THE LONG-TERM PROJECT "INTEGRATION OF SYMBOLIC CALCULATOR IN MATHEMATICS LESSONS" – THE CASE OF CALCULUS

Hans-Georg Weigand, Ewald Bichler

University of Wuerzburg

A long term project (2003 – 2011) was started to test the use of symbolic calculators (SC) in grammar schools in Bavaria (Germany). The project was firstly done in grade 10. During the 2006/07 school year the project was implemented in grade 11. 732 students at 10 Bavarian grammar schools took part in an empirical investigation. The content taught was calculus: basic properties of functions, limits, continuity, derivatives, and applications of calculus. The evaluation of the project was intended give answers to the following questions: how basic mathematical skills (algebraic transformations, solving equations) changed; how the students used the SC, how they evaluated the use of the new tool. This article presents the results of this project for school year 2006/07.

1. BACKGROUND

In the past, many empirical investigations concerning the use of CAS or symbolic calculators (with CAS) in mathematics teaching have been published (see Guin, Ruthven and Trouche, 2005). The central results of these projects have meanwhile been confirmed by other investigations world wide. The use of a CAS brings a greater meaning to work with diagrams, reinforces experimental work, in which the assumptions were obtained through systematic testing and CAS appears to bring an increase in computer cooperative forms of work. The effects are primarily long term. It is therefore necessary to develop a namely educational concept to evaluate the changes in knowledge and abilities over a longer time period. However, many investigations in this area restrict themselves to the applications of the computer over "just" a few weeks (Schneider, 2000, Drijvers, 2003, Pierce and Stacey, 2004 and Guin et al, 2005) and do not show the long-term effects on the knowledge and ability of the students.

In the school year 2003/04 we started a long term project to test the use of symbolic calculators (SC) – the TI-Voyage 200 and the TI-Nspire – in grammar schools (Gymnasien) in Bavaria (Germany). The project was done in grade 10 and has been repeated in the following two school years with a greater number of classes and with – concerning the use of new technologies – inexperienced teachers. An overview of the empirical investigation and especially of the theoretical background of this project gives Weigand (2008). On account of the positive results of this project, the Bavarian Ministry decided to continue the project. The follow-up project was started in September 2006.

2. THE TEACHING PROJECT – GRADE 11

2.1 The participants

During the 2006/07 school year the project was implemented in grade 11. A total of 732 students at 10 Bavarian grammar schools took part in this project. 412 students in 16 classes acted as the “pilot classes”, working with Voyage 200 and/or TI-Nspire. Schools could apply for the participation in the project. The pilot schools have been chosen by the Bavarian Ministry. They are spread over the state. In addition, 320 participants from 11 classes – from the same schools as the pilot classes – formed a “control group” for the purposes of quantitative statistical investigation. The students had different previous experiences; some students had been exposed to the SC in the previous grade 10, but other students came into contact with these systems for the first time during this project.

2.2 The teachers

The project was mainly taught by teachers with little experience of tuition using computer algebra systems (CAS). The project teachers held two three-day meetings during which examples of possibilities and opportunities for SC use were discussed. The teachers jointly prepared a number of suggestions for a range of teaching units intended to highlight the possibilities of using SCs; during the year, the teachers were offered additional learning units[1] by the coordinator (Ewald Bichler). However, there was no uniform overall concept according to which teaching was to be organised in all classes. The personal experience, attitudes and circumstances at the individual schools were too different for this to be possible.

2.3 The learning contents

In grade 11, calculus is taught (in Germany). The content taught was subdivided into the following:

- basic properties of functions (symmetry, monotonicity, variations in function terms and their impact on graphs, …)
- limits, continuity
- differentiability, derivation rules, derivation function(s)
- applications of differential calculus (“classical” functions discussion, extreme value problems)

2.4 Teaching methods with the SC

During the meetings with the teachers at the beginning and in the middle of the school year a theoretical frame of the use of the SC in the classroom was discussed

[1] One sort of learning units developed during the project is called “Minute Made Math”, more information on www.minute-made-math.com

with the teachers. Especially a short insight into the theory of instrumentation was presented and explained with examples (Artigue 2002, Trouche 2005).

Concerning the integration of the SC into the problem solving process we distinguished using the SC

- in the beginning of the problem solving process or a concept formation process (the SC as a "discoverer"),
- in the middle of the process (the SC as "solver") and
- at the end of the process (the SC as a "controller").

