STUDENTS' UTILIZATION SCHEMES OF PANTOGRAPHS FOR GEOMETRICAL TRANSFORMATIONS: A FIRST CLASSIFICATION♦

Francesca Martignone*, Samuele Antonini**

* Department of Mathematics, University of Modena e Reggio Emilia

** Department of Mathematics, University of Pavia

The activities with the Mathematical Machines are very rich from educational and cognitive points of view. In particular, the use of pantographs has revealed educational potentialities for the acquisition of some important mathematical concepts and for the emergence of argumentation and proving processes, at any school level. In this paper, we propose a cognitive analysis of the processes involved in the manipulation of the mathematical machines, providing a first classification of utilization schemes of pantographs for geometrical transformations. This classification can be efficiently used to observe, describe and analyse cognitive processes involved in the exploration of mathematical properties incorporated in the machines.

Keywords: Mathematical Machines, utilization schemes, pantographs, geometrical transformations and cognitive processes.

INTRODUCTION

The Mathematical Machines Laboratory (MMLab: www.mmlab.unimore.it), at the Department of Mathematics in Modena (Italy), is a research centre for the teaching and learning of mathematics by means of instruments (Ayres, 2005; Maschietto, 2005). The name comes from the Mathematical Machines (working reconstruction of many mathematical instruments taken from the history of mathematics), the most important collection of the Laboratory. These machines concern geometry or arithmetic:

> "a geometrical machine is a tool that forces a point to follow a trajectory or to be transformed according to a given law"…"an arithmetical machine is a tool that allows the user to perform at least one of the following actions: counting; making calculations; representing numbers" (Bartolini Bussi & Maschietto, 2008).

The MMLab research group carried out various activities with the Mathematical Machines, namely: laboratory sessions in the MMLab, long-term teaching experiments in classrooms, workshops at national and international conferences and also exhibitions (see chapters 2 and 5 of the forthcoming volume by Barbeau and

♦ Study realized within the project PRIN 2007B2M4EK (Instruments and representations in the teaching and learning of mathematics: theory and practice), jointly funded by MIUR, by University of Modena e Reggio Emilia and by University of Siena.

Taylor, from ICMI Study n. 16) in collaboration with the members of the association “Macchine Matematiche” (http://associazioni.monet.modena.it/macmatem).

The laboratory sessions in the MMLab are designed in order to offer hands-on activities with mathematical machines for classes of students in secondary schools (an average of 1300-1500 Italian secondary students a year come with their mathematics teacher to experience hands-on mathematics laboratory), groups of university students, prospective and practicing school teachers (Bartolini Bussi & Maschietto, 2008). As the Mathematical Machines activities in school classrooms concerns, the MMLab research group organized different long-term teaching experiments in primary and secondary schools (Bartolini Bussi & Pergola, 1996; Bartolini Bussi, 2005; Bartolini Bussi, M. G., Mariotti M. A., Ferri F., 2005, Maschietto & Martignone, 2007).

All the activities quoted above are based on two fundamental components: the idea of the “mathematics laboratory”[1] and the didactical research on the use of tools in the teaching and learning of mathematics (Bartolini Bussi & Mariotti, 2007).

The MMLab researches aim at the development of different activities that should foster, through the use of the mathematical machines, the acquisition of some important mathematical concepts and the emergence of argumentation processes.

In order to implement the studies on MMLab laboratory activities, and to set up new teaching experiments, we consider important to carry out a cognitive analysis of the processes involved in the manipulation of the Mathematical Machines. The aim of our research is identifying Mathematical Machines utilization schemes and the connected exploration processes, providing a first classification. In the paper we shall present the first steps of this new research.

THEORETICAL FRAMEWORK

According to the educational goals that the activities with Mathematical Machines intend to realize, we investigate students cognitive processes involved in exploration of open-ended problems (in particular the problem of identifying the geometrical laws that explain how a machine works), in generation of conjectures and argumentations and in concept formation (for example: the concepts of geometrical transformations, of conic, of central perspective…). First of all, to analyse deeply these processes we propose a classification of Mathematical Machine utilization schemes [2]. This classification is suitable not only for describing the interactions between machines and subjects but also for analysing both their exploration and argumentative processes.

