THE ROLE OF CONTEXT IN STOCHASTICS INSTRUCTION

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This report focuses on a research project that combines two aspects of a stochastics curriculum related to teachers' classroom practice, and their students' stochastical knowledge and beliefs. Data were collected with questionnaires. The development of the questionnaires derived from results of a qualitative research project will be sketched. Afterwards, some results concerning the role of the context will be discussed.

Keywords: stochastics teachers, students' learning, beliefs, role of the context

INTRODUCTION

One central aim of the teaching of stochastics in school is to prepare students to deal with real stochastic situations in their lives (Jones, Langrall, & Mooney, 2007). This aim involves two goals, the students' comprehension of stochastical concepts, and the students' awareness that it is possible to use stochastics to cope with specific real situations. There is a wide consensus between researchers into stochastic education that to achieve these two goals, students must explore stochastical concepts on the basis of realistic situations instead of exploring solely pseudo realistic situations (cards, urns, dices) or learning stochastics in a formal and abstract way (e.g. Jones et al., 2007). While there is a consensus about the *role of the context* for the teaching and learning of stochastics, there is, however, still little insight into the daily teaching practice of "conventional" stochastics teachers. In this report, the results of a research project involving a quantitative survey concerning the classroom practice of German stochastics teachers will be discussed. The main focus is the role of the context based on the following aspect:

1. The teachers' beliefs about the goals of teaching stochastics,
2. the students' beliefs about the usefulness of stochastics, and
3. the impact of the teachers' beliefs on the students' beliefs.

The research project discussed in this report is part of a larger research project involving a qualitative designed investigation of stochastics teachers' classroom practices and the impact of the latter on students' learning (Eichler, 2008a; Eichler, 2007). The results of the qualitative part of the research that provides the basis for the quantitative survey will be sketched in the following.

RESULTS OF THE QUALITATIVE RESEARCH

The first step of the qualitative research comprised an interview study with eight stochastic teachers (Eichler, 2007a). This study yielded four types of (individual) statistics curricula that are similar concerning the content, but considerably differ with regard to the teachers' objectives or beliefs. The distinction between the four types is characterised by differences of the teachers concerning two dimensions. The first dimension can be described with the dichotomous pairs of a static versus a dynamic view of mathematics or stochastics. The second dimension can be described with the orientation on formal mathematics versus mathematical applications. The four types of statistics teachers were characterised with reference to their main objectives as follows (Eichler, 2007a).

(dimension 2)	static view of mathematics	dynamic view of mathematics
application	*Application preparers*: their central goal is to have students grasp the interplay between theory and applications. Students firstly must learn stochastical theory in order to cope with mathematical applications later.	*Every-day-life preparers*: their central goal is to develop stochastical methods in a process, the result of which will be both the ability to cope with real stochastic problems and the ability to criticise.
formal	*Traditionalists*: their central goal is to establish a theoretical basis for stochastics. This involves algorithmic skills and insights into the abstract structure of mathematics, but it does not involve applications.	*Structuralists*: their central goal is to encourage students' understanding of the abstract system of mathematics (or stochastics) in a process of abstraction which begins with mathematical applications.

(dimension 1)

Figure 1: Four types of stochastics teachers

The second step of the qualitative research comprised the observation of the classroom practice of four teachers (Eichler, 2008a). One central result of this step of observation was that the instructional practice of the teachers provides strong evidence that they pursue their main objectives. Concerning the role of the context, the traditionalists and the every-day-life-preparers represent the extreme positions. The students of the traditionalists predominantly explore stochastical concepts on the basis of formal or pseudo realistic situations (cards, urns, dices). They seldom explore realistic situations. In contrast, realistic situations are crucial in the classroom practice of the every-day-life-preparers. Their students predominantly explore stochastical concepts on the basis of realistic situations or real problems, which arise, for instance, from articles of newspapers.

The third step of the qualitative research comprised an interview study with five students of each of the four teachers who were observed before. In this step the construct of statistical knowledge (Broers, 2006) and the distinction of declarative knowledge, procedural knowledge, and conceptual knowledge (Hiebert, & Carpenter, 1992) was used to describe the students' knowledge (Eichler 2008a). A central result of the third step of the qualitative research was that the students differ in their knowledge and beliefs. The differences consist between the students of one teacher, and between sets of students of different teachers. The students also differ concerning the role of the context. Thus, the students differ in the use of stochastic situations (formal, pseudo realistic or realistic) to explain stochastical concepts. Further, the students differ considerably concerning their beliefs about stochastics and mathematics referring to their relevance for society and their relevance for the own life (Eichler, 2008a).

