ON SITUATED MATHEMATICS

Over the last thirty years, researchers have investigated how mathematics in everyday practices differs from what was taught at school and in academic institutions. In this endeavour Lave (1988) found that mathematics practice in everyday settings is structured in relation to ongoing activities. Based for example on the use of shoppers' "best-buy" strategies, she points out that mathematical practices in work places do not require any imposed regulation. Rather, adults use any available resources and strategies which could potentially help to solve a problem. Also, in a collection of studies related to informal and formal mathematics, Nunes, Schliemann and Carraher (1993) found that there was a discrepancy between street mathematics and school mathematics. This is demonstrated through a mathematical test which was given to the same children who performed better out of school than in a school setting. This discrepancy is due to the fact that at school children tried to use formal algorithms whereas in real situation they did arithmetic based on quantities. It should be noted though that the requested arithmetic procedures were quite simple. In results from a study related to college mathematics and workplace practice, Williams, Wake and Boreham (2001) found that the conventions of school and workplace graphs might be different. Indeed, in a chemical industry, school graph knowledge was not enough to allow a college student to interpret a graph of chemical experiments. However, the college student was able to interpret it with the help of an experienced employee. In a recent study Naresh and Presmeg (2008) followed a bus conductor in India in his daily practice, where they observed that though he performed significant mental mathematical calculations the bus driver's attention was fully concentrated on the demands of his job, making his mathematical work more or less invisible to him.

From the results of the above studies, we conclude that when it comes to solve a particular problem, the way mathematics is used at work is different, however logically organized (Abreu, 2008), compared to how it is used in academic institutions. At a workplace the problem solvers keep the meaning of the problem in mind while solving it in the real situation. In contrast, in the academic institution, the meaning of the problem is often dropped because of the imposed curriculum regulation where the problem solver is expected to employ certain mathematical symbols and conventions.

Researchers have also studied mathematical concepts and processes that are used in different workplace settings. In a study on mathematical ideas of a group of carpenters, Millroy (1992) found that not only are many conventional mathematical concepts embedded in the everyday practices of the carpenters, but their problem solving is enhanced by their stepwise logical reasoning similarly used in mathematical proofs. Abreu (1999) also found that Brazilian sugar cane farmers used indigenous mathematics to control their income. However, over time, technological innovations in measuring quality requested change to more school-like problem-solving strategies which made farmers prone to abandon traditional units of analysis and value their children's success at school mathematics. A study by Massingila (1994) revealed that mathematical concepts and processes are crucial in carpet laying practices such as estimation and installation activities. Furthermore, she found that measuring and problem solving are two major processes in the carpet laying practice. In their exploratory study related to how mathematics is used and described in workplaces in the context of employees in an investment bank, paediatric nurses, and commercial pilots, Noss, Hoyles and Pozzi (2000) found that practitioners use mathematics in unpredictable ways. Hence, their "strategies depend on whether or not the activity is routine and on the material resources at hand" (p. 17).

A common point to all these studies is that mathematical strategies that are used at workplaces differ to those taught at academic institutions. A mathematical strategy for solving a problem refers to a 'roadmap' that consists of identifying the problem to be solved and the appropriate technique(s) that allow solving that kind of task. However, in the above mentioned studies mathematical strategies are described as applied by workers without details about how they are or may be underpinned by mathematical justifications. Mathematics is seen as a tool to mediate human activity through the lens of workers' goal achievement. None of them looked at mathematics through the lens of its knowledge organisation, including types of problems worked on, as well as methods used to solve them and their justification (cf. Bosch & Gascon, 2006). To fill this gap the current study emphasises mathematical practices and its justifications embedded in mathematical activities found in specific Rwandan workplaces and their relation to academic mathematics.

MATHEMATICS AS TOOL TO MEDIATE WORKPLACE ACTIVITIES

Human activity is always goal-oriented and characterised by two major parallel actions: thinking and acting. The action is shaped by thinking and inversely through available socio-cultural tools for goal-oriented activity. Human mind and activity are always unified and inseparable. This means that the "human mind comes to exist, develops, and can only be understood within the context of meaningful, goal-oriented, and socially determined interaction between human beings and their material environment" (Bannon, 1997, p. 1). In activity theory, social factors and interaction between agents and their environment allow us to understand why tool mediation plays a central role. Tools shape the ways human beings interact with reality and reflect the experiences of other people who have tried similar problems at an earlier time (Bannon, 1997). Tools are chosen and transformed during the development of the activity and carry with them a particular culture. In short, the use of tools is a means for the accumulation and transmission of social knowledge. At the same time, they influence the nature of external behaviour and the mental functioning of individuals.

Engeström's (1993) model of basic human activity systems comprises six main elements: subject, object, tools, rules, community, and division of labour. He also suggests that such systems always contain "subsystems of production, distribution, exchange, and consumption" (ibid., p. 67). The present study is located in the subsystem of production which is mainly characterised by interactions between subject, tools and object. Within the production activity, subjects chose and transform useful tools that match a prior defined object to achieve a desired outcome.

However, our study will not elaborate on the production process as such. It will rather focus on the sub-production related to the selection and transformation of useful mathematics that facilitates the concerned subjects to achieve their goal on their respective workplaces. In other words, the study will investigate how the selected mathematics is organised so that the workers may interpret it in terms of the outcome of their activities. At that stage, it was imperative to add a complementary theory which explains deeply about the organisation of mathematical knowledge.

We will thus use a theoretical model from the anthropological theory of didactics (ATD), viewing teaching and learning as an activity situated in an institutional setting (Chevallard, 1999; Bosch & Gascon, 2006). By engaging in this activity, the participants elaborate a target piece of knowledge for which the activity was designed. This perspective sets a focus to the knowledge itself as an organisation system (a *praxeology*), including a *practical block* of types of tasks and techniques to work on these tasks, and a *theoretical block* explaining, structuring and giving validity to work in the practical block (Barbé, Bosch, Espinoza, & Gascon, 2005). This praxeological organisation of knowledge can be used to describe very systematic and structured fields of knowledge (such as mathematics or any experimental or human science) and its related activities, with explicit theories, a fine delimitation of the kind of problems that can be approached and the techniques to do so. Considering the mathematics teaching and learning process, we can find two different (intimately related) kinds of praxeologies: mathematical ones, corresponding to the subject knowledge taught, and didactical ones, corresponding to the pedagogical knowledge used by teachers to perform their practice. For the purpose of the present paper we will look into the mathematical praxeologies (or *mathematical organisations*) observed at the different workplaces.

Aims and research questions

The study reported in this paper is from the first part of an ongoing research project aimed at finding ways to contextualise school mathematics within cultural mathematical practices in Rwanda. In this project, the researcher documents the rationale and characteristics of mathematical practices in local workplace settings, to serve as a source to design contextualised mathematical activities for student teachers in a teacher education programme. From the experiences of working on such problems, the student teachers will design tasks contextualised in the local culture for secondary school students, whose work on these tasks will then be analysed. In this three-stage process, the didactical transposition (see Bosch & Gascon, 2006) of the workplace mathematical practice, via the mathematical tasks designed for and solved by student teachers, to the school students' contextualised mathematical work will be analysed.

The general question about why and how mathematics is involved in specific Rwandan workplace settings was split into specific research questions. First it was important to clarify what motivates the workers to involve mathematics in their daily activities (the why-question). In this regard, the interest was on what problems workers solve at their workplaces. Next there was a need to look at how those mathematical problems were solved. The answer to these questions raised the issue of justification of mathematical techniques used (the level of logos in the mathematical organisation observed). Using the ATD framework the following research questions were thus set up: What types of mathematical problems do workers solve at their workplaces? What techniques do they use to solve their mathematical problems? How are the techniques used justified?

THE EMPIRICAL STUDY

Method

In this interview study the data-collection was performed by the first author who is familiar to the field. Four workers from the three workplace settings volunteered to participate in the study, a female restaurant owner, a male constructor and two male taxi drivers. Three visits were conducted to each workplace. The purpose of the first visit was to inform the participants why and how he wanted them to be involved in the research. On this occasion, they agreed that he was permitted to observe and interview them about the use of mathematics in their daily activities. On the second occasion, after three weeks, the purpose was to observe and conduct the first semi-structured interview in order to understand how mathematics helps the workers to achieve their goals in their respective work sites. Three months later, a third visit was conducted to strengthen the understanding of the mathematical organisations. On that occasion, supplementary semi-structured interviews and observations were conducted. The interviews were performed in Kinyarwanda, a common language to all involved parties. Field notes were taken and interviews were tape recorded and transcribed at all visits. In the analysis we have used ideas from activity theory in which we draw on the object of activity to elucidate mathematics as one among the involved mediating tools in the activity. The analysis does not encompass the whole activity system; rather it focuses on the subsystem of production. The reason is that the purpose of the study is specifically to shed light on mathematics as a tool to help the participants to achieve their outcome. This part of the analysis illuminates the mathematical problems that are embedded in the workers' activity. Regarding how mathematics is used by workers on workplaces, the analysis draws on ideas of ATD, especially on its notion of mathematical organisation (MO). To perform this analysis we will build on a reference MO (Bosch & Gascon, 2006, p. 57), based on our own knowledge of academic and applied mathematics, in order to be able to analyse the observed MO in the workplace settings and on the interview data.

Findings

Due to space limitations detailed data on the observed mathematical organisations will be reported only from the taxi driving workplace. We will provide knowledge about the mathematical basis they use to determine the estimated transport fee charged to the customer. The taxi driving profession in Rwanda is mostly exercised by citizens with limited school background. The majority of taxi drivers consider the driving license as their core means of generating income. Some of them drive their own cars whereas others are employed. Taxi driving is mostly done in towns where you find financially potential people able to use taxi as a means of transport. Rwanda has not yet any explicit policy or norms and regulations that taxi drivers should follow to charge their customers. Because of lack of taximeters in the cars, the cost is negotiated between the taxi driver and the costumer.

