

# LINGUISTIC ACCOMPLISHMENT OF THE LEARNING-TEACHING PROCESS IN PRIMARY MATHEMATICS INSTRUCTION

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*The linguistic accomplishment of a mathematics lesson, based on an illustrative example of an everyday lesson in a Hamburg fourth grade class, was analyzed via the person instructing. The linguistic accomplishment of instruction, for the purpose of analysis and with the help of qualitative procedures of interpretative classroom research of German mathematics education (Krummheuer/Naujok 1999), was analyzed on the basis of three hierarchical levels, developed from an existing theory. The results of these analyses grant on the one hand a hypothesis of the learning opportunities for a multilingual pupil body in German classes. On the other hand the results in the sense of local theory genesis can be integrated into a theory concept, which the author designates Implicit Pedagogy.*

## 1 Introduction

If one looks into the classrooms of German schools, one notes that the pupil body is increasingly becoming shaped by multilingualism and various cultural backgrounds; currently, almost a third of all pupils in the German educational system hold a migrant background. Despite the increasingly linguistic and cultural diversity in German schools, instruction seems to be only slightly flexible and adapted to the needs of the diverse pupil population. Students with a migrant background or students who grow up in a semi-illiterate environment perform worse, according to the findings of international and national scholastic achievement tests, in comparison to their classmates who grow up in a monolingual German environment (compare the results of PISA 2000 and 2003, as well as IGLU 2003). It appears to be indisputable, that the origin of this poor performance is in a not insignificant manner to be found in an insufficient mastery of the language of instruction. However, these differences in the mentioned studies are often gladly categorized as unchangeable via school and their cause legitimized by the socio-economic background and/or language of the family. The goal of the article at hand is thus to demonstrate the underlying reasons for the poorer performance of students with a migrant background and/or who grew up in a semi-illiterate environment. The achieved results will then be subsequently explained with the assistance of theoretical approaches and in this manner demonstrate possible consequences or potential for change. On the basis of this, further studies may be able to develop concrete possibilities of how to fit instruction better to students affected by lingual-cultural plurality.

Linguistic accomplishment of instruction constitutes a substantial aspect of the adjustment of instruction to suit the needs of multilingual pupil bodies. In accordance with some approaches in the field of mathematics education, language and communicative competence both have a special significance for the learning of

mathematic content. Above all, Maier (compare e.g. 2006, 2004 and 1986) was concerned with research in the field of language and mathematics within German-speaking countries. Maier (2006) justified, that language holds a special relevance in Mathematics instruction, as objects in Mathematics, “... do not have a material nature and thereby are not accessible through the senses” (p.137, translated by the author). This consequently accounts for the significant focus of Maier’s works on the observation of technical terminology in Mathematics instruction. In the international community there are several authors who can be mentioned, who concern themselves with the relevance of language in the learning of mathematics. In the following, it should be initially referred back to Pimm (1987) who understood Mathematics as a social activity that is structurally and closely connected with verbal communication. From this, Pimm introduces the metaphor “Mathematics is a language?” (ibid, p.XiV) as a question of whether Mathematics could be evaluated not in the sense of a natural language, but as its own style of language. He compares, for this purpose, teachers as a role model of a “native speaker” of Mathematics and other people, for whom Mathematics appears to be incomprehensible, as per a foreign language, to which they are not empowered (ibid, p.Xiii).

The empiric material of the underlying research consists of transcripts from video recordings of an everyday primary lesson. The video recordings took place over a time period of four months in three classes of the fourth grade in two Hamburg primary schools with an approximate 80% migration contingent amongst its pupils.