METHOD

In regard to the characterisation of the four types of teachers (figure 1), a questionnaire including four parts was developed. The first part concerns the instructional contents of stochastics courses. The other three parts of the questionnaire concern the teachers' objectives of statistics and mathematics instruction. In each of the latter three parts of the questionnaire the teachers were asked to rate typical statements of the teachers who represent one of the four types (from full agreement to no agreement, coded with 1 to 5). In these three parts respectively two statements of every type have to be rated.

The questionnaire for the students involves items concerning declarative knowledge and conceptual knowledge. Concerning their *declarative knowledge*, the students were asked to rate a list of 28 statistical concepts whether they: are not able to remember the statistical concept (coded with 0), are able to remember the statistical concept (coded with 1), are able to approximately explain a statistical concept (coded with 2), are able to exactly explain a statistical concept (coded with 3).

Concerning the conceptual knowledge, the students were asked to indicate interconnections into the consecutively numbered concepts (category *declarative knowledge*)

Four parts of the questionnaire comprise the role of the context. Thus, the students were asked to indicate

- stochastic situations of the classroom (category *application*).
- statistical applications along with related statistical concept (category *connections*).
- real situations (outside of the classroom), for which stochastics may be useful (category *benefit*).
- the benefit of stochastics for students' future life, the benefit of stochastics for the students' professional career. These two categories were linked with a single item, in which the students are asked to rate the relevance of stochastics for their lives from high relevance (coded with 5) to no relevance (coded with 1, category *relevance-life,* and category *relevance-profession*).

A random sample of 240 German secondary high schools was selected. These schools were asked to conduct the survey. 166 of these agreed. Two teachers' of each of these schools and three students per teacher with different statistical performance were asked to fill out the questionnaire (January to July 2007). The completed questionnaires of 107 teachers and 315 students were analysed. The stochastics courses last between three and six month with three to five hours a week.

RESULTS CONCERNING THE TEACHERS

The statistics curriculum is dominated by the so called classical block of probability (see table 1).

Block	Topics and percent of teachers teaching the topic (n=107)
Classical block of probability	Frequencies (98%), Laplacean probability (97%), statistical probability (72%), probability tree (100%), Bernoulli experiment (99%), binomial distribution (100%), expected value (95%), standard deviation (95%)
Inferential statistics	Hypothesis testing (89%), confidence intervals (51%), Bayesian statistics (27%)
Conditional probability	Conditional probability (81%), (in)dependence (80%), theorem of Bayes (74%)
Distributions	Normal distribution (79%), hypergeometrical distribution (49%) Poisson distribution (49%)
Descriptive statistics	Frequencies (98%), mean (87%), spread (74%), median (52%), regression and correlation (16%)

Table 1: Percentage of teachers teaching different instructional content

Factor analysis concerning the objectives of the teachers' statistics curricula in the responses to questionnaires yield three interpretable factors (table 2) which include 15 of the 24 items referring to the objectives of the statistics curriculum. For each factor the number of items and the Cronbach's Alpha is shown in table 2.

Factor	Factor 1 (5 items, $\alpha = 0.689$)	Factor 2 (6 items, $\alpha = 0.725$)	Factor 3 (4 items, $\alpha = 0.779$)
Interpretation	Traditional curriculum, little reference to real data	Curriculum with high reference to real data	Curriculum with high reference to process

Table 2: Factors concerning the objectives the statistics curriculum

In the following the main focus is on the first two factors or rather on the teachers with a high acceptance to the items of one of these two factors. These items are shown in the following table. The items involve a statement of a teacher who represents one of the four types of stochastic teachers (figure 2). The type is indicated in the brackets (T: traditionalists; S: structuralists; A-P: application-preparers; E-P: every-day-life-preparers).

Factor 1	Factor 2
- The objective of teaching stochastics is to establish a theoretical foundation of stochastics (T). - Students must learn to deal successfully with abstract and formal systems (S). - Algorithmic skills constitute the basis of learning statistics or mathematics	- The main goal of the teaching of stochastics is the students ability to understand decision-making processes in our society (E-P) - Students must explore stochastical concepts solely on the basis of real stochastic situations (E-P). - Students must learn to use stochastical or mathematical theory to be able to argue referring to real problems (A-P).

