A FRENCH LOOK ON THE GREEK GEOMETRICAL WORKING SPACE AT SECONDARY SCHOOL LEVEL

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Based on the geometrical paradigms approach, various studies have shown some tension in French Geometrical Working Space between institutional expectation and effective implementation. In this paper, we examine the Greek system from this point of view and we find the same kind of tension but in a certain sense stronger than in France even if both countries have an ancient Euclidean tradition.

FROM SPECIFIC FRENCH CASE TO THE PARTICULAR GREEK CASE

Since several years, it seems that curricula and syllabi converge to promote a close link between mathematics teaching and the "real world". The idea of "mathematical literacy" is especially strong in the PISA evaluation which aims to organize this general trend among European countries. At the same time and close to this conception of mathematics, the constructivist approach is favoured by national educational institutions and teachers are asked to substitute "bottom up" teaching methods to the traditional "top down" entrance in mathematics.

In France, till today, and at lower secondary school level the prominent way suggested by the intended curriculum is based on "inquiry methods" and "activities" and relationships between mathematics and other scientific or technological domains are always pointed up. But the link to sensible world is only mentioned and the emphasis is put on the logical rigour of mathematics. The relationship to the "real world" seems really far off and into everyday classroom, inquiry based methods are left aside.

In the special case of geometry, we were concerned with the contradiction between official expectation and the crude reality of the classroom. To understand and explain the phenomenon, the notion of geometrical paradigms (Houdement and Kuzniak, 1999) and of geometrical working spaces (Kuzniak, 2007) have been used to explicit the different meanings of the term geometry. The field of geometry can be mapped out according to three paradigms, two of which – Geometry I and II – play an important role in today's secondary education. Each paradigm is global and coherent enough to define and structure geometry as a discipline and to set up respective working spaces suitable to solve a wide class of problems.

This first idea is completed by the following hypothesis on the possible influence of these paradigms in geometry education and on the poor implementation of new teaching method. The spontaneous geometrical epistemology of teachers enters in contradiction with mathematical epistemology embedded in the new teaching methods. In other words: the geometrical work done and aimed by teachers could be of another nature than the institutional expected one. The teacher's geometrical thinking is led by another geometrical paradigm as the paradigm promoted by the institution. Moreover this way of thinking leads to prefer pedagogical methods in contradiction with inquiry based methods.

Our investigation work has its roots in the French context but some comparative studies showed us that such a tension could exist in other countries. Houdement (2007) has presented in CERME 5 a comparison of magnitude measurement problems in Chile and in France. The social and economical contexts are quite different in both countries and so, we were interested to have a look on other European countries to verify if this kind of tension really exists and how it was managed. We have had the opportunity to work with Greek colleagues and to be aware of a great change in the curriculum based on the real world and turning back to the Euclidean tradition. We present the first part of our work which gives our analysis of the Greek situation through our viewpoint.

GENERAL FRAME OF THE STUDY

The theoretical frame we used has been soon described in detail in former CERME sessions (Houdement and Kuzniak 2003, Houdement 2007) and we refer to these papers for complements. We retain only here some particular elements used in our description of the Greek situation.

As we are interested in the awkward relationships between reality and mathematics education, we will focus on the role the reality plays in the different paradigms. In the first one, Natural Geometry or Geometry I (GI), the validation depends on reality and the sensible world. In this Geometry, an assertion is accepted as valid using arguments based upon experiment and deduction. The confusion between the model and reality is great and any argument is allowed to justify an assertion and convince. This Geometry could be seen as an empirical science and it is possible to build empirical concepts depending on the experience of the "real world". Natural Axiomatic Geometry, or Geometry II (GII), whose archetype is classic Euclidean Geometry is built on a model that approaches reality. Once the axioms are set up, proofs have to be developed within the system of axioms to be valid. In the formal Axiomatic Geometry, or Geometry III (GIII), the system of axioms, which is disconnected from reality, is central and leads how to argue. The system of axioms is complete and unconcerned with any possible applications in the world. In that case, the system creates its reality. Concepts are given *a priori* and come "from the Book" and so "top down" form of mathematics education seems well fitted to this conception. The study of Greek mathematical education will show that this dichotomy GII / GIII is not so simple.