In section 2 of this article, the analytical findings of the analysis of interactions within a selected instructional episode will be presented. In connection, a methodologic indexing of the procedure of the underlying research will be taken as preparation of further analysis. The selected episode will be used in section 3 as an illustrative example to demonstrate how lingual accomplishment of primary mathematics instruction lends itself to be described and analyzed with the here-accepted theoretical perspective. To this, three hierarchical levels are developed from this theory, by which the linguistic accomplishment of the lesson in the selected episode will be deeply analyzed. In section 4, the possible outcomes will be described, that yield from the results of the analysis to learning opportunities for pupils in German primary school classes. Furthermore, the results will be presented for the purpose of local theory development in a theoretical concept developed by the author from the entire research.

## **2 An Episode from the Lesson Sequence “LCM”**

In the following a short transcribed episode of an everyday primary school mathematics lesson during the introduction of a new mathematic concept will be looked at.

### **2.1 Prehistory and Transcript of the Lesson Episode**

At the beginning of the scene “LCM” Ms. Teichmann along with 25 female and male pupils, 17 of which have a migration background, are situated in the classroom. In

this lesson the introduction of a new mathematic concept should take place: the LCM- the Least Common Multiple.

It is Wednesday morning Ms. Teichmann asks initially what the abbreviation LCM stands for. Thereafter she allows the multiples to be calculated. Finally she draws two circles on the board, that she divides into four and three segments respectively, with an addition symbol between them and an equals sign. She marks for each circle one of the segments in pink. While one pupil very quietly says, “ $1/3$  plus  $1/4$ ,” Ms. Teichmann asks the pupils which equation stands on the board. The pupils begin to guess and first give the answer, “ $1$  plus  $1$ ,” or, “ $2$ ,” and then somewhat later label the segment with  $1/3$  and  $1/4$ . The teacher notes this in the drawing on the board and adjusts the fractions from  $1/3$  and  $1/4$  to  $4/12$  and  $3/12$ . Several pupils offer many creative solutions for their addition, such as for example “ $2/7$ ”. In closing, her generalization of the procedure follows.

241	16:30	<L:	right/ you may not- add a large piece of pizza [points to the left circle]
242		>L:	and a small one and a smaller -.one together [points to the right circle]
243		L:	that is not equal right/
244		<L:	you must practically...
245			chop them into such pieces that they are equal\
246		<L:	[makes a chopping motion with her hand]
247		>L:	..right/these pieces are equal\ [points to the left circle]
248		<L:	[points to the right circle] These pieces as well\
249			only here it is less\ right/ here there are only three-
250		>L:	and here there are four pieces. [Points to the left circle]
251		S:	ah now I understand it
252	16:57	L:	and for that reason one need this\. if you at all want to (add) fractions-
253			so that you can add together such pieces of cake together\
254			right/one can not simply
255			say three and four is seven and from above
256			we will take two and then I have two sevenths\
257	17:11		Two sevenths is something completely different
258			no that doesn't work\

## 2.2 Concise Analysis of the Interaction

At the end of the episode the teacher attempts to show the pupils a generalization of the addition of fractions. She uses for this purpose the everyday example of the division of a pizza, respectively cake and makes the division of them visual through gestures. Hereby both levels of the illustration on the basis of the everyday and the generalization of the rules of fractional arithmetic meld together. This is shown in the statement by Ms. Teichmann in <252-258>. The reference to “LCM” seems to have been completely lost, or left as implicit. Alone the, “...and for that reason one needs this...” in <252> from Ms. Teichmann gives us the idea that there is still a reference to the “LCM”, since one needs an “LCM” in order to find the least common denominator for the addition of the two fractions. Ms. Teichmann does not further explain this connection. Also the final generalization by hand of the cake example <252-258> can barely be accounted for as a further clarification of the procedure, since Ms. Teichmann says that one may not simply add three and four together and means thereby apparently the denominators of one third and one fourth. Through the selected example, however, pupils did indeed have to add three and four in order to

ascertain the solution of the task – though, on the level of the numerator. They added  $\frac{3}{12}$  and  $\frac{4}{12}$ . Moreover, the addition of the numbers three and four are everyday tasks for primary pupils in basic arithmetic operation. Why one may no longer carry out this arithmetic remains unexplained. Since one cannot assume, that the pupils are competent to differentiate between numerators and denominators, one can classify the statement of the teacher as contradictory. Consequently, pupils in the end of this episode were merely able to solve an addition task, which they were already capable of solving before and whose correctness would now be put into question.