(T). - Students must be well prepared concerning final exams and studies (T). - Students must learn a precision in reasoning in order to deal successfully with abstract and formal mathematics (S).	- Students must understand that stochastics or mathematics is part of the general ability of problem solving (E-P). - Students must learn to solve real problems either for their own or in a team (E-P). - Students solely will be motivated if they understand that stochastics or mathematics is applicable in the reality (A-P).

Table 3: List of the items included in factor 1 and factor 2.

The correlation coefficient between factor 1 and factor 2 is - 0,1. For the distinction between teachers with high acceptance to the items of one factor and low acceptance to the other, two clusters were defined by the medians concerning the value of the two factors. Cluster 1 includes those teachers with high acceptance to factor 1 and low acceptance to factor 2. Cluster 2 includes those teachers with high acceptance to factor 2 and low acceptance to factor 1. Cluster 1 includes 39 teachers, cluster 2 34 teachers.

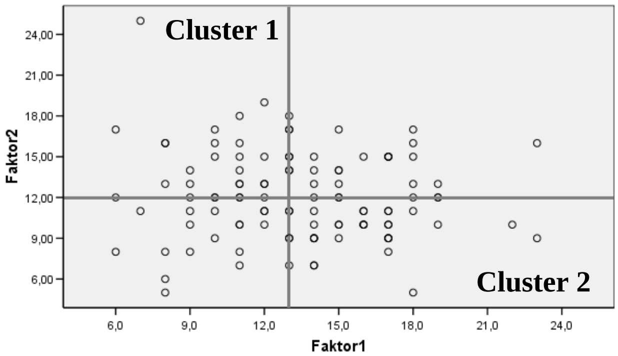

Figure 2: Clusters of teachers concerning factor 1 and factor 2

RESULTS CONCERNING THE STUDENTS

Figure 3 shows the results concerning five categories:

1. the students' self estimated ability to explain the 28 different stochastical concepts (the students' declarative knowledge),
2. the number of connections between two different stochastical concepts as part of the students conceptual knowledge (for instance: if a student indicated the connection between the three concepts of expected value, variance and standard deviation, the number of possible connection is 3 over 2 or rather 3)
3. the number of stochastic situations of the classroom (*application*).
4. the number of pairs of applications and statistical concept (*connections*).
5. the number of real stochastical situations (*benefit*).

Due to the fact that different teachers indicated different numbers of stochastical concepts taught in the classes, figure 3 shows the results concerning the category *knowledge weighted*. For this category the students' self estimated knowledge is divided by the number of concepts taught by the teachers. This category alludes to a restricted sample, which involves the set of completed questionnaires of one class (some of the completed questionnaires allude only to the teachers or only to the students).

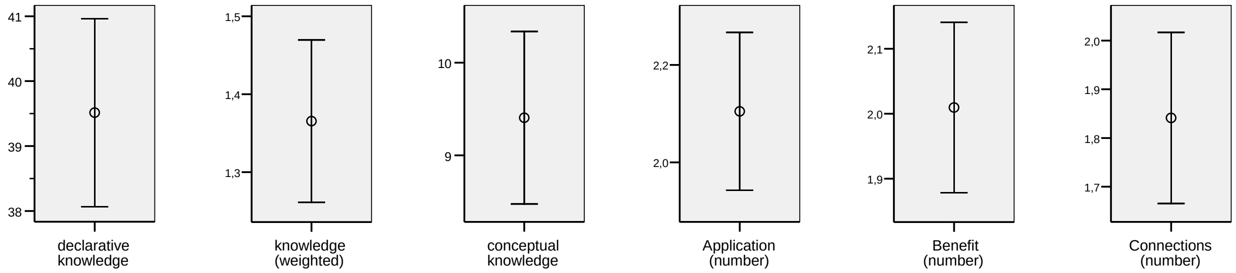

Figure 3: Results concerning the students knowledge and beliefs (average and 95%-interval)

The interpretation (only for the averages) is as follows: The sum of the students' self-estimations concerning the 28 given stochastical concepts is in average about 39. In average, the students rate their knowledge about the stochastical concepts taught by their teachers with about 1,4. The students indicate more than 9 connections between different stochastical concepts, they indicate about 2,1 stochastical situations of the classroom and about 2 stochastical situations outside of the classroom. Finally, the students indicate in average about 1,9 connections of a stochastical situation and a specific stochastical concept.