The processes through which a subject interacts with a machine have been studied by Rabardel in cognitive ergonomics: he grounded his research in constructivist epistemologies, primarily in *activity theories*, but also in the Piagetian and post-Piagetian developmental approach to the cognition-action dialectic (Rabardel, 1995; Béguin & Rabardel, 2000).

Rabardel proposed an original approach blending anthropocentric and technocentric approaches: as a matter of fact, in line with activity theory, he conceived the instruments as psychological and social realities and studied the instrument-mediated activity. According to Rabardel (1995) an *instrument* (to be distinguished from the material -or symbolic- object, the *artefact*) is defined as a hybrid entity made up of both artefact-type components and schematic components that are called *utilization schemes*.

> "What we propose to call " utilization scheme" (Rabardel, 1995) is an active structure into which past experiences are incorporated and organized, in such a way that it becomes a reference for interpreting new data" (Béguin & Rabardel, 2000)

An artefact only becomes an instrument through the subject's activity. This long and complex process (named *instrumental genesis*) can be articulated into two coordinated processes: *instrumentalisation*, concerning the individuation and the evolution of the different components of the artefact, drawing on the progressive recognition of its potentialities and constraints; *instrumentation*, concerning the elaboration and development of the utilization schemes (Béguin & Rabardel, 2000).

For the importance of these schemes, for their specificity in interacting with Mathematical Machine and for the limits that this paper has to respect, we focus here on utilization schemes in the case of pantographs.

METHODOLOGY

The method used for investigation was the clinical interview: subjects were asked to explore a machine and to express their thinking process aloud at the same time. In particular, after having explained to the student that the machines to be explored are pantographs for geometric transformations, we asked:
1. To define the mathematical law made locally by the articulated system.
2. In particular, to justify how the machine "forces a point to follow a trajectory or to be transformed according to a given law" and then to prove the existing relationship between the machine properties (structure, working…) and the mathematical law implemented.

The interviews were videotaped and the analysis is mainly based on the transcripts of the interviews. The interviews were analysed with special attention to verbal tracks and hands-on activities in order to detect mental processes developing during the exploration of the machines. Every protocol is analysed in a double perspective: as bearer of new information about possible exploration processes and as evidence for the existence of recurrent schemes.

The subjects were three pre-service teachers, two university students and one young researcher in mathematics. The choice to interview subjects which are familiar with (Euclidean) geometry and with problem-solving has allowed us to collect observations of complete machine exploration: namely, the generation of conjecture about the mathematical law implemented by the machine and, subsequently, argumentation and proof of mathematical statements that can explain the functioning of the machine. Moreover, the subjects were new in working with this environment: in this way we could assume that they did not have an a priori specific knowledge about these machines.

The artefacts selected for this first research are machines concerning geometry, in particular pantographs: for the axial symmetry, for the central symmetry, for the translation, for the homothety and for the rotation. These machines establish a local correspondence between points of limited plan regions connecting them physically by an articulated system; they were built to incorporate some mathematical properties in such a way as to allow the implementation of a geometrical transformation (i.e. axial symmetry, central, translation, homothety, rotation).

CLASSIFICATION OF THE UTILIZATION SCHEMES

In this paper we present the first part of our research that aimed to introduce a classification of utilization schemes observed during the explorations of pantographs for geometrical transformations. The identified utilization schemes were divided into two large families: utilization schemes linked to the components of the articulated system (as the constraints, the measure of rods, the geometrical figures representing a configuration of rods, etc.) and utilization schemes linked to the machine movements. As regards the first family, we have identified the following utilization schemes: the research of fixed points, movable points (with different degrees of freedom), plotter points and straight path; the measure of rods length; the research of geometric figures representing the articulated system or some part of it; the construction of geometric figures that extend the articulated system components; the individuation of relationships between the recognized geometric figures; the analysis of the machine drawings.