From the transcripts of the interviews conducted with two taxi drivers, an employed (A) and a car owner (B), their main concern seems to be a non fixed level of profit and to avoid the risk of loss. Due to the difficulty of determining the number of customers every day, the estimation of costs depends mainly of considering control of factors such as road condition (good/bad), trip distance (in kilometres), quantity of petrol that the car consumes for a given trip (measured by money spent), waiting time (if necessary), and the time of the day (different day and night tariffs). Following an agreement between driver A and the employer, A was not responsible for expenses such as taxes, insurance, spare parts and so on. Also, A and his employer had agreed that A must deposit 5000 Frw every day to B and A's monthly salary was 30000 Frw. When the drivers were asked about their mathematical reasoning process while estimating costs, they always referred to authentic examples like pre-fixed estimations and rounded numbers without detailed calculations. In the interview, A gives an example of how he calculated the costs for a trip Kigali – Butare on a high quality tarmac road.

Interviewer: Ok.. let's take an example. Has it happened to you that you have taken a client from here [Kigali] to Butare?

Driver A: Yes, many times.

Interviewer: Could you explain to me how you have estimated the price?

Driver A: A one way of that trip is about 120 kilometres. The estimated cost for that trip was 30000 Frw. It means that I considered the cost of the petrol about 12000 Frw and I remained with 18000 Frw …

But sometimes it happens that while I am on my way of returning back, I meet customers and depending on how we negotiate the cost I charge him

3000 or 5000, it depends … But when estimating the price with the customer before the departure, I ignore this case because there is no guarantee to have this chance

This extract shows that the estimation of cost was made with respect to the cost of petrol and the driver's profit only. Road conditions were probably not mentioned as both interviewer and interviewee were assumed to be familiar with it. Transports between Kigali and Butare are frequent as contacts between the National University in Butare and official administrators or foreign aid agencies and others in Kigali take place on a daily basis. The next example is taken from a less frequented distance.

Interviewer: OK. Ok let's take the case of a Kigali – Bugesera trip. Although the road is now becoming macadamized it was always used as a non macadamized road. How much do you estimate for instance when you bring somebody there?

Driver A: …distance is almost 50 kilometres…then the return trip is 100 kilometres. But because of the poor road conditions, the cost is estimated at 15000 Frw. In that case I assume that the car is going to consume petrol for 5000 and I remain with 10000.

In the above extract, the estimation of the trip cost was made according to road condition, cost of petrol and the driver's profit. A seems to assume that more petrol is needed if the road is of bad standard but looking at Example 1 the same unit (10 km for 500Frw) is used. However, in Example 2 the driver does not seem to expect to be able to pick up a new passenger for the return trip.

In the second interview with B, the owner of the taxi, he explains how he estimates costs in relation to distance, price of petrol and time.

Interviewer: Let me ask you one explanation… for example when you charge a customer a cost of 1500Frw … what is your basis for that price?

Driver B: Do you remember I told you that with the petrol of 1000 Frw, I usually go 20 kilometres? Now when the customer tells me the destination I start to think of the number of kilometres to reach there. Then you say this time one litre of petrol costs for example 550 Frw… Approximately my car consumes 50 Frw to go one kilometre. This means that to go a distance which is not more than 10 kilometres for a return trip my car uses 500 Frw. So if I transport the customer to that destination without any waiting time I should have 1000 Frw for a work time less than 20 minutes... Do you get my point?

Like driver A, B calculates with rounded thirds, one third for petrol, one third for time spent and one third as a profit. As he is the car owner he could also have calculated with taxes and other costs involved with keeping a car.

Analysis of the observed MO

To characterise the MO observed in this taxi driving workplace setting, the type of problems involved could be described as varying versions of calculating the value of a function symbolically written as $W = F(x, y, z, t) + P$, where W is the estimated cost that the driver suggests to the customer. This cost consists of a non-fixed profit P and a cost F for the driver, estimated from all or a few of the four variables road condition (x), covered distance (y), petrol consumption (z) and time (t). Referring to the examples shown above, in the case of waiting for the customer the problem simplifies to $W = F(t) + P$, while the case with a short distance on a bad road will increase both the time and petrol needed: $W = F(z(t(x))) + P$. When the road is good but the distance longer it is the distance which is the deciding variable, $W = F(z(t(y))) + P$, which in the case of also a bad road changes to $W = F(z(t(x, y))) + P$. The techniques used by the drivers to solve these different types of problems are based on rounded estimations of basic costs, without providing a rationale of the amounts mentioned, and when needed elementary arithmetic operations are performed on these rounded numbers. For example, for the Kigali-Butare trip the model $W = F(z(t(y))) + P$ was used, with $y = 2 \times 120\,\text{km}$ and $W = 30000\,\text{Frw}$ with $z = 12000\,\text{Frw}$ and $P = 18000\,\text{Frw}$. In the case of the Kigali-Bugesera trip the road was not macadamized and thus in a bad condition and the model $W = F(z(t(x))) + P$ was applied, where $W = 15000\,\text{Frw}$ and $P = 10000\,\text{Frw}$ with $y = 2 \times 50\,\text{km}$. Technologies included number facts of addition and subtraction of natural numbers, and simple multiplication facts such as doubling. All numbers used were contextualised with units of distance and currency and no justification of the mathematical techniques used was referred to. Rather, it could be described as silent knowledge, adopted by experience and exchange with colleagues.

CONCLUSIONS

In Rwandan society as well as elsewhere in the world, the utility of mathematics is recognized through several activities. Those activities are seen on the one hand in academic institutions such as in schools and universities, where mathematics is used and learned for the purpose of developing knowledge about the subject per se; and on the other hand at different workplaces, where mathematics is used as a mediating tool to facilitate production within the workplace. The present study is partly an answer to policy departments' demands for a more contextualized mathematics education with a move away from using pseudo-problems to more culturally adapted problems. However, one aim is also to meet a theoretical challenge that attempts to combine sociocultural theories with Chevallard's anthropological theory of didactics. The latter makes possible an analysis of the observed knowledge organisation of workplace mathematics (in this case of taxi driving in Rwanda) that deepens the understanding of the purpose and function for the worker of using mathematics.

In the current study our focus was on taxi driving. A pre-determined common object for the drivers was to avoid any risk of loss while generating their income. The taxi drivers chose an appropriate mathematical organisation (MO) among other tools to mediate their activities, as described above. The observed techniques used by the subjects build on basic arithmetic related to addition and subtraction. Taken-for-granted cultural knowledge is seen in the example when the drivers request a higher profit for the distance Kigali – Butare as most local people travel this distance by frequently running minibuses. Taxis are for those who can pay. For community members the return fee to Kigali is subject to negotiation.

The way in which elementary arithmetic is applied should be understood in the context of continuous control of changing situational and cultural factors which make up a fundamental basis for the drivers' success. The observed MO is characterised by techniques which are functional to the problems at hand, the cultural constraints and the educational background of the drivers. As long as they are pragmatic for the goals of the activity, no further justification of the techniques is needed, resulting in a MO with undeveloped logos. This is reflected in the evident fact the drivers' goal is not to develop knowledge in the discipline of mathematics. What is functional at workplaces may in some cases be less functional in an educational context, where levels of justification often play an important role. However, these sets of constraints will form a background to the series of didactic transpositions that will occur before workplace mathematics can be used to contextualise school mathematics. This is a challenge for continuing research in this field. Moreover, the documentation of constraints and possibilities with which taxi drivers operate contribute to the ecology of mathematical and didactical praxeologies.

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PARENTS' EXPERIENCES AS MEDIATORS OF THEIR CHILDREN'S LEARNING: THE IMPACT OF BEING A PARENT-TEACHER

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This article discusses the way parents' past experiences influence the construction of their mathematical identities, their representations and their valorizations of current school mathematics, and how these factors mediate involvement with their children's mathematical learning. Two different groups of parents, with and without teaching experience, were interviewed. Participants within the groups showed similarities in the ways they constructed their own mathematical identities, and differences in how they constructed representations and valorizations of current school mathematics. Whilst those with teaching experience generally held more positive representations of current practices, the way they valued these practices changed according to their perceptions of their child's needs, and the various roles they adopted.

Key words*: parents; home-school; identities; representations; valorizations*

INTRODUCTION

The William's Report argues that parental involvement in schooling is a powerful force, and that 'parents are a child's first and most enduring educator, and their influence cannot be overestimated' (Department for Children, Schools and Families, 2008, p.67). However, research indicates that parental involvement in their children's education is complex. In a study reported by the Department for Children, Schools and Families (2007), it was found that whilst 73% of parents feel it is extremely important to help with homework, confidence amongst parents to become involved has decreased in recent years. Barriers to successful interaction may be particularly evident when parents and children work together over mathematics homework (Abreu & Cline, 2005; O'Toole & Abreu, 2005). Societal and cultural changes (e.g. National Numeracy Strategy, UK, 1999; immigration) are among the factors which have resulted in very different experiences of mathematics learning by both parents and children (O'Toole & Abreu, 2005). Abreu and Cline (2005) found that many parents were confronted with differences between their own ways of tackling mathematics and methods their children learned at school. Parents developed sophisticated representations of these differences, the most common concerning teaching methods and tools (e.g. calculators) available in the classroom. They also found that even when parents share knowledge of different methods to approach calculation, they may have a different understanding of how these methods are valued, and it is the position they adopt towards these shared representations that may affect how they organize mathematical practices for their children (Abreu & Cline, 2003). Abreu (2002, 2008) proposes that it is *participation* in particular practices which enables individuals to master cultural tools, and to understand how these are socially valued. For parents whose experience of learning mathematics was algorithmic rather than conceptually based, new methods of learning may remain inaccessible and they may be expected to support their children's learning in ways that don't make sense to them (Remillard & Jackson, 2006).