To find a possible tension or contradiction between the institutional expectation and the teacher's approaches, we will describe what we call the personal teacher's Geometrical Working Space (GWS) faced to the GWS expected and promoted by the national institution in charge of mathematics education. More precisely (Kuzniak

2006), the Geometrical Working Space (GWS) is the place organized to ensure the geometrical work. It makes networking the three following components: the real and local space as material support, the artefacts as drawings tools and computers put in the service of the geometrician and a theoretical system of reference possibly organized in a theoretical model depending on the geometrical paradigm. To ensure that the components are well used, we need to focus on some cognitive processes involved into the geometrical activity and particularly the visualization process with regard to space representation and the material support, the construction process depending on the used tools (rulers, compass, etc.) and on the configuration, and finally reasoning in relation to a discursive process.

THE NEW CURRICULUM IN GREECE

Since 2007, a new curriculum for compulsory education is implemented in gymnasium (grades 7 to 9) in Greece and summarised in a list of ten highlights. It is presented as cross-thematic (1st and 5th highlights) and aims to connect the academic disciplines, everyday life, working world, history, technological improvement, etc. Within the flexible zone (4th highlight), some hours are planned for reaching this specific goal. Primary school learning explicitly rests on the Bruner's constructivist theory and assessment is now an essential part of the learning process (8th highlight). Sources and goals of connection with realty are in the 9th highlight, “A Broad Spectrum of Literacies”:

> Successful living in post-modern times presupposes that one is fully literate in many areas, such as reading, science, technology and mathematics in order to face international evaluation (PISA, TIMS, etc.) which demand more connections between school knowledge and the life reality.

The present mathematical syllabus expands the ancient one with no change in the content. It is written in a three columns table where some more detailed mathematical sections appear into the traditional blocks (arithmetic, algebra, geometry). Mathematical skills, which have to be learned by pupils, are described in the first column, the main mathematical notions are in the second and in the third one some activities are proposed, often to introduce some mathematical notions.

New textbooks are conformed to syllabus with no surprise since they are chosen by the curriculum designer Pedagogical Institute, one for each level. Textbooks structure is quite the same as the syllabus structure and activities coming from the syllabus third column can be found with few changes in textbooks. For these reasons, institutional GWS means the GWS presented by the curriculum including the official textbooks.

A SO FAR REALITY

We will highlight some internal slides into the institutional GWS itself. First, in spite of the curriculum demand, new technologies have to be used (7th highlight), syllabus and textbooks do not mention software, computers or Internet. Beside this slide inside the curriculum, the reality is concerned by a second and less obvious one.

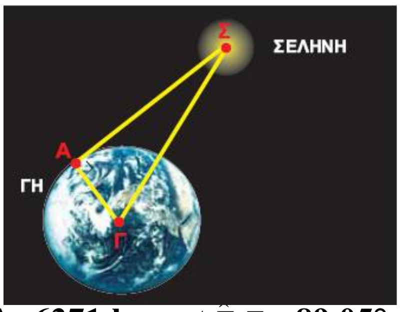
According to the cross-thematic curriculum, reality and everyday life have to be embedded in the learning process. But when everyday life is mentioned in syllabus it is without any details and only one syllabus activity could be described as real : *measure the width of the street and pavement in front of the school*. But the difficulty to follow this curriculum directive is more obvious in textbooks. This real activity in syllabus does not appear in the A' textbook (grade 7), and if there are numerous activities based on a "real picture", they are not relevant for this purpose for several reasons:

- The 3D/2D problem: angles and distances on the textbook are not the good ones. For these kind of activities, geometry does not seem to be able to give the right answer!

Students have to find the lower distance between the point A at the house to the water. (A' page 184)

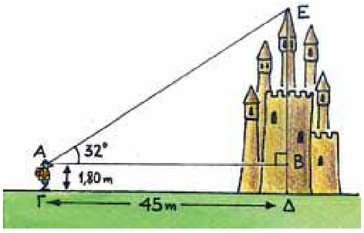
- A lot of activities refer to the macro-space but authors represent reality – probably under editorial constraint – with an image or photography. On these pictures, most of the time, some geometric element are placed and the reality is already mathematized. However, we often find activities and exercises with geographic maps, as it is stated in syllabus. But reality is once more already mathematized.

Why an airplane realize a lower distance than a boat to go from Athens to Samos? (A' page 164)

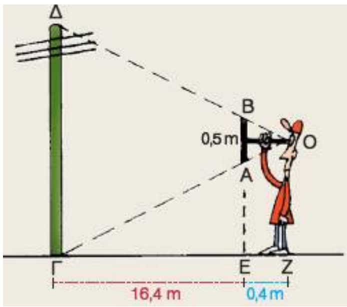


ΓA=6371 km, $A\hat{\Gamma}\Sigma$ =89,05°, find ΓΣ. (B' page 151)

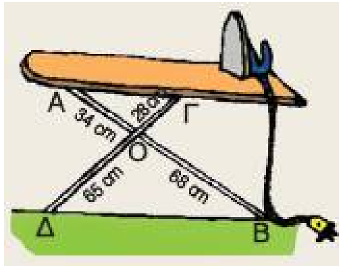
- Activities and exercises are most of the time based on a picture of a real problem with a geometric diagram with all the measures needed to solve the problem, no more no less. Reality is not the point and is viewed through a picture already turned into a geometrical task support.