As regards the utilization schemes linked to the machine movements [3], we distinguish between the movements aimed at finding particular configurations obtained stopping the action in specific moments and the continuous movements aimed to analyse invariants or changes. We summarize this classification in a table:

Linkage Movement that stops in	**Movements description:**
Generic Configurations	Movement that stops in a configuration which is considered representative of all configurations observed (that does not have "too special" features)

Particular Configurations	Movement that stops in a configuration that presents special features (i.e. right angles, rods positions…)
Limit Configurations	Movement that stops in configurations in which the geometric figures that represent the articulated system become degenerate
Limit zones	Movement that stops in the machine limit zones: i.e. the reachable plane points

Linkage Continuous movements	**Movements description:**
Wandering movement	Moving the articulated system randomly, without following a particular trajectory
Bounded movement (For example: Movements by fixing one point or one rod…)	Moving the articulated system, blocking particular points or rods
Guided movement	Moving the articulated system, forcing a point to follow a line or a specific figure
Movement of a particular configuration	Moving the articulated system, maintaining a particular configuration
Movements between limit configurations	Moving the articulated system so that it can successively assume the different “limit Configurations”
Movement of dependence	Moving (in a free, guided or bound way) a particular point and see what another particular point does
Movement in the action zones	Moving the articulated system in a such a way that all the possible parts of the plane are reached

A PROTOCOL

In this paragraph we present the first part of one clinical interview transcripts dealing with the exploration phase (i.e. the beginning of the machine exploration, before the identification of the geometrical transformation made by the machine), where we can identified some of the utilization schemes described in the previous paragraph [4]. The subject of the protocol, Anna, is a pre-service teacher graduated in mathematics and she explored the pantograph of Scheiner (see Fig. 1-2).

Anna: (*she touches a rod which seems to remain blocked*) all motionless!...(*she moves the articulated system*) Ah, no, only a single fixed point … I saw that leads are useful, and then… … (*opening and closing the linkage, she draws lines that converge in the fixed point*) … then (*she turns the machine and she draws again "concentric lines"*)…

She starts controlling which part of the linkage is pivoted to the wood plane (*research of fixed points)* and then, in order to explore the linkage movements, she puts the leads in both plotter holes (*individuation of plotter points*) and draws curves produced by the linkage closing movement (*guided movements that end in a limit configuration*: see Fig. 3)

Anna: I do not see anything then………(*she is looking the motionless machine and the curves drawn*)…(*she moves the linkage and she stops in a generic configuration*) well, this is a parallelogram, I would say… That is… then, parallelogram, and in a vertex there is a lead… (*with the ruler she measures two rods: in the fig. 2 CQ and CP*)… are congruent (*she points them out)*

The analysis of the drawn curves does not seems to help her to discover what transformation the machine makes, therefore she starts an analysis of the linkage structure (*research and individuation of a generic configuration and recognition of particular geometric figures in the linkage structure*): at first she identifies a parallelogram (see Fig.4), and then she focuses on other linkage rods (those parts that do not form the parallelogram). She recognizes the parallelogram without using the ruler (probably the visual perception of congruence has been supported by the previous exploration of movements during which the rods remained parallel). Differently, to discover the other characteristics of the linkage geometric structure, Anna feels the need to *measure the rods length*, so she discovers that there are two congruent rods (CQ and CP).

Anna: … so this (*she looks at the linkage and she uses two fingers to show the "virtual segment" PQ that completes the triangle PQC:* see Fig. 5) is an isosceles triangle

The identification of these congruent rods arouses the construction of a new geometric figure (an isosceles triangle) created completing, with an imaginary segment, the sequence of the congruent rods (*extending and individuation of geometric figures in the linkage structure*).

Anna: but I will not see anything… but it doesn't say anything to me at this moment…… (*she moves the machine*, *drawing always concentric lines*) well they are always circumferences…(*she is looking at the drawings*) I do not understand if they are or not circumferences …

Also the exploration of linkage characteristics does not seem to help her, for this reason she comes back to the previous strategy: she starts again to draw lines that follow the machines closing movement (*guided movements that end in a limit* *configuration and analysis of these drawings*), but, as before, she is not aware of the drawn lines characteristics; therefore, not knowing which properties designed curves have, she can not understand how they are transformed by the machines.

Anna: (*she makes a zigzag movement*) well, but it seems to me that they trace the same thing (*she makes the zigzag movement in another area of the paper*)… (*she points the zigzag drawing and she moves away the linkage*)… the leads then trace the same, the same image, it seems to me, but I dare say that (*she makes a gesture*: see Fig.6)…that it is reduced in scale.