We also emphasized the "rule of three" while working with representations: If possible a problem or the solution of the problem should be represented on a symbolic, graphic and numeric level.

2.5 Research questions:

In the following we concentrate on a selection of the research questions (RQ) of the project:

RQ1. Can any differences be ascertained in terms of core mathematical abilities (substitutions, interpretation of graphs, solving equations, working with tables, and working with formulae) between the pilot and the control groups after one year?

RQ2. Can different effects of SC use be ascertained with "good", "average" and "weak" students?[2]

RQ3. To what extent have students mastered the SC at the end of the year?

RQ4. In which phases of a problem solving activity do the students use the SC?

2.6 Test instruments

For the purpose of answering the 1st and 2nd questions we took a (classical) pre- and post-test-design – the tests using paper and pencil but no calculator – in pilot and control classes.[3]

For the purpose of answering the 3rd and 4th questions the pilot classes took a test using a SC in February 2007 and June 2007 in which they were asked to record their working methods with the SC in a questionnaire which they completed immediately *after* the test.

---

2 The performance criteria used relate to the results of the pre-tests at the beginning of the school year.

3 See: www.dmuw.de/weigand/2009/CERME6/

3. EVALUATION OF PRE- AND POST-TESTS

3.1 The questions

The pre- and post-test-questions (PP-questions) can be divided into the following groups:

- Questions 1 and 2: doing "classical" simplification of terms
- Question 4 and 5: solving equations
- Question 5: understanding the concept of root functions
- Questions 6 – 8: seeing the correlation between graph and term
- Question 9: interpreting graphs

3.2 Comparison of results of pre- and post-tests

The post-test was the same as the pre-test. In the following diagram, the *differences* between the average scores achieved for each question in the pre- and post- tests for the pilot and the control group are shown. The "average performance increase" is therefore measured for each question.

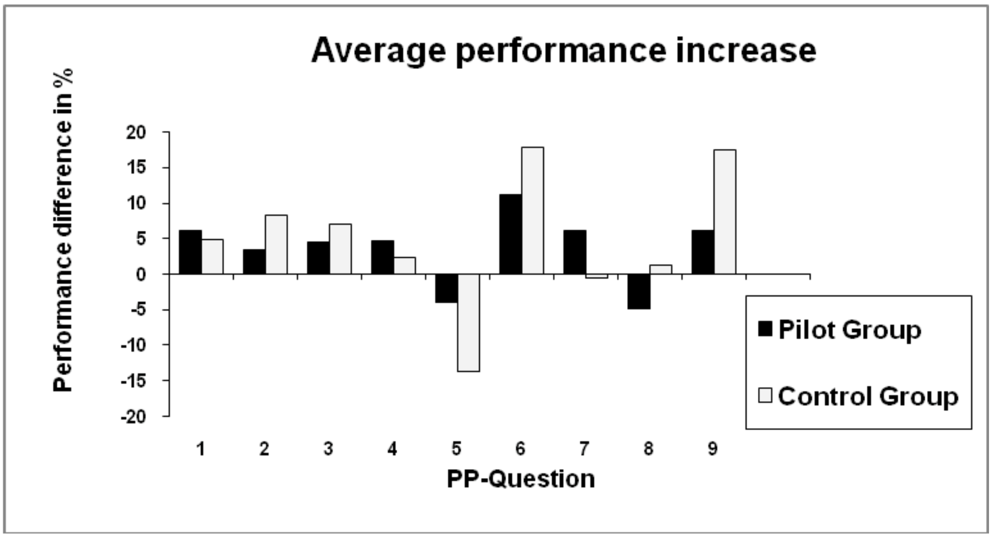

Figure 1: Average performance increase of the pilot and the control group

In PP-questions 5 and 7 the pilot classes' results are significantly better than than those of the control groups (t-Test: PP 5: 0.01, PP 7: 0.02). However, in PP-questions 6 and 9 they are significantly worse (t-Test: PP 6: 0.01, PP 9: 0.01).