Find ΔE. (B' page 139)



Find ΔΓ. (G' page 223)



Why it is not horizontal? (G' page 209)

As we notice it, the geometrical local space is almost always the micro-space of a sheet of paper which is sometimes a representation of a macro-space problem (geographic maps, pictures, etc.). Actually, the reality in textbooks appears from a relevant point of view only in the GI paradigm [1], on a sheet of paper. And so we can characterize this internal slide: everyday life is not taking into account and reality is only treated within the GI paradigm, inside geometry.

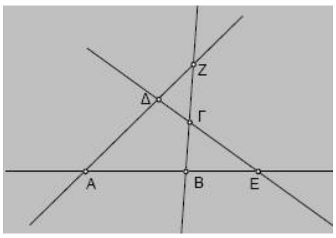
GYMNASIUM INSTITUTIONAL GWS

Since reality is not actually present in institutional GWS, except within the GI paradigm, we study the institutional GWS all along the gymnasium.

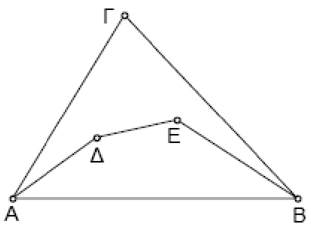
Artefacts, visualization and diagrams constructions: the GI paradigm

Geometric tools (ruler, compass, protractor, square, tracing paper) are only mentioned in syllabus at the A' class (grade 7). However, construction activities are present all along the gymnasium (much more at the first class). In the A' textbook, tools are pictured in many places, especially for showing how to construct. Tracing paper is used in many geometry sections, often to introduce a new concept. In the B' and G' textbooks (grades 8, 9) geometric tools are never drawn, sometimes mentioned.

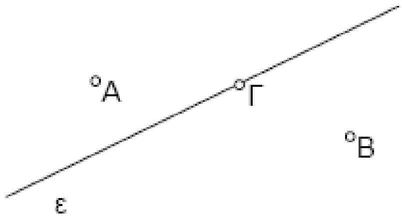
There is no freehand construction in syllabus, no freehand diagram in textbooks and we do not find any exercise where pupils have to draw such a kind of diagram. Some activities proposed in syllabus (third column of A' class) are in GI, excluding, or not, visualization:



How many angles?



Find in measuring the lower distance between A



Draw the perpendiculars to ε passing by these points.

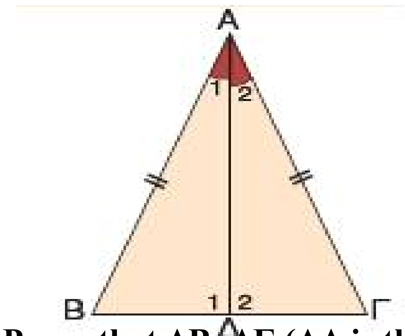
An aim of syllabus, at B' class (grade 8), section trigonometry, is to construct an angle whose sinus, cosine or tangent are known. But we do not find any activity on this topic in textbook. At the final class (grade 9) the section on dilation is directed by the GI paradigm with numerous drawing activities (7 exercises of the 9 at the end of the section ask for drawing).

Formal proofs: the GII paradigm

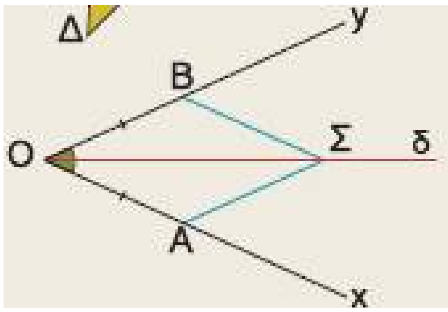
Proof process should start as it is written in syllabus preamble, but no formal proof is mentioned in the detailed table of mathematics syllabus. There are some theorems, definitions, properties.

Very few examples of formal proofs are given in the A' textbook (grade 7) and their solutions are always completely written. It is quite the same situation in the B' textbook (grade 8), except the proof that a dodecagon is regular (exercise 8, page 185). In the B' area section, a lot of exercises ask to "show that" but, in fact, the solution is always given by a calculation of an area or a length.