THE RESEARCH QUESTIONS

Many studies have examined the response to perceived differences in numeracy practices in minority cultural groups (Abreu, 2008; Abreu & Cline, 2005; O'Toole & Abreu, 2005; Quintos, Bratton & Civil, 2005; Civil & Andrade, 2002). In Abreu's previous studies, it was apparent that both parents' own experience of mathematical learning in a different cultural setting, and their lack of direct exposure to current school mathematics, impact on their understanding of their children's mathematical learning. This study seeks to understand further parental participation in their children's learning within the majority (White-British) cultural group, in terms of how this group experiences their children's mathematical learning in the context of historical changes between their school education and the education of their children. In addition, the study seeks to explore further the impact of parents' personal histories on their involvement with their children's learning, in terms of their experience of direct participation in current methods of learning. In this way, the study can shed light on issues that are specific to curriculum changes over time within a society, and issues that are more related to minority cultural groups. The study explores the experiences of two different groups of parents, those with teaching experience (direct participation in current teaching practices) and those without, with a view to determining similarities and differences in the way the participants in each group interpret their past experiences, construct current representations, and use these representations to mediate interaction with their child. The research questions investigated were: (1) What are the similarities and differences between the parents of these two groups in the way they construct their mathematical identities, and how does different adult experience affects these identities? (2) How do the parents from the two groups construct representations of current school mathematics, and how do they value perceived differences between current school mathematics and their own? (3) How do the parents from the two groups use their representations and valorizations of school mathematics to mediate interaction with their children's learning?

METHODOLOGY

Two groups of six White-British parents were interviewed. All participants had attended schools in the UK during the late 1960's - early 1970's, were university-educated, and all had children currently attending Primary schools. One group ('parent group') had no teaching experience, and were recruited through a Primary school in Oxford. The other group ('parent-teacher' group) had varying teaching experience. Four of this group had teaching experience prior to the National Numeracy Strategy, had taken a career break, and were selected from a Return to Teaching course organised by the Teacher Development Agency. These parents had undertaken recent placements in Primary schools which involved teaching numeracy, and could therefore compare their experiences of teaching numeracy both before and after the educational reform. The remaining two parent-teachers had recently trained as Primary teachers, and were able to draw on their experience of helping their children with their homework prior to their training.

Procedure and tools for data collection: An episodic interview (Flick, 2000) format was used as this method of questioning encourages participants to give their opinions about the subject matter, and to give concrete examples of situations in their past. The interview covered basic information, and explored the interviewee's biography in relation to their mathematics learning, current uses of mathematics, and their experiences of helping their children with school homework. For parent-teachers, their teaching experience was also explored. All participants were interviewed in their own homes for approximately 45 minutes, and interviews were audio-recorded.

Data analysis: The interviews were fully transcribed and analysed using thematic analysis (Braun & Clarke, 2006), taking into account the research questions, key concepts from the literature, and new information emerging from the data. The coding was supported by NVivo qualitative analysis software. Initial thematic maps grouped sub-themes together into super ordinate themes as described in Table 1. The data was then examined for similarities and variability between the two groups of participants.

Table 1. Superordinate themes and sub-themes.

Superordinate themes	**Sub-themes**
1. Parent's mathematical identities	1. Memories of mathematics learning - emotions 2. Perceptions of own ability 3. Social value of mathematics in family/peer group 4. Effect of parent's identity on child's identity
2. The effect of adult experience on identity	1. Effect of work experience on identity 2. Effect of teaching experience on identity
3. Parents' representations of school mathematics	1. Knowledge/understanding of current methods 2. Perception of own school mathematics as same/different 3. Effect of teaching experience on representations
4. Parents' valorizations of	1. Equivalence of/confidence in different methods

different practices	
5. How different representations and valorizations influence interaction	1. Effect of representations and valorizations on interaction 2. Valorization of methods by parent and child 3. Effect of teaching experience on interaction 4. Emotional aspect: frustration/fear of confusing child

FINDINGS AND DISCUSSION

Parents' mathematical identities

Three main themes were revealed in participants' perceptions of themselves as mathematics learners: their perception of their ability, memories of the emotive nature of their mathematics learning experiences, and their status as a learner amongst family and peer group. Participants in both groups were similar in that their assessment of their cognitive competence in the cultural tools of mathematics formed a significant part of the way in which they constructed their mathematics identity. The data also indicates that participants' view of their mathematics ability did not solely rely on their perception of their competence, but was strongly influenced by their feelings about their experiences. For example, Table 2 shows that there were parents from both groups for whom learning mathematics was remembered as a struggle and was associated with fear and panic. Tilda talks about 'feeling lost for ever, for ever after'.

Table 2. How emotions mediate mathematics identity.

Parent group	Parent-teacher group
P: I can remember saying, "I don't understand," and him trying to explain it, and I was none the wiser. I can actually remember saying, "Help!" I mean he tried but it was no good, and then I can just remember being lost for ever, for ever after … I think I was always quite good at just basic maths, but with algebra or anything like that, I'd always be frightened. [I felt] a sort of terror, fear. *Tilda, parent*	*P:* I think it got to that point where sometimes you'd go, "Oh, I can't do that!", and your brain freezes, and your brain would stop working and decide that it can't do this. *Rebecca, parent-teacher*

For both parents and parent-teachers, their mathematical identity relied strongly on how they were identified by significant others, for example, parents and teachers, and their perceptions of their ability in comparison to siblings and peers. Parents in both groups hoped that their child would construct a positive mathematics identity, and for many, it was more important that their child have a confident relation with mathematics, than be expert in the subject. The consequences of parents identifying themselves, or their children, as less competent, resulted in participants from both groups positioning themselves, or their child, as an ‘arts’ person rather than a mathematician. In positioning themselves in this way, they devalued mathematics as something not necessary to succeed. Consequently, this may have limited their capabilities in mathematics, or their expectations for their child. Many showed awareness of how their own parents’ mathematics identity had influenced the way they perceived themselves as mathematicians, and how this could, in turn, influence their children’s identity. As illustrated in Table 3, Tilda felt it was extremely important not to let her daughter know that she wasn’t a confident mathematician, whilst Clare understood that her own identity was interlinked with her father’s.

Table 3. How parents’ mathematical identity can affect their children’s.

Parent group	**Parent-teacher group**
P: I’ve got a friend that says, “I was crap at maths, so my kids are crap at maths”, that’s what she says. And she has a daughter who isn’t doing so well in maths, but she’s taking it as an absolute given that that is how it will be and I suppose I don’t … I’ve never said to [Lily] I wasn’t any good at maths because that would be a dirty little secret I would keep to myself! *Tilda, parent*	*P*: My dad was a maths teacher for a while, and he used to get really frustrated with me, helping me with maths, because he’s sort of mathematically-gifted, he sort of finds it easy. So there was this conflict in my relationship with my dad … and I didn’t see myself as a natural mathematician. *Clare, parent-teacher*

The effect of adult experience on identity

The research revealed that parents in both groups felt that they had developed a more positive relation with mathematics due to experience during adulthood (see Table 4).

Table 4. The effect of adult experience on mathematical identities.

Parent group	**Parent-teacher group**
P: I think it’s practical maths … because once you actually leave school and you start working, you have to use maths on a day to day basis, and suddenly it all starts to make sense, and depending on the kind of work you do … I’ve always learnt by rote, managed to get through, and then latterly actually as	*P*: It’s interesting actually as I think my own feelings about mathematics really changed when I did my teacher training … Suddenly I saw the beauty of numbers, it all fell into place and I could see how all the different parts of mathematics relate to each other … revisiting it I had this sudden enthusiasm for maths that I’d never had before … I’m not suddenly a better mathematician because I’m doing more

you get older, you realize why that goes with that, and it's a late discovery. Suddenly it's like, "Oh! Oh yes!" *Lisa, parent*	advanced level maths, I'm a better mathematician because I understand the basics in a different way. *Clare, parent-teacher*

Often, those parents who described a change in their mathematical identity, experienced a transformation of their understanding of activities through participation in different contexts for mathematics practice. A number of parent-teachers experienced transition from being an anxious mathematics learner, to a confident teacher of mathematics, through participation in different contexts for mathematics *learning*. Clare reveals that the experience of 'revisiting' mathematics during teacher training allowed her to acquire an understanding of the concepts of mathematics she felt she lacked as a child. Many parent-teachers attributed this greater understanding to current conceptually-based methods, in comparison to the algorithmic approach they had experienced themselves.

Whilst participants in both groups had experienced changes in their relation with mathematics during adulthood, there was variability between the groups in how the participants constructed their relation to mathematics due to the differing nature of these experiences. Those in the parent group tended to associate the change in their mathematics identity with maturity, or to using mathematics in daily life. Those in the parent-teacher group, however, were more likely to associate change with the opportunity to revisit mathematics, and participate in practices which differed from those they were familiar with.

Parents' representations of their children's school mathematics

Whilst having clear memories of certain aspects of their own learning, many participants, particularly in the parent group, had unclear ideas of how their children were currently learning mathematics. As Table 5 shows, this lack of knowledge sometimes produced a strong emotional response. Lisa, for example, talked of feeling 'closed' to the new methods because they didn't make sense to her, whilst Karen experienced frustration and could not view the school's methods in a positive light.

Table 5. The effect of parents' lack of knowledge of current methods.

Parent group
P: I know I'm not open, I feel that I'm quite closed to these new methods because I look at them and they don't make sense to me. I get the impression that they're trying to make maths meaningful and I just think it isn't meaningful, it only becomes meaningful if you start to use it in life. And if you're one of those people that it's not obvious to, the way they're doing it, it's not making it more obvious, it's actually making it more obscure. *Lisa, parent*
I: Can you show me any ways that you think they're doing it?