Concerning the role of the context it is important whether the indicated stochastical situations to the categories application, benefit, and connections refer to realistic situations or pseudo realistic situations (the pseudo realistic situations include games of chance). Table 4 shows the distribution of the indicated stochastical situations (with the number of indications in brackets) for the first two categories:

Application		Benefit	
realistic situations (255)	pseudo realistic situations (385)	realistic situations (359)	pseudo realistic situations (270)
quality control (48)	game of chance (100)	economy (63)	game of chance (100)
forecasts (30)	lottery (91)	quality control (45)	lottery (78)
elections (28)	dice (66)	elections (39)	poker (13)
statistics (24)	urns (33)	statistics (37)	bets (18)
clinical diagnostic (23)	coins (23)	polls (32)	dice (14)
polls (16)	cards (15)	clinical diagnostic (26)	bingo (13)
economy (16)	poker (13)	further education (26)	
weather (11)	lots (10)	weather (17)	
		stock market (16)	
		insurance (12)	
other situations with less than 10 indications		other situations with less than 10 indications	

Table 4: Distribution of stochastical situations and number of indications in brackets

The stochastical situations are topics: the situation *economy* includes, for instance, market research, promotion and some more specific situations. Although some of the stochastic situations were indicated for both categories, application and benefit, it is obvious that

- concerning the category benefit, the pseudo realistic situations are restricted to existing games of chance, and
- concerning the category application, the majority of situations refers to pseudo realistic situations.

Some of the indicated situations stem from typical tasks in German textbooks, in particular quality control, elections, and clinical diagnostic. Students predominantly use these three different situations connecting a stochastical situation with a specific stochastical concept. The students, however, more often use pseudo realistic situations for connecting a stochastical situation with a specific stochastical concept, and, in this case, predominantly dice, urns and lottery (see table 4).

Realistic situations (157)		Pseudo realistic situations (341)	
Situation	Connected stochastical concepts	Situation	Connected stochastical concepts
Quality control (85)	hypothesis testing (17), binomial distribution (6), confidence interval (5), Bernoulli experiment (4), conditional probability (4) normal distribution (3), expected value (2), spread (2), probability tree (1), combinatorics (1) 2 x 2 table (1)	Dice	Laplacean probability (36), Bernoulli experiment (14), probability tree (9) random experiment (7), expected value (5), binomial distribution (4) probability (2), statistical probability (2), normal distribution (2), hypothesis testing (1), variance (1) simulation (1), combinatorics (1)
Clinical diagnostic (33), elections (9)		Urns (79), lottery (53)	

Table 5: stochastical situations and related stochastical concepts

Obviously, students remember predominantly connections between pseudo realistic situations and specific stochastical concepts. Further, the variation of indicated stochastical situations concerning the category connections is much lesser than the variation of indicated situations concerning the categories application and benefit.

Although the students estimated their declarative knowledge by themselves, these estimations give evidence of the students' factual knowledge. Thus, the correlations between the students' declarative knowledge and other categories discussed above are shown in table 6:

	conceptual knowledge	Application		benefit		connections	
		realistic situations	pseudo realistic situations	realistic situations	pseudo realistic situations	realistic situations	pseudo realistic situations
declarative knowledge	0,418**	0,172**	-0,233**	0,277**	-0,181**	0,269**	-0,177**

Table 6: Correlations between students' declarative knowledge and 5 other categories

The correlations are predominately weak, although they are significant different from zero. However, the correlations as a whole give evidence that the students' self estimated declarative knowledge measure in some sense the students' flexibility of dealing with statistical concepts. Further, there is evidence that the higher the students' flexibility of dealing with statistical concepts is the higher their reference to realistic statistical situations is, and the lower the reference to pseudo realistic situations is.

TEACHERS – STUDENTS

To prove possible interrelations between the teachers' orientation concerning the goals of the stochastics instruction and the students' knowledge and beliefs, the sample must be restricted. This was necessary, because sometimes a teacher sends his completed questionnaire back but his students not, sometimes the students send their completed questionnaires back, but the teacher not. Two strategies were used for the following analysis. Firstly, the correlations between the factors, i.e. factor 1 and factor 2 (or rather the sum of ratings the teachers given to the items of the two factors), and the categories concerning the students (knowledge weighted, application, benefit, and connections). Secondly, the clusters of teachers defined above (figure 2) were used to split up the sample of the students. The averages of the two new samples concerning the categories knowledge weighted, application, benefit, and connections were compared by a t-test.