### **2.3 Methodology**

After having summarized the analysis of the scene, I would like to offer as preparation of further analysis a few explanatory notes to the methodological situating of the underlying research. The underlying research to this article is qualitatively oriented and grounded in interpretative classroom research. More exactly: in the domain of the interactionist view of interpretative classroom research in the field of mathematics education. Through the analysis of the units of interaction in the videotaped instructional episodes, I oriented myself to a reconstructive-interpretative methodology and on a central element of the research style of Grounded Theory- the methodic approach of comparative analysis. The goal of interpretative classroom research is to pursue a local theory genesis through “understanding” of interactions of individuals in concrete instructional practice. The scope of this concept theory is related to the interpretative classroom research, however, to be decidedly restrained, since this is in many areas mostly globally and universally connoted. The theoretical results of research of such a reconstructive-interpretative procedure present hypothetical outcomes, which do not follow the claims of the development of globalizing and universalizing theoretical approaches (compare Krummheuer/Naujok 1999, p. 105). These hypotheses stay arrested to the fact, that they are directly connected to the respective context of the researched field of study and are thereby rich in empirical elements and feature inner consistency. A universality of underlying results does not lend itself to be understood as, “is always applicable,” rather may be related to only a limited scope of classes, who are taught and will learn under similar conditions.

## **3 The analysis of the linguistic accomplishment**

Here subsequently follows the analysis of the linguistic accomplishment of the instruction on the basis of the selected instructional episode on three hierarchical levels.

### **3.1 Technical terminology versus everyday language**

Since objects of Mathematics are according to Maier (1986, p.137) of an abstract nature, the introduction of new mathematic concepts allows for particular attention to the technical language of Mathematics, as objects of Mathematics can ultimately be handled and represented only on a linguistic-symbolic level (compare *ibid*, p. 137). The question, which should be answered in the following sections, is how these technical terms of Mathematics are introduced into the analyzed lesson. To this, Maier (2004) refers to the fact that in the technical language of Mathematics, as well as in

other technical languages, there is a problem of ambiguity within the technical language, since it interferes with the everyday language of the pupils (compare *ibid*, p. 153). The problem of ambiguity within the technical language of Mathematics, according to Maier (2004, p.153), carries a significant relevance in the verbal actions of teachers. Maier writes, that teacher language moves in a stress-ratio between technical linguistic “*Hypertrophy*” and accordingly “*Hypotrophy*”<sup>i</sup>. The goal should be, according to Maier, to have the teaching language, which moves on a scale between these two extreme points of Hypertrophy and Hypotrophy, positioned “in the middle.” Thus a necessary technical language development of the pupil body can be assured and on the other side the pupils can be given the opportunity to comprehend mathematic phenomena with their own language. In which forms the usage of mathematic concepts let themselves be differentiated from the usage of everyday language concepts in instruction follows as next in the first level of hierarchisation.

### The analysis of the selected episode

In the underlying episode the teacher attempts to give a generalization for the addition of fractions. She stresses here the relevance of LCM for the addition of fractions in line <252> in saying, “*and for that reason you need this.*” In this statement she uses the place holder “*in addition*” and “*this*” instead of the technical terminology. In her entire generalization she uses a multiplicity of everyday language concepts such as, “*a piece of pizza*” <241>, “*pieces*” (of a pizza or cake) <245, 247, 248, 250>, “*chopping*” <245>, “*pieces of cake*” <253>. From the terminology she used, the following language can be found in everyday language as well as in technical language: “*to add together*” <241-242, 253>, “*not equal*” <243>, “*equal*” <245, 247>, “*less*” <249>. Only the expressions of “*fractions*” <252> und “*two sevenths*” <256, 257> suggest, on the other hand, technical linguistic terminology. With this analysis in mind, the procedure of the above-mentioned teacher would surely be described, according to Maier, more in terms of technical Hypotrophy, since the teacher through the generalization of the procedure, where the greatest level of abstraction could have been conjectured, reverted only minimally back to technical terminology.