Anna changes the *guided movements* (zigzag movements) and, this time, the *analysis of the drawings* leads to the recognition of the transformation (the homothety). Therefore it seems that what lets Anna to do the discovery of the transformation incorporated in the machine, is the drawings analysis more than the machine structure; but not all the drawings seem to be successfully: in fact each of them gives only partial information about the transformation. In particular, for Anna is determinant the choice to change the movement (and consequently, the drawing): as a matter of fact in the zigzag lines it can be seen that the correspondent segments are modified, while the angles are not (in the previous drawings these proprieties are "hidden", while it came out the presence of a fixed point).

In conclusion, it is interesting to underline that also in a brief excerpt, it is possible to see the variety, the complexity of their relationships and, in particular, the plot of the different utilization schemes. After the individuation of the schemes, we can make a cognitive analysis of the exploration processes linked to these schemes. For example, we intend to examine closely how (and then why) Anna swings between two different strategies that remain separated (the drawing/analysis of lines and the study of linkage structure). This analysis brings important information for the understanding of subsequent processes: in fact, in the continuation of this protocol, the lack of interweaving of the information acquired through the different utilization schemes used, seems to be an obstacle in the Anna's proof construction (about how the machine incorporates the transformation properties). This part of the research is still in progress, but the first results raise the hypothesis that successful strategies are those that maintain a tension and integration between the analysis of the articulated system proprieties, the drawings and the invariants of the movement.

CONCLUDING REMARKS

The studies on the interaction between a subject and a machine have to take into account an intriguing complexity because several components are involved. From a cognitive point of view and with educational goals, in this paper, we have presented a study to better understand the exploration of some geometrical machines: in particular, we have proposed a first classification of utilization schemes of pantograph for geometrical transformations and we have shown an analysis carried out through this classification. In this analysis we have underlined the importance of the identification of the different schemes in describing the aspects of mathematical machines exploration.

Further researches are needed in two directions. On the one hand, we will study how these schemes are intertwined with the processes involved in conceptualisation, in argumentation and in proving; on the other hand, we will explore the evolution of the utilization schemes and its relationship with argumentation processes and subject's cultural resources.

Moreover, this study will be developed to offer teachers tools that could be efficient to set up activities with educational goals and to intervene in students' interactions with the machines, promoting those processes that are considered relevant for the activities with the mathematical machines.

NOTES

1. "A mathematics laboratory is a methodology, based on various and structured activities, aimed at the construction of meanings of mathematical objects. (…) The mathematics laboratory shows similarities with the concept of Renaissance workshops where apprentices learned by doing and watching what was being done, communicating with one another and with the experts" http://umi.dm.unibo.it/italiano/Didattica/ICME10.pdf.

2. In literature there are not previous cognitive studies of this type on mathematical machines. A classification of utilization schemes of instruments of different nature is proposed in Arzarello et al. (2002) where different modalities of dragging are discussed.

3. In addition to the linkage movements, there are also the movements of the machine wood base (on which the linkage is set): i.e. the rotations of the base that permit to look the machine from other points of view.

4. In these extracts there are not all the utilization schemes identified during our research. For the limit of this article we should not make an example for each of the utilization schemes previously listed.

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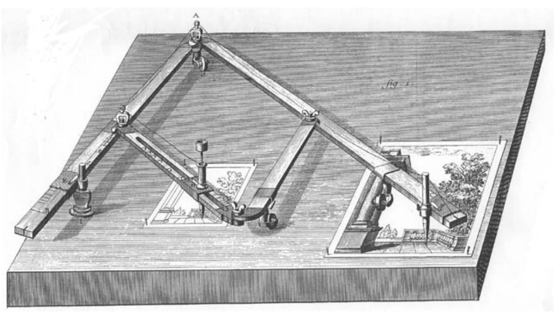


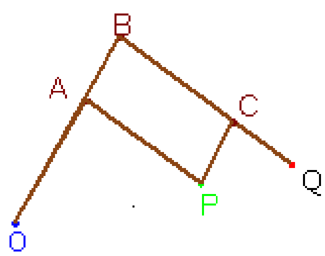
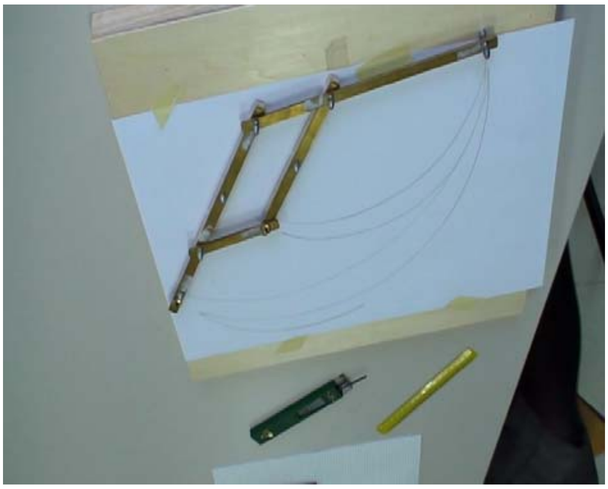