P: Oh, God, I can't. I mean, no, I can't. I must be really honest here, I don't actually understand how the mathematics is taught or why the mathematics is taught in the way it is … And the point is, I don't actually know whether there are advantages to the way they do it, I just don't know, because I don't understand it, and I don't know how they're teaching it. *Karen, parent*

Although many felt unclear about the new methods, all participants remembered their learning as very different to the ways their children learn now, and these differences were explained as historical changes within Primary education. The representations of these differences were similar in the two groups of parents in terms of teaching methods used, and different mathematical strategies for calculation. Current methods were viewed by participants in both groups as having a greater emphasis on underlying meanings and relationships, whereas a significant feature of their learning had been the repeated practice of 'rules' or 'formulae' for calculation. The groups differed, however, in their conception of whether current or old methods placed a greater focus on mental strategies. Indeed, it became clear that what was meant by 'mental strategies' was quite different to the groups. Those in the parent group tended to equate mental strategies with basic mental arithmetic, and felt strongly that there was less emphasis on this in current teaching. The participants in the parent group valued the repeated practice which had allowed their mental skills to become 'second nature'. The participants in the parent-teacher group, however, viewed current methods as having a *greater* emphasis on mental strategies, but saw this in terms of children having more opportunities to discuss concepts, and have a greater range of mental strategies to tackle calculation. Although parents from both groups talked about valuing mental mathematics, how they constructed their representations and valorizations of mental mathematics was quite different.

Parents' valorizations of different mathematical practices

Whilst participants in both groups shared the view that current school mathematics was different to their own school mathematics, the way the groups valued different practices was quite diverse. The parent-teacher group participants had a clearer idea of the purpose of the new methods, saw the changes as predominantly enhancing children's global abilities in mathematics, and as providing them with a more solid platform for later mathematical study. They spoke positively of children talking about mathematics, developing a greater ability to reason, and a greater understanding of the concepts of mathematics. They were more likely to value conceptually-based learning, and less likely to value an algorithmic approach. Their view of current mathematics was often in comparison with how they remembered their own experiences of learning which whilst enabling them to perform calculation procedures well, had also meant they adopted an 'automatic approach' without understanding how numbers worked together. Their accounts of the way in which they learned may have been mediated by their greater knowledge of the aims of current methods and their current perceptual frameworks.

Most of the parent group participants, on the other hand, saw the changes predominantly in terms of confusion and complexity. They described the new methods as too numerous and more complicated, and were anxious that the focus on understanding the concepts of mathematics was at the expense of rigorous training in the acquisition of basic mental skills. They viewed that this would result in a gap in their children's cognitive skills, particularly if they perceived their children to have a less confident relation to mathematics. Amongst the participants in this group, differences in methods were not described in neutral terms, and were not treated as equal alternatives. Parents used language such as 'simple', 'straightforward' and 'logical' to describe their own form of mathematics, and 'long-winded', 'complicated', and 'obscure' when describing new ways. Parents in this group were more likely to value an algorithmic approach, and less likely to value an emphasis on conceptual understanding. As they possessed less knowledge of the new methods, they were more likely to feel new methods inadequate or confusing, and to feel closed towards them.

How different representations and valorizations influence interaction

The data revealed that many of the parent group participants experienced difficulties in understanding practices in which they did not have direct participation, and were often dependant on children's explanations about how they use particular procedures. That children themselves were often unable to explain clearly often resulted in a breakdown in communication between parent and child. Table 6 shows that Karen felt frustration that her incomplete knowledge prevented her from helping in anything more than a checking role, whilst Susie described how lack of information made her feel there was nothing she could do, and compromised the amount of effort she was prepared to invest. Not only did those parents who lacked knowledge of current methods feel excluded from helping their children, they couldn't judge their child's competence in comparison with their own ability at a similar stage, and felt they did not know what could be expected of their child.

Table 6. The effects of parents' lack of information on interaction.

Parent group
I: …Do they think they're good at maths? *P*: Yes, I think so. The problem is it's difficult for me to know whether they're good … obviously they seem to get their maths homework right …but I don't know what that means, are they good beyond that? Are they capable of more than that? … I sort of feel like, and this is my lack really, I feel I should be more sort of involved with their mathematics ... I feel I'm not involved enough, because I basically just sit and look at it and any that are wrong I'll check them, but only from a distance really … So I do find it difficult to support them as much as I could. I don't feel I can get as

involved as I would if he was learning in the same way as I did. *Karen, parent*
P: Well, a lot of the time if I don't understand what method is to be used, I just throw up my hands. There's nothing I can do. I don't feel ... I don't feel anything really, it's a waste of energy really. There's nothing I can do, but I sometimes feel sorry for Molly, because she gets really upset and there's nothing I can do. *Susie, parent*

As parents talked about the way in which they interacted with their children, it became clear that many children valued school's methods more highly than methods their parents showed them. This was not necessarily because the school's methods were better or clearer, but that children perceived them to be the 'right way of doing it'. Parents in both groups talked of how their children 'revered' school more than their parents, and of their child's resistance to being shown other ways. This often resulted in discordance between parents and child, and led to homework as a source of conflict. The data also revealed that responses to mathematical practices differed according to which practices parents valued more highly. Whilst not wanting to undermine school methods, many in the parent group displayed frustration that their own tried and tested methods were being devalued, whilst they perceived other methods as resulting in confusion for their children. Those in the parent-teacher group, on the other hand, generally had more favourable representations of current school mathematics, and were more willing to support methods which they viewed as enabling their children to achieve a positive relation with mathematics. They reported that their teaching experience had enabled them to develop a greater understanding of current school mathematics, and this allowed them to be more confident in assessing their child's ability, and in participating in mathematics homework. However, the data also revealed that although most of the parent-teachers understood and appreciated the use of multiple methods, they adopted different positions towards these approaches if they perceived their own child was confused and this, in turn, affected how they organized mathematical practices for their children (see Table 7).

Table 7. Parent-teacher's valorizations of their own methods.

Parent-teacher group
P: Milly, I know, knows one method, and if something else is being taught, then I'm afraid I'm saying to her, ignore it, because I'm worried that she will mix it as well. I'm saying forget what Mrs Woods tells you, I keep telling her, which is very naughty, but stick to what you know, because you can do it that way. *Jane, parent-teacher*
P: I think we took the right decisions for Luke at the time, but I think potentially it could have been even more confusing to him, because I could explain to Luke, yes, you can do it these different ways at school, but you know if Dad's shown you this way and you're happiest with that way, then you do it that way. *Cathy, parent-*

teacher

Although some parent-teachers were unwilling to devalue the school's methods, others felt they were right to encourage their children to use only one method, if their child continued to be confused. Jane talks about actively encouraging her own daughter to ignore the school's insistence on multiple methods because of her fear that she will become confused. The effect of teaching experience, then, was generally positive in terms of parents' representations and valorizations of current school mathematics. However, although, many parent-teachers recognized that multiple methods may enhance understanding by providing 'the bigger picture', they constructed different representations of new methods as too numerous and too complex if they perceived their own child to be confused by them. Even with a good knowledge and understanding of new methods, and sympathy towards the aims of the National Numeracy Strategy, their position in relation to the numeracy practices changed according to the particular role, as professional or parent, they had to adopt at any given time. It was the position parents adopted towards these representations which affected how they interacted with their children's mathematical learning.

CONCLUSIONS

The research set out to explore how parents' past experiences influence the way in which they construct their mathematical identities and their representations of different mathematical practices, and how these factors influence the ways they interact with their children's learning. The findings illustrated that both those with, and those without teaching experience, construct their mathematical identity in similar ways and this identity was shown to evolve through participation in different contexts of mathematical practice and learning. Participants in both groups were similarly aware that their own mathematical identity could affect the way in which their children approached mathematics.

The study revealed that both those with and without teaching experiences perceived current school numeracy practices to be very different to those they had experienced when learning. Varying levels of knowledge, and different levels of participation in current methods resulted in the participants from both groups valuing different mathematical practices in different ways; those with teaching experience tending to attribute a higher value to current methods than those without teaching experience. However, the study indicated that although in many areas, those with teaching experience were able to bridge the gap between differing mathematical practices more easily, when confronted with their child's continuing confusion about mathematics, parents may revert to the methods they formerly depended on, despite holding positive representations of current methods. Parents' perception of their child's ability in relation to certain mathematical practices was, therefore, a more significant resource for parents, and contributed more significantly to the way in which they interact with their children, than their overall representations of current methods.

This research indicated that it is the opportunity for participation in different mathematical approaches which allows parents to construct more positive representations of varying practices, and in turn, to understand how they are socially valued. This has implications for how schools communicate the way they approach mathematics, and the opportunities they offer to parents for understanding these practices and for raising confidence amongst parents to become involved. The study also explored the transitions parents experience between their roles as parent and teacher, and how the subjective knowledge they developed during these transitions is adapted for each role. Further study of teachers' representations and practice in the classroom, in the light of the interaction they experience with their own children, would contribute to research on the ways in which valorization of numeracy practices affect support both within the home and at school.

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BATIKS:
ANOTHER WAY OF LEARNING MATHEMATICS

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This paper examines a micro-project that was developed in an 8th grade class. Students elaborated batiks and then they discussed mathematical tasks based in their batiks' elaboration process. This research is based in two research projects: Interaction and Knowledge (IK) and IDMAMIM. We assume an interpretative approach and a case study design. Results illuminate the potentialities of these classroom practices, illustrated through the analysis of some video taped peer interactions. The focus of analysis is in the didactic contract, based in collaborative work, and in the nature of the tasks that were part of this micro-project.

THEORETICAL BACKGROUND

Portuguese schools are multicultural settings (César, 2007; César & Oliveira, 2005). Considering Nieto's definition (2002), culture is "(…) the ever-changing values, traditions, social and political relationships, and worldview created and shared by a group of people bound together by a combination of factors (…), and how these are transformed by those who share them" (p. 53). According to this definition, in school we find a great diversity of cultures. Not only origin cultures but also many others, including the school's culture, or some teenagers' group culture.