In G' textbook (grade 9) there is a great change with a lot of exercises where pupils have to prove. At the section on triangle congruence, the 21 exercises at the end of the section ask for a formal proof and the theoretic system of reference, with the three criteria of triangle congruence, is clearly directed by the GII paradigm. In this section, there are four solved exercises (pages 191, 192) which ask for a formal proof on triangle congruence (see below, for example, the figure on the left). At the end of the section (pages 194-196) some similarly exercises are given (see below, for example, the figure on the right). One could thought that the solutions of the four solved exercises could give a proof model to students to solve exercises at the section end.



Prove that ΔB=ΔΓ (AΔ is the bisector of Â). <u>With solution</u>. (G' page 191)



Prove that AΣ=BΣ (OA=OB, Oδ is bisector of Ô). Exercise <u>without solution</u>. (G' page 194)

The diagrams similarity section is also in GII paradigm (half of the exercises ask for a formal proof, the others are on ratio and length calculation).

Gymnasium paradigm

At the first class A', both in curriculum and textbook, the main paradigm is GI and it is generally well assumed. However, the paradigm in which pupils have to work is not always clear. For example, the following syllabus activity starts in GI and finishes, with questions g) and h), necessarily in GII:

a) Let O a point and a line ε and the point A so that OA is the distance from O to ε.

b) Let B another point on ε, find the symmetrics A' and B' of A and B through O and let ε' the line A'B'.

c) Which is the symmetric of ε through O ?

d) Which is the symmetric of the angle OÂB ?

e) How are the angles OÂB and OÂ'B'?

f) How is the angle OÂ'B'?

g) How are ε and ε' with respect to AA'?

h) How are ε and ε'?

Didactic contract is not very clear for the intermediate questions c), d) and e): GI, with tools or visualization, or GII paradigm? This activity is given in textbook with only one question and a complete solution below. The task paradigm is clearly GII: the answers corresponding to questions e) to h) are formal proofs. This example is a non explicit slide from GI to GII in a class where GI is the main paradigm [2].

Artefacts and diagrams constructions are used in many activities to discover geometrical properties, as it is written in the curriculum according to the bottom-up point of view: from the GI paradigm arises the GII paradigms. Some activities given in the third column of syllabus are in GI, to construct, to observe a property (sometimes in first class with the use of tracing paper and folding). This kind of activities can be find in all gymnasium textbooks (grades 7 to 9).

In gymnasium, from grade 7 to 9, geometrical tasks are very different. The GWS depends on the class and the section. In the first class GWS is clearly directed by GI but there are some slides in favour of the GII paradigm. In the last class, the GWS of the triangle congruence section is directed by GII while it is directed by GI in the section on dilation. In this last class, there are several very different GWS which seem not to be connected.

EUCLIDEAN PRESSURE ON TEACHER'S PERSONAL GWS

This section is supported by six secondary teachers' interviews where we focussed on the new curriculum and more specifically on reality, geometrical tools, diagram constructions and formal proofs in textbooks and in classrooms. We turn out to teacher's personal GWS which is quite different from the institutional one as we will show it. Before studying the GWS teachers, we point out the particular importance of Euclidean Geometry in the Greek syllabus and for Greek teachers.

The paradoxical place of Euclidean geometry

According to the Lyceum syllabus, students have to learn a geometry based on axioms with formal reasoning (grade 10) and measurement of magnitudes becomes the main geometric topic at grade 11. The unique geometry textbook is entitled "Euclidean Geometry" and it is used in the two first classes (grade 10 and 11). Its content is close to the syllabus and to the classical Euclidean Geometry with a strong axiomatic point of view, except for measurement. In textbook, and for lyceum teachers, geometry starts from zero with Euclidean axioms. Construction problems are of theoretical nature with letters and magnitude, such as AB=a, without any measure: geometrical tools are virtual and consist of compass and ruler according to the Euclidean tradition.

If Geometry is taught in compulsory education and during the two first lyceum classes (till grade 11), geometric knowledge is not assessed at the very important lyceum final test: the University where students will enter depends of this final test. Students know this fact and are less concerned with geometry than the others mathematics domains and do not work geometry especially in the numerous private institutes (*frontystiria*) where they could follow additional and expensive courses after the class time. It is a quite great contrast: a lot of geometry teaching times for nothing at the end? Teachers we interviewed told us that geometry is not important in the curriculum because of the hidden curriculum and, finally, “geometry is taught for culture, for Euclid”.