According to the statements of Maier one could reason, that such a procedure enables pupils to describe mathematic phenomena with their own language, but also endangers the development of technical language. Since, however, these attempts to explain multiplicity are through everyday language concepts and the usage of placeholders, the general principle remains implicitly hidden (see section 2.2) and it is doubtful, that pupils are in a position to shift into their own language to describe this mathematic phenomena.

### **3.2 The embedding of mathematic concepts in a mathematics register**

The second level of analysis of the linguistic accomplishment of instruction via the teacher by the introduction of a new mathematic concept lends itself to a reference of the statements of Pimm (1987). Pimm compares teachers as a role model of a “*native speaker*” of Mathematics (*ibid*, p. Xiii) and other people, for whom Mathematics ap-

appears to be incomprehensible, as per a foreign language, to which they are not empowered (ibid, p.2). In this context, Pimm (1987) is speaking of a “*mathematics register*” (p. 74). With the term register, Pimm is referring to Halliday (1975). Halliday understands a register as an assemblage of meanings that are intended for a particular function of language, that together with the words and structures are able to express these meanings. Halliday subsequently talks of the mathematics register only when a situation is concerned with meaning, that is related to the language of Mathematics, and when the language must express something for a mathematical purpose. Mathematics register in this sense can be understood as not merely consisting of terminology and that the development of this register is also not merely a process to which new words can be added (Halliday 1975, p. 65). The task of the pupils to learn mathematical concepts in their lessons contains, according to Pimm (1987), more a deeper learning of linguistic competence than is the case by Maier (e.g. 2004). In Maier’s approach the focus lies on the acquisition of technical linguistic competence through a well-balanced application of technical linguistic terminology and everyday language concepts in the linguistic accomplishment of instruction via the teacher. Pimm (1987, p.76) sees the task of pupils, however, as to become proficient in a mathematics register and in this way to be able to act verbally like a native speaker of Mathematics. The second level of hierarchisation of the linguistic accomplishment of instruction falls into what extent the newly learned mathematic concepts in the researched lesson were integrated into a mathematics register or if they were to be introduced and regarded as isolated units.

### The analysis of the selected episode

In the selected episode the teacher appears to attempt to explain the mathematic concept “LCM” in connection with the addition of fractions. In the beginning of this episode the teacher produced for this purpose a reference to the concept of multiples in allowing pupils to calculate them. According to the theoretical perspective of Pimm (1987) the attempt by the teacher to reconstruct the concept of “LCM” only allows itself to be incorporated, not as an isolated conceptual unit, but through its connection with other mathematic concepts in a mathematics register. According to Pimm, it should be the goal to make pupils competent native speakers of Mathematics. In the introduction by the teacher, however, there was no time point in the entire scene in which the mathematic concepts of denominator, numerator, fractions, fraction strokes, or multiples were verbally and content-wise clarified in the official classroom discourse. They remain implicit and are integrated without reflection in the already familiar calculation routines. Even the teacher herself seldom uses the concepts to be learned actively, such as is shown in the first analysis, rather reverts back predominantly to the everyday language concepts. Pupils must extract the meanings of the new concepts by themselves from the illustration on the board. Pupils are then additionally given only the possibility to calculate the multiple as an active manner in which to solely understand the meaning of the concept of a multiple. That pupils are able to extract the concepts, without a verbal contextual explanation of the concepts by the teacher seems questionable. For example, in the analysis at the beginning of

the scene there were alternatives for interpretation, in which the pupils interpreted the fraction stroke as minus sign. Pupils must extract the subject with this *implicit procedural method* from their everyday background or from that which they already know from their lessons and will thus be able to take no decisive steps in the direction of becoming a native speaker of Mathematics.