Sometimes the school culture is so far away from students' cultures that they focus their energies on other directions (Säljö, 2004). School needs to facilitate the emergence of "thinking spaces", a construct coined by Perret-Clermont (2004) that stresses the role played by securing spaces in which students may discuss doubts, conjectures, solving strategies, learning difficulties, developing their critical sense, learning autonomy, but also their "sense of identity" (Zittoun, 2004), of belonging to that particular learning community. As César (2007) claimed, becoming a legitimate participant in a learning community, namely in formal educational settings, facilitates students' engagement in academic tasks but also their construction of identities and the management of the dialogical I-positioning (Hermans, 2001) that are often conflictive when the student's culture is much different from the school's culture.

Schools also need to be more inclusive (Ainscow, 1999; César, 2003, 2007, 2009; César & Santos, 2006) and to promote interactions among community members and cultures. Intercultural (mathematics) education facilitates the emergence of dialogical interactions, namely among students from different cultures (D'Ambrósio, 2002;

Favilli, César, & Oliveras, 2004; Peres, 2000; Powell & Frankenstein, 1997; Teles & César, 2007). Ouellet (1991) has already stressed that this education is for everyone, based on the comprehension, communication, and promotion of interactions. Collaborative work among students (and with the teachers) was studied by many authors. It acts as a facilitator and mediator for student's knowledge appropriation when it is part of a negotiated and coherent didactic contract (César, 2007; César & Santos, 2006; Schubauer-Leoni & Perret-Clermont, 1997; Teles & César, 2005), and it also facilitates transitions (Abreu, Bishop, & Presmeg, 2002; César, 2007, 2009).

The development of intercultural (and interdisciplinary) microprojects related to handicraft activities promotes students' performances and academic achievement (Favilli et al., 2004). They underline the cultural dimension these activities give to the learning processes, also contributing to the mobilisation/development of competencies. Their social marking of the tasks, i.e., the possibility of connecting them to students' daily experiences and social life, plays an important role on students' engagement and mathematical performances (Doise & Mugny, 1981; Teles, 2005). It also plays an important role when teachers aim at changing students' social representations about mathematics. Social representations are often stated as being an important contribution for students' performances and school achievement (Abreu & Gorgorió, 2007; César, 2009).

METHOD

We assume an interpretative approach, inspired in ethnographic methods. This study is based in two research projects: *Interaction and Knowledge* (*IK*) and *IDMAMIM*. The first one was developed during 12 years (1994/95-2005/06) and its main goal was to study and implement social interactions in formal educational scenarios (for more details see César, 2007, 2009). The didactic contract that was negotiated in this class was clearly shaped by this project's features. Teachers' practices, based in collaborative work, were also shaped by this project's pedagogical ideals. *IDMAMIM* project was developed in some towns of Spain (Granada), Italy (Pisa) and Portugal (Lisbon). Its two main goals were: (1) to identify didactic needs in order to develop an intercultural mathematics education; and (2) to elaborate intercultural didactic materials, like the ones based in the batiks elaboration, and its later exploration in mathematics classes (Favilli et al., 2004). The mathematical tasks used in this class were part of this project.

This case is part of a broader study including 4 case studies. In all these case studies students developed an intercultural microproject, based on the elaboration of batiks. Batiks are a handicraft from Java, that was then developed in other parts of the world, namely in Cape Verde, where we collected information about how to elaborate them. Batiks assume different ways of being produced in different parts of the world, according to the native cultures of each country, and also to their economic conditions. In Cape Verde, as it is a very poor country, they use flour, water and lime, instead of wax in order to make the production process cheaper. Thus, even discussing the different ways of production of batiks, that students discover in the internet before elaborating them, it is a way to explore a critical mathematics approach. This is complemented by the discussion of the video we made in Cape Verde in which batiks are being produced. This way of approaching the microprojects also allows them to be explored in a multidisciplinary way, as teachers from different subjects may participate and, for instance, explore the texts from the internet in English language subject, the production process in Chemistry, the evolution of batiks around the world in History, the elaboration of the templates in Arts. In this paper we focus in the one of the mathematical tasks that was solved after elaborating the batiks. Thus, the research question that we analyse in this paper is: What are the contributions of intercultural and collaborative microprojects to students' mathematical knowledge appropriation?

The participants were the students from a 8^{th} grade class (13/14 years old), their mathematics teacher, external observers and evaluators. This class had 21 students, one of them categorized as presenting special educational needs (SEN). There were 12 girls and 9 boys. These students were from different cultures and some of them were born, or had families that were born, in other countries. But even Portuguese students belonged to different cultures and socio-economical backgrounds. The mathematics teacher described this class as "(…) a working, engaged, interested and challenging class" (Teacher's final report, p. 7), as some of these students experienced underachievement in previous school grades in mathematics. Thus, many of them presented a negative social representation about mathematics in the beginning of the school year (September), according to the data of the *IK* project (students' protocols – for more details about the first week procedure, see César, 2009 or Teles, 2005). Some of these students usually did not participate in mathematics activities during classes, in previous school grades. They did not disturb the class work. They simply did not do anything and just waited for the end of the class to go to the break. Thus, many of these students never went to the blackboard after solving mathematical tasks, or participated in the general discussion. For these reasons, one of the main teacher's practices aims during the first month of classes was to promote students' participation in mathematical activities, and to avoid having only three or four of them – always the same ones – participating. The dyad whose peer interaction we chose to discuss is a paradigmatic one: J. was one of the students who experienced underachievement in mathematics in previous school grades while her peer loved participating in mathematics classes. Thus, the teacher tried to promote J.'s participation and, in this episode, we can see that she is no longer silent, or just trying to be unnoticed. She is already able to go to the blackboard, during the general discussion, after dyad work, and to explain to the whole class her dyad's solving strategy. Thus, this dyad illuminates some of the processes that could be observed in many other excerpts from the videos, and that were shaped by the collaborative work these students developed during the whole school year in mathematics classes.

Data was collected through observations, questionnaires (*IDMAMIM*), interviews (*IDMAMIM*), the teacher's and external evaluators' reports and students' protocols. In this paper we focus in the analysis of some video excerpts, the teacher's report and in students' protocols.

In this episode, students were solving a mathematical task in dyads, after elaborating their batiks. A batik is a pure cotton wrap tainted with colours where a drawing is contrasted. This elaboration process uses mathematical knowledge that can be explored further in later mathematics classes (for more details, see Favilli et al., 2004; Teles, 2005). They were discussing about the following situation:

Ms. Bela made a batik. It was in a square piece of cotton whose side measured 60 cm. Mr. Evaristo is interested in buying a batik. But he wants one with the double of the size.

- Ms. Bela, how much is a batik like that with the double of this size? – asked Mr. Evaristo.

- Look, Mr. Evaristo, this batik costs 18€. And I can sell you the other batik at the same price each m^2.

- Then, I offer you 36€! Do you accept my offer?

1.1. What do you think: Should Ms. Bela accept Mr. Evaristo's offer? Explain your reasons.

1.2. Complete the table below, considering the correspondence ***f*** that associates a square batiks' side (*x*) to its area (*y*).

Length of the side of the batik,		20		6
Area of batik, cm^2 (*y*)	0		1600	

Figure 1: Batik

RESULTS

This episode is an excerpt of an interaction between two students: J (a girl – 13 years old) and N (a boy – 12 years old). They are both Portuguese, but their family cultures are differentiated: N. comes from a highly literate family, whose parents have an university graduation; J. comes from a family whose parents have jobs related to commerce and services. From the economical point of view their families are from a class that is not very high or very low. They could be characterised as paradigmatic teenagers, with the hobbies, dressing code, language, and friendships of most of the teenagers in Portugal. J and N are on 8th grade for the first time but they have different previous experiences with mathematics. J does not like mathematics. In the first term she still experienced some underachievement (she got Level 2, a mark that is negative, in the Portuguese educational system, in which students' marks vary from Level 1 – the lowest - up to Level 5 – the highest). But during the next two terms she was engaged in mathematics classes and she was able to achieve. N is a student with a calm and pleasant relationship with mathematics. He always succeeded in this subject. He shows a high self-esteem, namely an academic one, while J was less confident about her abilities and competencies, in particular in mathematics and during the first months of the school year. It was precisely their differentiated characteristics as mathematics students, and when they addressed the mathematics tasks – in the beginning of the school year J tended to give up very easily or even not try at all to solve them – that were the criteria for choosing them to be discussed and analysed in this paper, as they both represent many other similar students we had in this class, and even in the other three cases from the *IDMAMIM* project.

In this episode, they are solving the question 1.2. It is N who starts the interaction writing on his notebook his reasoning in order to explain it to J.

b) Preencham a tabela:

Comprimento do lado do batique, em cm (x)	0	20	40	60
Área do batique, em cm^2 (y)	0	400	1600	3600

Figure 2: J and N resolution (Question 1.2.b))

1 N: It is: 20, 40, 60. It is half of 1600 [He understood that 20 is half of 40; then the table should be completed with half of 1600, i.e., 800]. It is 800. It is the double of this [He points]. Then, here it is 40 is 1600; then 20 is 800.

2 J: A little confusing!

3 N: What is the part you don't understand?

4 J: This part [she points to the sum]. Why is this plus this?

5 N: Because… This plus this equals 1600. Teacher!?

[The teacher approaches them]

6 N: Could you see if my reasoning is correct?…

7 J: So, what do you [turning to J] think about his reasoning?

This piece illuminates the role of the didactic contract of this class (César, 2003; César & Santos, 2006; Schubauer-Leoni & Perret-Clermont, 1997; Teles & César, 2005): students can start their resolution of the task by individual work but they need to explain their reasoning to his/her colleague from the same dyad. They need to discuss the solving strategies they used in order to find a consensus. But they also need to understand each other's solving strategy because one of them may be asked to represent their dyad in the general discussion and to explain to their colleagues their solving strategies. As they are both engaged in this type of didactic contract, they know that just having an answer produced by one of them is not enough. Thus, J is trying hard to understand her peer's solving strategy and this is exactly what her teacher aimed: to improve her participation in the mathematical activities, during mathematics classes. Their teacher was trying to create what Perret-Clermont (2004) designates as thinking spaces, facilitating students' reflection upon their solving strategies and some mathematical concepts.