Fig 1: Encyclopédie ou dictionnaire raisonné des sciences, des arts et des metiers (1751-	**Fig 2**: An image from Scheiner pantograph graphic animation: Four bars are pivoted so

1780)	that they form a parallelogram APCB. The point O is pivoted on the plane. It is possible to prove that the points P, Q and O are in the same line and that P and Q are corresponding in the homothetic transformation of centre O and ratio BO/AO.



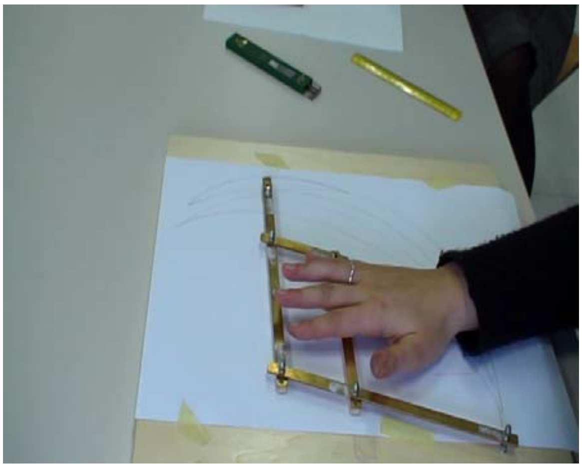
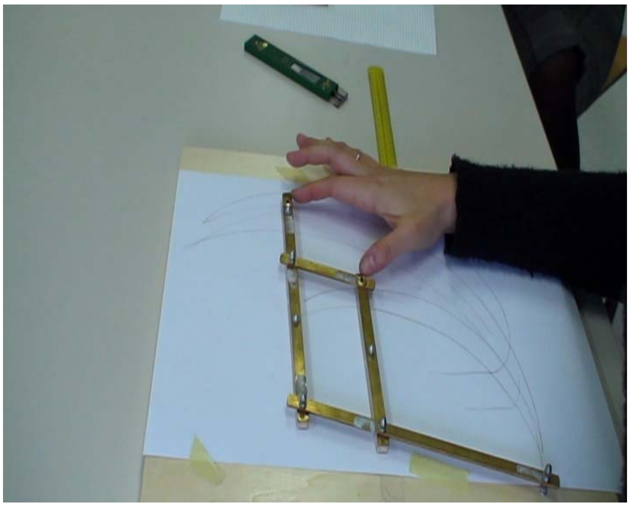


Fig. 3: Anna's drawings	**Fig. 4**: Anna identifies the parallelogram



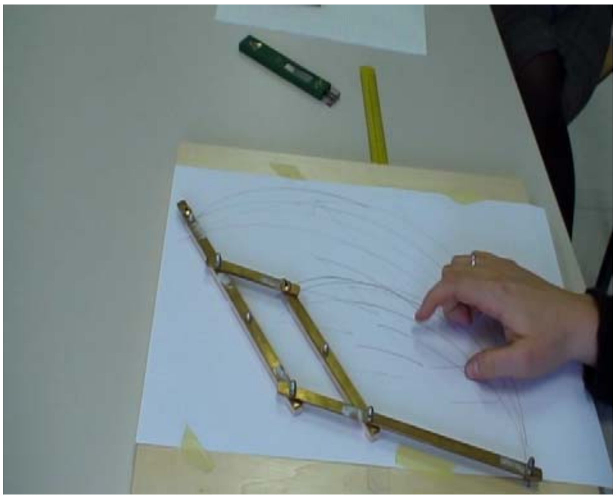


Fig. 5: Anna shows the isosceles triangle	**Fig. 6**: Anna's gesture for indicating the "reduction in scale" of the zigzag lines