### 3.3 The embedding of the mathematic concepts in a formal language register

The third level of analysis of linguistic accomplishment of instruction unfolds from the reference of the theoretical explanations of Bernstein (1977), Gogolin (2006), and Zevenbergen (2001). According to Gogolin (2006), pupils in German schools are submitted to the normative standard, that they are receptively and productively in command of the cultivated linguistic variations in class. This language of school- described by Gogolin as “*Bildungssprache*”<sup>ii</sup> (ibid, p.82 ff., according to the concept of “*Cognitive Academic Language Proficiency*”, Cummins 1979)- has on a structural level more in common with the rules of written linguistic communication. It is in large part inconsistent with the characteristics of the everyday verbal communication of many pupils.

Bernstein (1977) and Zevenbergen (2001) target, with their discussion of the language of instruction, the children from the working and middle class for differentiation. According to them, the linguistic abilities of formal language that are required in schools set a line of demarcation in everyday language, that is more in accordance to the abilities of the middle class, than to those of the working class. This formal language of instruction stands out through its precise grammatical structure and syntax as well as through its complex sentence structure. Through proficiency in this formal language, pupils develop - those in the middle class in particular - a sensibility in regards to the structure of objects and the structure of language, that helps them to solve problems in life and in school in a relevant and goal-oriented manner. Successfully receptive in “*being (a) part (of)*” and productive as in “*taking part (in)*” (Markowitz 1986, p.9, translated by the author) a linguistic discourse of instruction is something that is only possible for pupils, according to the above-mentioned authors, when they have competence in the formal language or the *Bildungssprache* of instruction. In this way it is possible for them to understand abstract concepts independent of concrete context and to be able to transfer them into written decontextualized form. In the third level of hierarchisation of the linguistic accomplishment of primary mathematics instruction there follows the question, to what extent, and how pupils are introduced during instruction to a formal *Bildungssprache*.

#### The analysis of selected episode

In her attempt to make a generalization, the teacher says in <241-242> “*Right/ you may not add a small piece of pizza and a small one and smaller one together*” <241 – 242>. She also uses the comparative form of the adjective “small” for this purpose, but does not go into the “Least Common Multiple” more explicitly. However, it is not self-explanatory that all pupils- most especially those who have grown up

multilingual- are familiar with the comparative forms of adjectives in the German language. It is not self-explanatory that pupils will be able to differentiate between “Small Common Multiple” and “Least Common Multiple”. This interpretation is supported by analysis of previous episodes, in which pupils used the incorrect comparative form when attempting to use the term “Least Common Multiple”. Another correlation to this can be seen in the procedure at the beginning of the scene where the teacher allowed the pupils to calculate multiples. At no point in time did the teacher explain the connection between the terms “multiple” and “Least Common Multiple”. In this way it is made difficult for students to be able to recognize that the “Least Common Multiple” is really a subset of all “multiples”. It is not attempted on the part of the teacher to integrate the new concept into a related text. Hereby the question may be asked if and how the students should be empowered to understand such abstract concepts independent of concrete examples and to be able to transfer them into written form.

### Summary of the analysis of the linguistic accomplishment of instruction

In the underlying research of this article there were 15 different episodes in total which were analyzed<sup>iii</sup>. These episodes with the help of comparative analysis were systematically compared. The comparison thereby of the three hierarchical levels of the linguistic accomplishment of instruction resulted in the following structure characteristics:

In the case of the first level, the application of technical terminology or everyday language by the teacher in instruction, allows no structural commonalities to be reconstructed. A unified procedure by the usage of mathematics register and everyday language does not seem to make a difference in the episode. The teachers use either predominantly everyday language concepts or several new and unexplained mathematic concepts. Unlike the first level, the results of the analysis of the other two levels behave in a different way. The implicitness of learning content, as a phenomenon in the introduction of a new mathematic concept, allows itself to be reconstructed as the common basic structural characteristic of the linguistic accomplishment of instruction via the teacher. The implicitness of the learning content defeats itself by the usage of different mathematics and formal linguistic registers. In this introduction of new mathematic concepts one can reconstruct through mathematics register, that the meanings of the concepts, just as the content references between the new mathematic concepts to be learned or the already known everyday language concepts is not made clear or only implicitly. The meanings or connections are not explicitly taken up in the instructional discourse and find thus no consideration in the classroom discourse. The meaning or the reference are explicitly assimilated by the teacher into the instructional discourse and thus find no consideration in the interaction of the classroom discourse. The formulated goal of Pimm (1987, Xiii; see Ch. 2.4) that students should learn to speak Mathematics like a native speaker, will be difficult for students to achieve, as the native speaker of Mathematics - the teacher - does not exemplify this active speaking themselves. A



similar picture shows itself in the way the teachers commit themselves to linguistic particularities of formal linguistic register. Also here there is an implicitness that rules the teaching. The teacher only refers back to the grammatical structure implicitly, in which the mathematical concept is embedded, or to that which characterizes the meaning carrying elements. With which linguistic methods the complex and abstract mathematic concept, in the sense of the conceptual writing, is expressed to a connected text is left, as regards content or implicitness, in the end of the attempted explanations, unconnected. An integrated embedding of the mathematical concept in a *Bildungssprache* is not noticeable.

#### 4 Implicit Pedagogy and its consequences

In the basis of the research the reconstructed procedures of the teacher in the linguistic accomplishment of the lesson alone was with mathematics teaching approaches not enough to explain, and for this reason further pedagogical, sociological and linguistic approaches were expanded into the theory genesis (compare Bourne 2003; Bernstein 1996; Walkerdine 1984). Through this opening of the theoretical framework of the underlying research, there allows for the procedure of the teacher to be conceptualized under the concept of “*Implicit Pedagogy*” (compare “*Implizite Pädagogik*” Schütte 2009). This displays itself in the introduction of new mathematical concepts, in the manner, that decisive aspects of meaning negotiating of the individuals and the thereby possible constructions of enduring, non-situational bodies of knowledge for the individuals, remain concealed. One such Implicit Pedagogy is attached to the main idea, that students alone on the basis of the abilities they bring along with them can unlock meanings. Not the lesson, the qualifications of the teachers, nor their efforts can bring a deciding influence on the possible educational success of students in school, but rather, and above all else, the abilities that the children have brought with them decides this. The linguistic accomplishment of the instruction via the teacher, that follows such fundamental ideas, would not appear to make enough adjustments to the existing relationships of linguistic-cultural plurality in the classroom, since the procedure as it stood only served to reproduce existing social relationships in the educational system. The consequence of such an implicit procedure by the teacher can be, for example, that the comprehensive development of the relevance of the new concepts to be learned, on the side of the students, can be hindered. On the other hand it is a possible consequence that the students could be hindered by, or could refuse to participate in, a formal linguistic educational discourse in their lessons. Additionally, the opportunity is taken away from them to participate actively, that means productively, in the lesson, and through this accomplish the lesson. This happens for the main reason that the teacher, through her primarily implicit procedure, presents no model for her students to follow in her interactions with the formal linguistic *Bildungssprache*.

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<sup>i</sup> The excessive use of almost “*pure technical language*” (ibid) by teachers and instructional media is viewed by Maier (2004, p.153) as technical linguistic hypertrophy. The excessive use of almost

“*pure colloquial language*“ (ibid) by teachers and instructional media is characterized by Maier (2004, p. 153) as technical linguistic hypotrophy.

<sup>ii</sup> *Formal linguistic instructional language* (translated by the author).

<sup>iii</sup> This episode under consideration deals primarily with a shortened extract from the original episode, since for reasons of space limitations no analysis of the entire episode was possible. The detailed analysis of this episode can be found in Schütte (2009).

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