Figure 4: Students' weighted knowledge and students' procedural knowledge. f1F2: teachers, who have low acceptance to factor 1 and high acceptance to factor 2, F1f2: teachers, who have high acceptance to factor 1 and low acceptance to factor 2

Most parts of the analysis give no evidence of an interrelation between the teachers' orientation and the students' knowledge and beliefs. For instance, concerning the clusters of teachers, who have low acceptance to factor 1 (traditional curriculum) and high acceptance to factor 2 (curriculum with high reference to real data) or who have low acceptance to factor 2 and high acceptance to factor 1 (see figure 2), the distribution of the students' weighted knowledge and the students' ability to indicate connections between stochastical concepts (figure 4).

Although there are differences in detail, these differences are statistically not relevant. Thus, there is little or no evidence that a teacher's orientation towards a traditional curriculum (factor 1) or a curriculum that includes real data (factor 2) promote (or impede) students' learning in reference to the students' declarative knowledge, the students' conceptual knowledge, and the students' beliefs concerning the relevance of statistics except the category benefit. For this category t-test give some evidence that the students of teachers with high acceptance to factor 2 and low acceptance to factor 1 use more often realistic situations than pseudo realistic situations to explain the relevance of stochastics for the society. However, the differences are not significant (table 7).

Benefit	Realistic situations (F1f2)	$\bar{x} = 1{,}14$	Psudo realistic situations (F1f2)	$\bar{x} = 0{,}66$
	Realistic situations (F1f2)	$\bar{x} = 0{,}83$	Psudo realistic situations (F1f2)	$\bar{x} = 1{,}00$
		$\alpha = 0{,}121$		$\alpha = 0{,}063$

Table 7: Difference of the students concerning the category benefit

In contrast to the low interrelations between the teachers' objectives concerning the statistics curriculum and their students' knowledge and the students' beliefs, there is stronger evidence that the amount of contents has an impact on the students' knowledge. So, the greater the number of statistical concepts taught by the teachers is, the lower the declarative knowledge of the students seems to be (Pearson's correlation coefficient r = -0,43**).

CONCLUSION

The results of the quantitative survey concerning the curriculum of statistics teachers and the learning of students give evidence that:

- "The traditional way of teaching statistics, with its heavy emphasis on formal probability" (Broers, 2006, p.4) is still existent in German secondary high schools;
- the teachers' instructional contents are similar, but the teachers' objectives differ considerably;
- the quality of students' declarative knowledge affects their conceptual knowledge and their beliefs concerning the relevance of statistics;
- the students predominately indicate few realistic situations to explain both the relevance of stochastics for the society and connections between stochastical situations and specific stochastical concepts;
- the teachers' orientation towards a curriculum with high reference to real data seems to affect the students' ability to use realistic stochastical situations to explain the relevance for the society.

However, the latter interrelation between the teachers' orientation and the students' beliefs is weak. Above all, there is no evidence for the impact of the teachers' orientation and the students' knowledge and beliefs. The lack of statistical relevant interrelations between the teachers teaching and the students learning may be caused by the fact, that there are only small differences of the teachers' stochastics teaching with the emphasis on probability. It is possible that a stronger orientation to a data driven curriculum has a stronger impact of the students' knowledge and beliefs concerning the role of the context. Further it is possible, that the quantitative survey discussed in this report is not able to measure possible differences concerning the students' knowledge and beliefs. There is some evidence that qualitative research can show differences in detail between students' of teachers who have different goals concerning the role of the context (see Eichler, 2008a).

However, the stochastics teachers' teaching is determined by the teachers' fundamental orientation, i.e. the teachers' system of objectives (or beliefs) concerning stochastics teaching. Pajares (1992) stated that it could be difficult to change the teachers' central beliefs. One approach to change these central beliefs may start by the teachers' conviction that a changed curriculum actually will promote students' stochastical knowledge. For this reason it would be desirable to have more research into the stochastics teachers' curricula, the students' stochastical knowledge and beliefs, and, in particular, the relations between stochastics teachers' curricula and the students' stochastical knowledge or beliefs.

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