Overall there is not a significant difference in the average performance increase between the pilot and control classes. For the comparatively worse result of the pilot classes compared with the control classes (especially for questions PP 6 and PP 9), there are two possible hypotheses. On the one hand it could be due to the fact that the students in the pilot classes were no longer adequately challenged or motivated to tackle this type of “traditional” question with enthusiasm, as they had tackled much more interesting questions during lessons – due to the SC. On the other hand the poor results of the pilot classes when determining functional equations from specified graphs (question 6) could be due to the fact that the students in the pilot classes had seen a large number of graphs – compared with the control group – during the course of the year and were therefore overtaxed by the diversity. However, the students in the control class have probably worked more often with the sine function graph which had been introduced in grade 10.

If, however, the range of performance increases is considered, an interesting picture emerges.

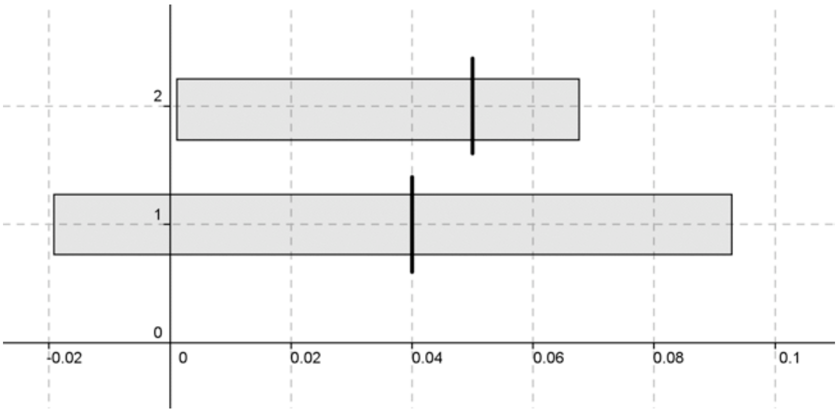

Figure 2: Average value and range of average performance increases in pilot (1) and control groups (2)

With an almost identical average value, it becomes apparent that the differences in performance are more varied with the students in the pilot classes than with the students in the control groups. Therefore, there are students in the pilot classes who benefit more from SC use than students in the control classes. However, there are also students whose results deteriorate compared with the initial test.

The test results can also be interpreted in a positive way for the pilot classes, as there are no differences in terms of classical technical and manual abilities and skills. However, this investigation has deflated hopes that the ability to interpret graphs and transfer between different forms of representation are automatically improved by the use of the SC.

3.3 Scores for “good”, “average” and “weak” test participants

In accordance with the results of the pre-test, we divided the test participants into “weak”, “average” and “good”.[4] The following result is produced when the performances of these groups are compared in terms of pre- and post tests.

[4] The “good” students form the upper performance quartile, the “weak” students the lower performance quartile, and the “average” students are represented by the two central performance quartiles.

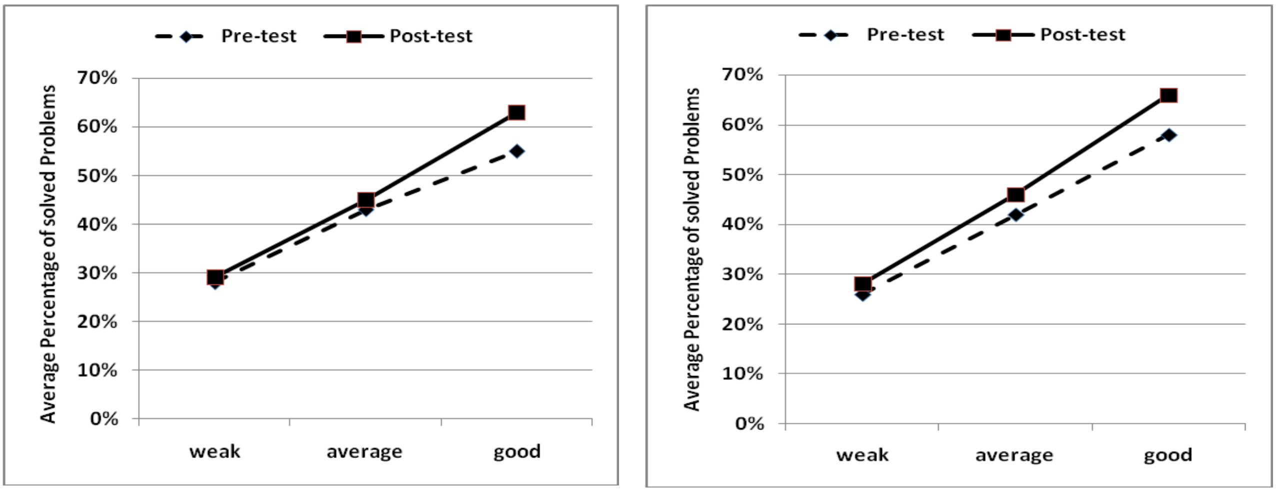

Figure 3: Performances of the pilot group (left) and the control group (right)