They also know that discussing their solving strategies is a way of learning for both of them. For the one who used this solving strategy as s/he has to clarify its steps in order to explain them and to answer to his/her peer's doubts and questions; and to the one who is, at that moment, acting as the less competent peer (Vygotsky, 1932/1978), as it helps him/her progressing in his/her mathematical performances and in knowledge appropriation. These features of collaborative work, that we can also see in other parts of this episode presented below, also help students develop their positive self-esteem – particularly clear in the way of acting of J, in this episode, namely when she goes to the blackboard during the general discussion and is able to explain her dyad's solving strategy without taking any sheet with their resolution in her hands (according to the video record, she acted like this due to her teacher's suggestion). Thus, it helps them to begin acting as legitimate participants and not as peripheral ones (César, 2007). This changing form of participation is illustrated by the ways J acts, during the different parts of this episode, as well as by the external observers reports, during the school year, and by the analysis of other episodes that were also video recorded.

In Turns 5 and 6 N asks for their teacher's help and assumes this dyad's leadership. He is assuming the role of the more competent peer (Vygotsky, 1932/1978). This happened in this dyad during the first month they worked together, as J considered N "much better than me" (questionnaire, January) and it took some time before she was able to express her opinions, solving strategies and arguments before listening to N. It must be added that while analysing many other pieces of videotapes from this class it was clear the teacher's effort in order to promote the positive self-esteem of J and to make her feel more confident. Her aim, according to the features of collaborative work, inclusive education and this particular didactic contract, was to be able to have the role of more competent peer assumed by each one of them, in different mathematical tasks, or even in different moments/steps of their solving strategies. But when one of the students usually performed much better than the other in previous school years, achieving this point takes time and needs a lot of knowledge about how to act from the teacher's point of view.

J considers N's resolution "A little confusing!" (Turn 2). Thus, N tries to realise what J did not understand. Then, he tries to explain J what she did not understand (Turn 5). But he is not very clear in his explanation. He realises that J is still confused and thus he asks for their teacher's help, trying to legitimate his reasoning (Turn 6). According to the didactic contract, their teacher does not answer him. Instead, she asks J's opinion about N's reasoning (Turn 7) and tries to promote a dialogical interaction between these students. The teacher assumes the role of a mediator of learning (Vygotsky, 1932/1978). She is more concerned with students understanding and with the interaction between them than just with the validation of students' answers. Their teacher's reaction illuminates how the expert other can facilitate students – in this case, J's – change from a peripheral to a legitimate participation (César, 2007, 2009; Lave & Wenger, 1991). As we stressed in other cases we analysed in other papers, this is an essential move in order to promote more inclusive formal educational settings, and an intercultural education (for more details see César, 2007, 2009; César & Santos, 2006; Teles, 2005).

8 N: It is 20…

9 T: But, I don't want that answer! [Points to Question 1] Well… explain! I said that we'll correct Question 1. So, I want you to explain me why you wrote this and…

10 N: 36€. 36€ is the double of batik that cost 18€. Ms. Bela's batik cost 18€.

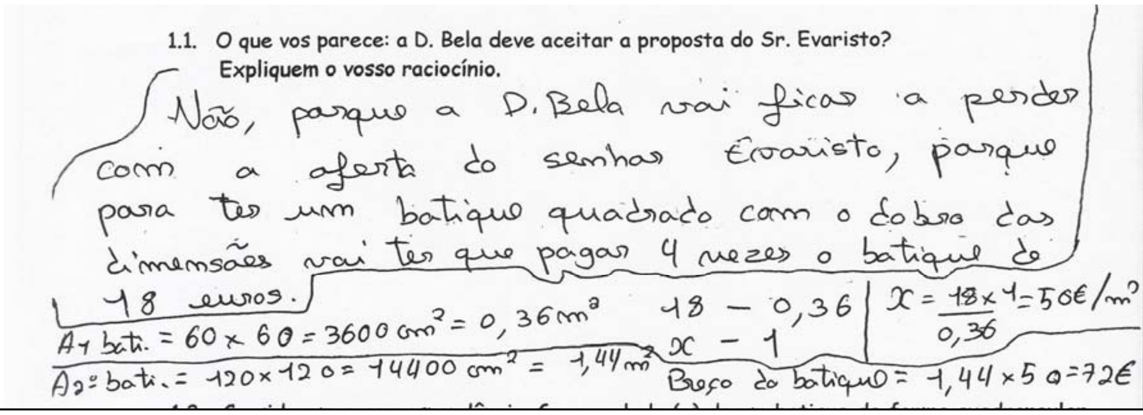
11 T: It measures 60cm in this side.

12 N: It is 60cm of side but we want the double of this batik…

13 T: You want a batik with the double of these dimensions [she points at each side of the batik].

14 N: Yes. Yes.

15 J: So, it is the double of this one.


1.1. O que vos parece: a D. Bela deve aceitar a proposta do Sr. Evaristo?
Expliquem o vosso raciocínio.

Não, porque a D. Bela vai ficar a perder com a oferta do senhor Evaristo, porque para ter um batique quadrado com o dobro das dimensões vai ter que pagar 4 vezes o batique de 18 euros.

A1 bati. = 60 x 60 = 3600 cm² = 0,36 m²   18 — 0,36   x = 18×1/0,36 = 50€/m²
x — 1
A2 bati. = 120 x 120 = 14400 cm² = 1,44 m²   Preço do batique = 1,44 x 50 = 72€

No, because Mrs. Bela would loose money with Mr. Evaristo's offer. Because in order to have a square batik with the double of the dimensions of the first one, he has to pay 4 times more, i.e., four times 18€.

Figure 3: J and N resolution (Question 1.1.) and students' answer translation

N starts the interaction with their teacher again (Turn 8), and explains the solving strategy they used to answer to Question 1. He answers the teacher's questions, but J also participates in this dialogue and concludes N's argumentation (Turn 15). But another interesting feature appears in Turn 9: these students, although engaged in solving the task, were not answering to the part their teacher had asked to be solved. This illuminates the importance of the teacher's role during classes, even when students are working in an autonomous way, it is only by observing closely what is going on that the teacher can help students to learn how to self-regulate their work in a more adequate way. In the excerpt, we understand that both students know the solving strategy they used and they can explain it because they co-constructed it together, according to the rules of the didactic contract (César, 2007, 2009; César & Santos, 2006; Teles, 2005). But in order to understand their different solving strategies students also need to establish an intersubjectivity that allows them to understand each other's arguments and solving strategies (Valsiner, 1997; Wertsch, 1991), as illuminated in the following piece:

16 T: Is it?

17 N: It is the same as we have another batik here, together.

18 T: Is it? I didn't think like this! Put two batiks together and confirm if it is a batik with 120cm of side.

19 J: We did 18x2.

20 T: I understood! But, I'm asking you if this is correct!?

21 N: Maybe!

22 T: Maybe? So, imagine that this is a batik. And you have another batik here …

23 J: It has 120cm of side.

24 T: Here [she points in their sheet of answers].

25 J: Yes.

26 T: And here? [she points again]

27 J: It doesn't. It is 60.

28 T: Ah… I want a square batik! 120 per 120. But, if you put two batiks together it has 120 per 60. Ah! Why? I said that I want the double of dimensions. The first one had 60 per 60 and this one has to have 120 per 120. Right?

An interesting point here is their teacher's care to avoid any evaluative comments on their work. She asks challenging questions as she seeks to encourage the students to realise their mistake (Turns 18, 20, 22, 24, 26, and 28). Their teacher wants these students to question themselves about what they did. Thus, she chooses to ask them questions and to pretend she does not understand what they did and why they did it this way (Turns 18 and 20). But her tone of voice is a kind one, she smiles from time to time, the interaction has an easy-going mood, and students, although paying attention, also have a smiling face.

As we can observe, J participates actively in this discussion, in spite of her usual introverted mood and her lack of confidence in her competencies (Turns 19, 23, 25 and 27). She believes on what she did with N.

29 J: Right! It is impossible!

30 T: Impossible!?

31 N: The teacher wants the double of this one. So, we have to add… we have to divide batik for all sides!?

32 J: What!?

33 T: To divide batik for all sides!? I don't understand.

34 J: I don't understand it either.

35 N: I don't understand it too.

J does not understand what their teacher told them, and thus she considers this problem impossible (Turn 29). Her attitude illuminates her lack of confidence and persistence in the activity, when she fails. This situation makes their teacher look for other alternative ways to promote students' interest and increase their positive academic self-esteem.

36 T: Let's think a little bit more. You are saying that … I think that you already understood that if you put another batik here… the other is the double, isn't it?...

37 J: If we put here (down side), it is not enough. It isn't 120.

38 T: [We can't understand]

39 J: But, here (down side) is not enough. It is 60.

40 T: And? You are about to have a square.

41 N: It is a square.

42 T: In the question they say that it is a square after we cut the batik. Think a little bit more.

Facing students' doubts and this impasse, their teacher decides to change the direction of the resolution because she wants them to go on trying to solve this problem. But, she starts from what she believes the students already understood (Turn 36). J's interest seems to increase during this interaction. She participates actively in the discussion. But, even more important, she goes on trying to solve this task when the teacher goes away again. Thus, although this episode ends without a resolution, students' discussion around that question continued. During the general discussion (whole group discussion) J went to the blackboard and was able to explain to their colleagues their solving strategy. She did it in a convincing way, explaining their solving strategy clearly and she was even able to answer to two colleagues doubts. Thus, J showed different I-positioning as mathematics student during this resolution. Basically, she passed from a non-confident I-positioning, typical of a low achieving student, to a confident I-positioning, that let her be considered a competent peer in the resolution of this task.

FINAL REMARKS

To get students' engagement a teacher needs some effort and creativity. Students' access to the rules of the didactic contract can help them understanding their role in that particular classroom and at school. It also facilitates facing the academic tasks in a confident and responsible way. As we could observe both N and J knew the rules of the didactic contract. They discussed their reasoning to find a consensus and they asked for their teacher's help only when they couldn't solve an impasse.