Teachers’ personal GWS

Gymnasium teachers think that pupils have to learn how to construct geometric diagrams, but they think that it is not the main point of mathematics learned in gymnasium. So as they have no time to teach all the syllabus, teachers often choose to teach very quickly diagrams constructions despite its importance and the fact that students have troubles with the use of drawing tools (especially the protractor) and with constructions. In the personal teacher's GWS, directed by GII, the aim of a diagram is to set a conjecture and the proof do not need an exact figure. That explains why teachers think that a freehand drawing is equivalent to a drawing with geometrical tools and the first one is done more quickly. Teachers’ local space could be anywhere they can draw a freehand diagram, for example a pack of cigarettes as two teachers told us. We see here a great difference between teachers’ beliefs and institutional content: in syllabus, nor in textbooks, there is none freehand drawing.

Another example of the prominence of GII in the personal teacher GWS is the importance they give to properties of quadrilaterals and triangles. They all think that these properties are fundamental even if they do not know the role of these geometric objects in mathematics class. As teachers rate highly Euclidean Geometry, a sufficient reason to teach triangles and quadrilaterals is given by their importance in the theoretic system of reference.

To conclude this part, we can say that the teachers’ GWS is clearly directed by a strong GII, almost GIII because of the axiomatic theoretic system of reference.

GWS TENSION

The new Greek curriculum demands to take into account reality. But the interviewed teachers told us how it is difficult for them: they do not know how to teach in a constructive way which is often opposite to their top-down learning conception. They concluded that Greek teachers do not like this new way of teaching and do not understand it. Teachers' learning beliefs agree with the internal slide we pointed out about the everyday life in curriculum.

In the case of diagrams constructions, teachers' GWS is clearly against the institutional GWS, and not only in considering freehand drawing. Teachers do not only prefer teaching others geometric topics but they give all the diagrams in tests too to go over the lack of their pupils [3]. The same opposition to the institutional GWS can be seen with the use of tracing paper. According to syllabus, tracing paper has to be used as a geometric tool in A' class (grade 7). It is used in many places with a particular and original graphical representation in the A' textbook and it is explained how to use it. But creativity stops at the school border and tracing paper is never used in class!

In gymnasium, formal proof is usually taught during the last class year (grade 9), more specifically, in a Euclidean section about triangle congruence. In order to know how teachers could initiate their students to the formal proof in one year, we asked them about the possible use of the four solved activities we spoke about in the "Formal proofs: the GII paradigm" section. They are indeed proof models and, for assessment, students have to learn ten lesson proofs by heart which one of them is asked in test. This proof process initiation is again opposite to the curriculum expectation.

In gymnasium, there is a distance between institutional and teachers GWS. That creates a tension which is supported by the different beliefs on learning and geometry among teachers and curriculum writers. Moreover, teachers do not really deal with the existing and remaining students' difficulties with diagram constructions and the proof process initiation is based on a learning by heart. This tension between institutional and teachers' GWS is specific to gymnasium, it completely disappears at lyceum, but what about pupils?

CONCLUSION

Geometry positions in Greece and in France are closed even if we point out some main differences. In both countries, even if curriculum emphasizes its place, reality is not taken into account. Similarly, the transitions between paradigms GI and GII are most of the times ambiguous and implicit and give rise to fuzzy GWS.

The GI paradigm seems to be more assumed in Greece than in France and in France formal proofs are taught all along the junior high school. But the main curriculum difference takes place at the lyceum: in Greece, axiomatic Euclidean geometry is taught, not in France, and in France geometry is assessed in final test for some sections, not in Greece. Geometry is taught in Greece only for cultural reasons, for Euclid, whereas in France the geometrical work is oriented by the GII paradigm and university studies. However, according to the six teachers' interviews, the Greek teachers' GWS is quite different from the French teachers' GWS because of the axiomatic theoretic system of reference: GII paradigm is well structured and stronger in Greece than in France. In Greece, the cultural tradition of Euclid is more important than in France and geometry knowledge seems to come from the Book [4]. This last point strengthens the GWS tension in junior high school which seems to be stronger in Greece than in France.

NOTE

1. The exercise on a map are in GI, but it could be solved by visualization or measurement, pupils have to choose.

2. This non explicit slide can also be seen, for example, at page 227 of A' textbook, examples 1 and 2.

3. In the A' final test we studied there is no construction; lyceum pupils have problems with geometric diagrams constructions, even for the equilateral triangle whereas it is a skill of the A' gymnasium class (grade 7).

4. According to Toumasis (1990) the Book is not Euclid's Elements but Legendre's geometry elements.

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