Compared with tests carried out in recent years in grade 10 (see Weigand 2008), different behaviour was demonstrated here. Whilst the "weak" students achieved a greater performance increase than the "average" and "good" students in grade 10, the "good" students – both in the control and pilot groups – improved more markedly (by 8 percentage points) than the "average" and "weak" students (by 3 percentage points and 1-2 percentage points respectively) in the grade 11 test.

The differences between the "weak" and "good" groups can be found in the understanding of concepts (question 5) and the transfer between different forms of representation (between graph and equation - questions 8 and 9)). The lack of performance increase in the case of weak students is attributable to the greater cognitive challenges posed by calculus, which may have taken some students to the limits of their capacities so that they were no longer able to follow lessons ("dropout effect").

4. THE SYMBOLIC-CALCULATOR-TESTS (SC-TESTS)

4.1 Research questions

In February and in June the pilot classes took a test where they were allowed to use the SC. Use of the SC was optional for the students, i.e. they decided themselves whether or not they would use the calculator. The two tests consisted of four questions each.[5] In order to establish how calculators were used, we applied a new investigation method: the students completed a questionnaire on SC-use immediately *after the test*, giving details of whether and how they used the calculator. This test was intended to answer the following questions:

1 How do students use the calculator?

2 In which phases of a problem solving process do the students use the calculator?

[5] See: www.dmuw.de/weigand/2009/CERME6/

3 Which functionalities (symbolic – graphic – numerical) do the students use?

In addition, the teachers were presented with a questionnaire regarding the questions immediately *before the test*, in which they were intended to provide details of the difficulties expected in terms of the questions.

In the following, only a few spotlights of the results will be given.

4.2 Actual use of the SC

The following diagrams show how many students used the SC during the tests in February and in June – according to their own statements:

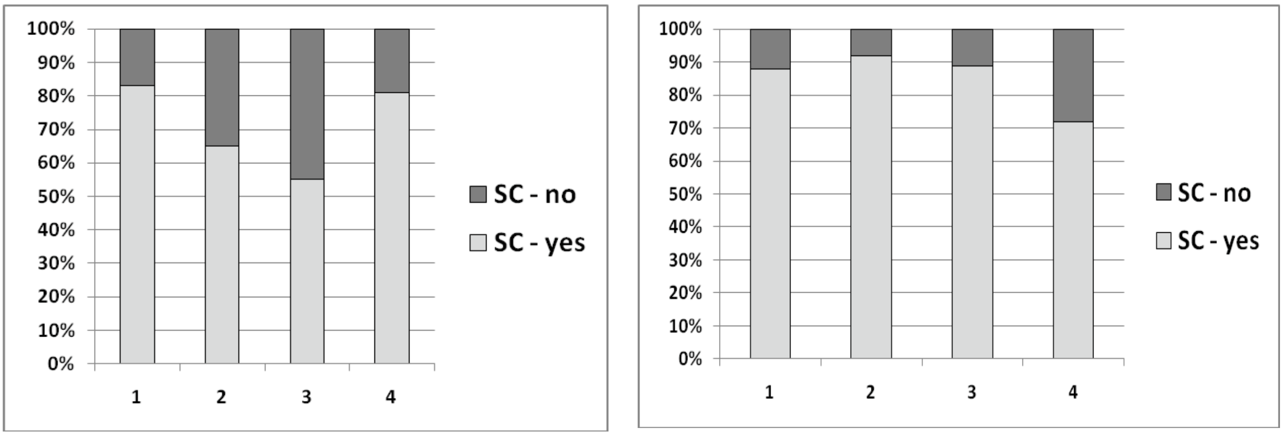

Figure 4: Results of the SC-test in February (left) and June (right) 2007

The difference between SC use in February and in June shows an increase in use of the calculator. More over, those students who used the SC in June when solving the questions scored significantly better than those who did not use it. We attribute this to the fact that it takes a full school year for students to acquire adequate confidence in the SC, as well as knowledge of the benefits of its use as a tool when solving problems, to be able to use these for the purpose of solving problems.