The teacher's role is another important feature. In this episode we could observe a teacher that assumes a mediating role. She did not tell students the right answer. She helped them to realise their mistake and she gave them assistance in order to facilitate their progress in their solving strategy. This teacher believed in the students' competencies and she aimed at facilitating the mobilisation and development of other students' competencies.

The nature of the task is another relevant feature to achieving students' engagement. In this episode the task was about batiks, which students elaborated in previous classes. The social marking of the task helped students' understanding of the task. As they elaborated batiks, they knew the process of elaboration and they were able to give a meaning to this mathematical task. Thus, the social marking of the task facilitated students' learning processes and also their knowledge transition from one situation (elaborating batiks) to another (mathematics class, solving problems).

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THE ROLE OF ETHNOMATHEMATICS WITHIN MATHEMATICS EDUCATION

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Abstract

This paper considers the field of enquiry called ethnomathematics and its role within mathematics education. We elaborate on the shifted meaning of 'ethnomathematics'. This "enriched meaning" impacts on the philosophy of math education. Currently, the concept is no longer reserved for 'nonliterate' people, but also includes diverse mathematical practices within western classrooms. Consequently, maths teachers are challenged to handle people's cultural diversity occurring within every classroom setting. Ethnomathematics has clearly gained a prominent role, within Western curricula, becoming meaningful in the exploration of various aspects of mathematical literacy. We discuss this enriched meaning of ethnomathematics as an alternative, implicit philosophy of school mathematical practices.

Key-words: Ethnomathematics, Diversity, Politics, Philosophy, Values.

INTRODUCTION

Until the early 1980s, the notion 'ethnomathematics' was reserved for the mathematical practices of 'nonliterate' – formerly labeled as 'primitive' – peoples (Ascher & Ascher, 1997). What was needed was a detailed analysis of the sophisticated mathematical ideas within ethnomathematics, which it was claimed were related to and as complex as those of modern, 'Western' mathematics. D'Ambrosio (1997), who became the "intellectual father" of the ethnomathematics program proposed "a broader concept of 'ethno', to include all culturally identifiable groups with their jargons, codes, symbols, myths, and even specific ways of reasoning and inferring". Currently, as a result of this change within the ethnomathematics discipline, scientists collect empirical data about the mathematical practices of culturally differentiated groups, literate or not. The label 'ethno' should thus no longer be understood as referring to the exotic or as being connected with race. This changed and enriched meaning of the concept 'ethnomathematics' has had its impact on the philosophy of mathematics education. From now on, ethnomathematics became meaningful in every classroom since multicultural classroom settings are generalized all over the world. Every classroom nowadays is characterized by (ethnical, linguistic, gender, social, cultural …) diversity. Teachers in general but also math teachers have to deal with the existing cultural diversity since mathematics is defined as human and cultural knowledge as any other field of knowledge (Bishop 2002). The shifted meaning of ethnomathematics into a broader concept of cultural diversity became meaningful within the community of researchers working on the topic of ethnomathematics, multicultural education and cultural diversity. Where the topic was absent at the first two conferences of the Conference of European Research in Mathematics Education (CERME 1, 1998; CERME 2, 2001), the topic appeared at CERME 3 (2003) as *Teaching and learning mathematics in multicultural classrooms*. At CERME 4 (2005) and CERME 5 (2007) the working group was called *Mathematics education in multicultural settings*. At CERME 6 (2009) the working group was called *Cultural diversity and mathematics education.* From now on, there is an explicit consideration to the notion of cultural diversity.

DEALING WITH CULTURAL DIVERSITY IN THE CLASSROOM

Ethnomathematics applied in education had a Brazilian origin, but it eventually became common practice all over the world. It has been extended from an exotic interpretation to a way of intercultural learning that is applicable within any learning context. Dealing with cultural diversity in the classroom is the universal context within which each specific context has its place.

The meaning of the *ethno* concept has been extended throughout its evolution. It has been viewed as an ethnical group, a national group, a racial group, a professional group, a group with a philosophical or ideological basis, a socio-cultural group and a group that is based on gender or sexual identity (Powell 2002, p.19). This list could still be completed but since lists will always be deficient, all the more because some distinctions are relevant only in a specific context, we use the all-embracing concept of *cultural diversity*. With respect to the field of mathematics, and in line with Bishop's (2002) consideration on mathematics as human and cultural knowledge, there appears to be a change in the meaning of ethnomathematics as diversity within mathematics and within mathematical practices. This view enables us to see the comparative culture studies regarding mathematics that describe the different mathematical practices, not only as revealing the diversity of mathematical practices but also to emphasize the complexity of each system. In addition there is interest in the way that these mathematical practices arise and how they are used in the everyday life of people who live and survive within a well-defined socio-cultural and historical context. Consequently there has to be a translation of this study to mathematics education where the teacher is challenged to introduce the cultural diversity of pupil's mathematical practices in the curriculum since pupils also use mathematical practices in their everyday life.

This application exceeds the mere introduction in class of the study of new cultures or – to put it dynamically – new culture fields (Pinxten 1994, p.14). These are the first 'ethno mathematical' moves that were made, even before dealing with cultural diversity arose. Diversity within mathematical practices was considered as a practise of the 'other', the 'exotic'. It was not considered relevant to mathematics pupils from a westernised culture. That is why the examples regarding mathematics (and adjacent sciences) are an enquiry of all kinds of exotic traditions such as sand drawings from

Africa, music from Brazil, games such as Patience the way it is played in Madagascar, the arithmetic system of the Incas or the Egyptians, the weaving of baskets or carpets, the Mayan calendar, the production of dyes out of natural substances, drinking tea and keeping tea warm in China, water collection in the Kalahari desert, the construction of Indian arrows, terrace cultivation in China, the baking of clay bricks in Africa, the construction of African houses. The examples are endless (Bazin & Tamez 2002). Notwithstanding the good intentions of these and similar projects, referring to Powell & Frankenstein (1997) we would like to emphasize that these initiatives may well turn into some kind of folklore while originally intending to offer intercultural education.

> However, we also stress that we are not advocating the curricular use of other people's ethnomathematical knowledge in a simplistic way, as a kind of "folkloristic" five-minute introduction to the "real" mathematics lessons. (Powell & Frankenstein 1997, p.254)

In line with the empirical research by Pinxten & François (2007) on mathematical practices in classroom settings, one can prove many appropriate examples that pupils' mathematical practices may be used in class, not as some kind of exoticism but as the utilization of a mathematical concept. Starting from pupils' mathematical knowledge and their everyday mathematical practices is a basic principal of the new orientation towards realistic mathematics education and the development of innovative classroom practices (Prediger 2007). The question remains how one can move from a teacher centered learning process towards a pupil centered learning process where pupils' mathematical practices can enter the classroom. Cohen & Lotan (1997) describe how interactive working can be structured and they also explain the benefits of interactive learning in groups to *deal with diversity*. For that purpose the *Complex Instruction* theory was developed which they implemented in education. Meanwhile this didactic has had an international breakthrough in Europe, Israel and the United States and it has been elaborated to the didactic of *Cooperative Learning in Multicultural Groups* (*CLIM*) (Cohen 1997: vii). This teaching method has been tested in a number of settings, in distinct age groups and with regard to different curricula (Cohen 1997, Neves 1997, Ben-Ari 1997). Besides the acquisition of mathematical contents was part of this. *Complex Instruction* is a teaching method with equality of all pupils as its main objective. This teaching method tries to reach all children and tries to involve them in the learning process, irrespective of their diversity (François & Bracke 2006). In order not to peg cultural diversity down to a specific kind of diversity Cohen (1997) in this context speaks of working in heterogeneous groups. Heterogeneity can be found in every group structure. Even a classroom is characterized by a diverse group of pupils where every pupil has in some way his or her everyday mathematical practices. If pupil centered learning is taken seriously, teachers are challenged to deal with this present mathematical practices while teaching mathematics. In this way, ethnomathematics became a way of teaching mathematics where cultural diversity of pupils' everyday mathematical practices art taken into account (François 2007).

ETHNOMATHEMATICS IN EVERY CLASSROOM

The extended notion ethnomathematics as dealing with pupils' everyday mathematical practices has equality of all pupils as its main objective. Ethnomathematics becomes a philosophy of mathematics education where mathematical literacy is a basic right of all pupils. The teaching process tries to reach all pupils and tries to involve them in the learning process of mathematics, irrespective of their cultural diversity. All pupils are equal. This notion of mathematics for everyone fits in with the ethical concept of *pedagogic optimism* that is connected with the theory of *egalitarianism*. This ethical-theoretical foundation on which the project of equality within education is based, assumes that the equality is measured at the end of the line. As reported by the justice theories of John Rawls (1999) and Amartya Sen (1992) pupils' starting positions can be dissimilar in such a way that a strictly equal deal will prove insufficient to achieve equality. A *meritocratic* position – which measures the equality at the start of the process – thus cannot fully guarantee equal chances (Hirtt, Nicaise & De Zutter 2007). An *egalitarian* position starts from a pedagogic optimism and it needs to take into account the diversity of those learning in order to give equality maximum chances at the end of the line.

By extending the notion ethnomathematics to cultural diversity and mathematics education, the distinction between mathematics and ethnomathematics seems to disappear. Hence the critical question can be raised whether the achievements of ethnomathematics will not become lost then. On the contrary the distinction between ethnomathematics and mathematics can only disappear by acknowledging and implementing the ethnomathematics' achievements in the mathematics education. The issue on the distinction between ethnomathematics and mathematics has been raised before within the theory development of ethnomathematics (Setati 2002). Being critical on the dominant Western mathematics was the basis out of which ethnomathematics has developed and now the time is right to raise the critical questions also internally, within the field of ethnomathematics itself. What exactly distinguishes ethnomathematics from mathematics? Setati raises this question in a critical review on the developments within the ethnomathematics as a theoretical discipline that dissociates and distinguishes from mathematics (Setati 2002). Setati sees mathematics as a mathematical practice, performed by a cultural group that identifies itself based on a philosophical and ideological perspective (Setati 2002). Every maths teacher is supposed to use a series of standards that are connected with the profession and with obtaining the qualification. The standards are philosophical (about the way of being), ideological (about the way of perceiving) and argumentative (about the way of expressing). Both mathematics and ethnomathematics are embedded in a normative framework. So the question can be raised as to whether the values of mathematics and ethnomathematics indeed are that distinctive.

It cannot be denied that ethnomathematics was based on an emancipatory and critical attitude that promotes the emancipation and equality of discriminated groups (Powell & Frankenstein 1997). This general idea of emancipation can also be found in the UNESCO's view on education. Moreover we see in its mission a tight connection with the socio-economic development, with working on an enduring and peaceful world, while respecting diversity and maintaining human rights. Education here is obviously connected with the political factor.

> UNESCO believes that education is key to social and economic development. We work for a sustainable world with just societies that value knowledge, promote a culture of peace, celebrate diversity and defend human rights, achieved by providing education for all. The mission of the UNESCO Education Sector is to provide international leadership for creating learning societies with educational opportunities for all populations; provide expertise and foster partnerships to strengthen national educational leadership and the capacity of countries to offer quality education for all. (UNESCO 1948)

Taking into account these general stipulations we have to conclude that the explicit values of the general education objective connect to the values of equal chances for all pupils which are central within ethnomathematics. Consequently the expansion of ethnomathematics as a way of teaching mathematics which takes the diversity of pupils' mathematical practices into account can be justified. There is a kind of inequality in every group and the real art is to learn to detect the skins of inequality and the skins of cultural diversity. Instead of a depreciation of the concept 'ethnomathematics' this extended notion could mean a surplus value in situations where heterogeneity and cultural diversity are less conspicuous.

Within ethnomathematics education two aspects are highlighted. First there is the curriculum's *content*. Often this is the first step when implementing ethnomathematics. Besides the mathematics that can be found in the traditional curriculum, there will now be additional space to be introduced to more exotic or traditional mathematics practices. Powell & Frankenstein (1997) also emphasize this aspect in their definition of the enrichment of a curriculum through ethnomathematics. Stressing other mathematical practices offers the opportunity to gain a better perception in the own mathematical practice and its role and place in society (D'Ambrosio 2007a). It also offers the opportunity to philosophize and critically reflect on the own mathematical practice. In language teaching it goes without saying that it is better to learn more than one language. It broadens the outlook on the world and offers a better adaptation to dealing with other people in this globalized world. Knowledge of several languages is undoubtedly an advantage and besides it broadens the knowledge of the mother tongue. This comparison could even be extended to the mathematics education where knowledge of mathematical practices of several cultural contexts and throughout time proves to be advantageous. A second aspect within ethnomathematics is the *didactics*, the way that the learning process is set up. Here an interactive approach is crucial (Cohen 1997, César 2009). The two aspects obviously have mutual grounds. An interactive approach results in contents being defined also by the learning with an active participation in the learning process. This aspect is strongly emphasized by researchers who investigate the integration of so-called traditional groups within the academic context. This is expressed as one of Graham's key questions in his enquiry into mathematics education for aboriginal children: *what do the children bring to school?* (Graham 1988, p.121). With the extended notion ethnomathematics as cultural diversity and mathematics education and with the emphasis on dealing with pupils' everyday mathematical practices, ethnomathematical practice is now closer to the social environment of the pupil and unlinked it from its original (exotic) cradle. Both the theory and practice of ethnomathematics have opened up the eyes and broadened the minds. It immediately answers the question as to what exactly could be of benefit to the highly-educated countries – with their outstanding results in international comparative investigations – regarding ethnomathematics as it originally developed, as a critical and emancipation theory and as a movement that aimed to give all pupils equal chances. In a final section about ethnomathematics we would like to link up mathematics education and politics.

ETHNOMATHEMATICS AS HUMAN RIGHT

D'Ambrosio, who is the mathematician and educationalist of the mathematics on which ethnomathematics is based, situates mathematics education within a social, cultural and historical context. He can also be considered the first to explicitly link mathematics education and politics. Mathematics education is a lever for the development of the individual, national and global well-being (D'Ambrosio 2007a, 2007b). In other words the teaching and learning of mathematics is a mathematical practice with obviously a political grounding. D'Ambrosio advances the political proposition that mathematics education should be accessible to all pupils and not only to the privileged few. This proposition has been registered in the OECD/PISA report, which is the basis for the PISA-2003 continuation enquiry.

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 2004, p.37)

This specification of mathematical literacy clearly implies that this form of literacy is a basic right for every child, such that it gets a chance to participate to the world in a full, constructive, relevant and thoughtful way. We will see this proposition recurs later in the essays of Alan J. Bishop (2006) where he demonstrates the link between mathematics, ethnomathematics, values and politics.

Mentioning mathematics education and education of values in one and the same breath does not sound unambiguous because mathematics is undeniably being perceived as non-normative.

> It is a widespread misunderstanding that mathematics is the most value-free of all school subjects, not just among teachers but also among parents, university mathematicians and

employers. In reality, mathematics is just as much human and cultural knowledge as any other field of knowledge, teachers inevitably teach values […]. (Bishop 2002, p.228)

It is predominantly within D'Ambrosio's' ethnomathematics research program that the link of mathematics and mathematics education with values is extended to the political domain, not in the least with the intellectual father of ethnomathematics. According to D'Ambrosio still too many people are convinced that mathematics education and politics have nothing in common (D'Ambrosio 2007a). He will take the edge of this cliché. In his recent work D'Ambrosio (2007a, 2007b) departs from the Universal Declaration of Human Rights where articles 26 and 27 highlight the right to education and to share in scientific advancements and their benefits.[1] This declaration concerning education is further developed and confirmed within the UNESCO's activities by means of the *World Declaration on Education for All* in 1990 and ratified by 155 countries. Finally the declaration has been applied in mathematical literacy in the OECD/PISA declaration of 2003. D'Ambrosio regrets that these declarations are not well-known by maths teachers since they play a key role in the emancipation process. In line with the World Declaration, 'mathematics education for all' implies a critical reflective way of teaching mathematics. According to D'Ambrosio, this way of teaching does not receive sufficient opportunities. Following Bishop (1997) he criticizes the technically-oriented curriculum with its emphasis on technique and drill and where history, philosophy and critical reflection are not given a chance. D'Ambrosio develops three concepts to focus on in a new curriculum regarding the usage of the international (UNESCO) emancipatory objectives - *literacy*, *matheracy* and *technoracy*.

Literacy has to do with communicative values and it is an opportunity to contain and use information. Here both spoken and written language is concerned but so are symbols and meanings, codes and numbers. Mathematical literacy is undoubtedly a part of it. *Matheracy* is a tool that offers the chance to deduce, to develop hypotheses and to draw conclusions from data. These are the base points for an analytical and scientific attitude. Finally there is *Technoracy* which offers the opportunity to become familiar with technology. This does not imply that every pupil should or even could get an understanding of the technological developments. This elementary form of education needs to guarantee that every user of a technology should get to know at least the basic principles, the possibilities and the risks in order to deal with this technology in a sensible way or deal not at all with it.

With these three forms of elementary education, which can be developed throughout the ethnomathematics research program, D'Ambrosio wants to meet the Universal Declaration of the Human Rights that relate to the right to education and the right to the benefits of the scientific developments.

CONCLUSION

This paper considered the shifted meaning of ethnomathematics and its role within mathematics education. Ethnomathematics is not longer reserved for so-called nonliterate people; it now refers to the cultural diversity in mathematics education. Math teachers are therefore challenged to handle pupils' diverse everyday mathematical practices. In line with the UNESCO declaration (1948) on education and the OEDC declaration (2004) on mathematical literacy, ethnomathematics clearly gained a more prominent role. Within Western curricula, ethnomathematics became meaningful to explore as an alternative, implicit philosophy of school mathematical practices. The extended notion of ethnomathematics as dealing with pupils' cultural diversity and with their everyday mathematical practices brings mathematics closer to the social environment of the pupil. Ethnomathematics is an implicitly value-driven program and practice on mathematics and mathematics education. It is based on an emancipatory and critical attitude that promotes emancipation and equality (Powell & Frankenstein 1997). Where the so-called academic Western mathematics still is locked in the debate on whether it is impartial or value-driven, the ethnomathematics' purposes stand out clearly right from the start. The historian of mathematics Dirk Struik postulated the importance of ethnomathematics. He validates ethnomathematics as both an academic and political program. There mathematics is connected to its cultural origin as education is with social justice (Powell & Frankenstein (1999). D'Ambrosio even puts it more sharply: *Yes, ethnomathematics is political correctness* (D'Ambrosio 2007a, p.32).

The implication for research is threefold. First, research has to reveal the (explicit and implicit) values within mathematics, mathematical practices and mathematics education. Second, research has to investigate thoroughly the use and integration of pupils' mathematical practices in the curriculum. Third, pupils' daily mathematical practices have to be studied.

NOTES

1. Article 26. (1) Everyone has the right to education. Education shall be free, at least in the elementary and fundamental stages. Elementary education shall be compulsory. Technical and professional education shall be made generally available and higher education shall be equally accessible to all on the basis of merit. (2) Education shall be directed to the full development of the human personality and to the strengthening of respect for human rights and fundamental freedoms. It shall promote understanding, tolerance and friendship among all nations, racial or religious groups, and shall further the activities of the United Nations for the maintenance of peace. (3) Parents have a prior right to choose the kind of education that shall be given to their children. Article 27. (1) Everyone has the right freely to participate in the cultural life of the community, to enjoy the arts and to share in scientific advancement and its benefits. (2) Everyone has the right to the protection of the moral and material interests resulting from any scientific, literary or artistic production of which he is the author. (United Nations Educational, Scientific and Cultural Organization. 1948)

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