4.3 The SC-use during the problem solving process

The students also provided information in the questionnaire as to whether they used the SC in the beginning, during or at the end of the problem solving process.

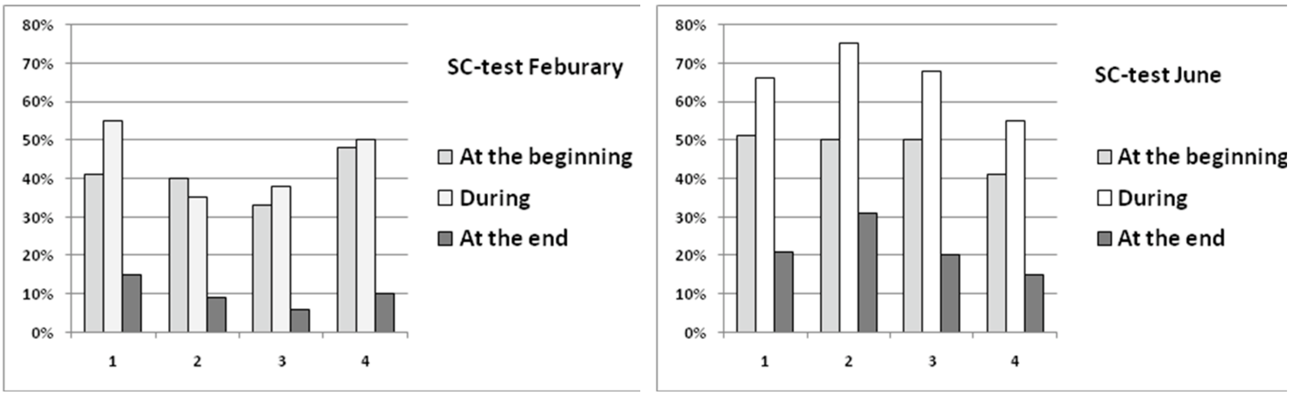

Figure 5: Use of the SC during the course of the solving process (according to statements made by the students themselves)

When students integrate the SC into their solving process, it is predominantly used at the beginning and during the solving process. If we compare the middle of the school year with the end, we can observe a clear increase in the frequency of positive responses to “during”. This allows us to conclude that the SC is more strongly integrated into the solving process by the students at the end of the school year. A slight increase can also be observed “at the end”, which makes us aware that the use of checking the solution is gaining in importance.

We also asked the students which representations they used while solving a problem with the SC. It appears that the students mainly use the symbolic and graphic possibilities of the SC. Numeric use is very limited. More over they are not familiar with the special advantages or diadvantages of the representations nor do they use the relationship between the different representations. The type of the used representation depends on the one hand very strongly on the way problems are given to the students. If it is asked for a “solution of an equation”, they mainly work on a symbolic level, if it is asked for an “intersection point of two graphs” they work on a graphic level. This shows that the SC is used in a very mechanical way, guided not by the type of problem but by the expressions used in the problem. On the other hand the type of use depends also very strongly on the classes and indicates the significance of the teacher and his or her didactic approach.

4.4 Teachers' predictions

Before each test was carried out, the teachers provided an assessment of the extent to which students would solve the problems. The question has been: “For each problem, a student gets 100 % of the marks for a completely right answer. What do you suggest will be the average score of marks your class gets for problem 1 (2, 3, 4)?” The results are as follows:

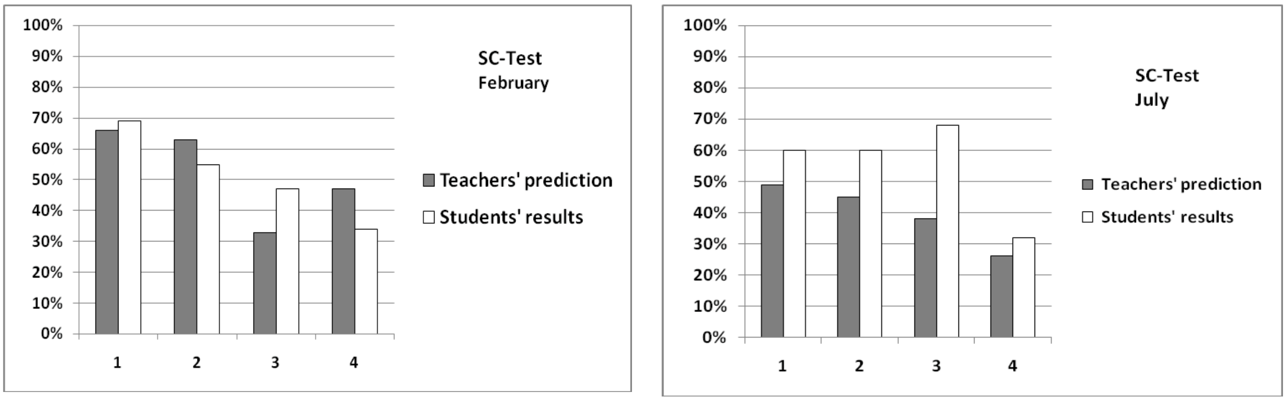

Figure 6: Comparison of teachers' predicted and student results in the SC tests

It is noticeable that the teachers underestimated the students in the June test.

5. Questions for the future

If we summarise the core results of this one-year school project there are some questions for up-coming investigations.

- **Methodology of pre- and post-tests**.

Hopes have not been fulfilled that students in the pilot classes would improve to a greater degree in terms of dealing with and interpreting graphs than students in the control classes. The hypothesis is that students in the pilot classes are not have been adequately challenged or motivated as the result of the largely traditional nature of the test problems. This raises the question whether the used pre- and post-test methodology is an adequate method to answer this question.

- **Polarisation**.

When working with new technologies, polarisation occurs in that some students benefit greatly from SC use, whereas for other students, SC use inhibits performance or even decreases performance. Two thirds of students are of the opinion that the SC was helpful and made them more secure and they classify lessons as “interesting”. Approximately one third of students do not share this view. Are there ways to get all students convinced of the benefits of the SC?

- **Calculator use**.

The reasons for non-use of the calculator are on the one hand the uncertainty of students regarding technical handling of the unit and on the other hand a lack of knowledge regarding use of the unit in a way which is appropriate for the particular problem. Is there a correlation between these two aspects?

- **Period of adjustment**.

The responses of the students confirm that familiarity with the new tool requires a very long process of getting used to it. It is surprising that it took almost a year to establish familiarity with this tool for students to use it in an adequate way. After one year of SC use, confidence in and familiarity with the SC grow. However there is still a large group of students who experience technical difficulties when operating the SC. Will there be ways to shorten this period of adjustment?

- **Solution documentation.**

Students have problems how to record solutions when using the SC. Difficulties with the type and manner in which to document the solution decreased over the year, but still remain at a high level. This latter point will continue to be a permanent challenge when working with the SC, as there is no algorithmic solution for the procedure. Are there documentation rules for all or a special type of problems?

REFERENCES

Artigue, M. (2002). Learning Mathematics in a CAS Environment: The Genesis of a Reflection about Instrumentation and the Dialectics between technical and conceptual Work, *International Journal of Computers for Mathematical Learning*, **7**, 245-274.

Drijvers, P. (2003). *Learning Algebra in a Computer Algebra Environment*, Design Research on the Understanding of the Concept of Parameter, Dissertation, Utrecht.

Guin, D., Ruthven, K. and Trouche L. (Eds.) (2005), *The Didactical Challenge of Symbolic Calculators*, New York: Springer.

Pierce, R. and Stacey, K. (2004). A Framework for Monitoring Progress and Planning Teaching towards the effective Use of Computer Algebra Systems, *International Journal of Computers for Mathematical Learning* **9**, 59-93.

Schneider, E. (2000). Teacher Experiences with the Use of a CAS in a Mathematics Classroom, *International Journal of Computer Algebra in Mathematics Education*, **7**, 119-141.

Trouche, L. (2005). Instrumental Genesis, individual and Social Aspects, in Guin, D., Ruthven, K. and Trouche L. (Eds.), *The Didactical Challenge of Symbolic Calculators*, New York: Springer, 197-230

Weigand, H.-G. (2006). Der Einsatz eines Taschencomputers in der 10. Jahrgangsstufe - Evaluation eines einjährigen Schulversuchs, *Journal für Mathematik-Didaktik* (2006), 89-112

Weigand, H.-G. (2008). Teaching with a Symbolic Calculator in 10th Grade - Evaluation of a One Year Project, *International Journal for Technology in Mathematics Education*, Volume 15, No 1, 19-32