WHAT WORKS IN THE CLASSROOM - PROJECT ON THE HISTORY OF MATHEMATICS AND THE COLLABORATIVE TEACHING PRACTICE

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This paper describes the project that was undertaken in the South East of England, and which aimed to introduce the history of mathematics at the primary and secondary level. The project was conducted through collaborative teaching practice (peer based network of teachers collaborating on research, planning, teaching in teams, and assessing the outcomes of lessons) and was based on the premise that the history of mathematics can improve both the motivation and attainment when used as a contextual background in the teaching of mathematics at this level.

THE PROJECT BACKGROUND

The project described here was one of the first few projects awarded the support by the National Centre for Excellence in the Teaching of Mathematics (founded in June 2006). Aims of the project were to:

- Introduce the history of mathematics into everyday teaching in order to
    - Encourage students to begin making the connections between mathematical topics
    - Increase interest and motivation by setting the problems in historical context
    - Enrich mathematical understanding through historical explorations
    - Assess the role of the history of mathematics in setting the new curriculum
- Introduce collaborative teaching practice as a model of continuing professional development, at the same time adopting an inquiry-led learning approach to the lesson development thus raising issues about
    - Teachers learning with pupils (simultaneously in some cases) and the effects this may have on his or her professional role
    - Training preparation for teachers in an inquiry-led learning environment.

The answers to these questions will be provided in this paper in two-fold ways: through the personal reflections of teachers who participated in the project, and through a synthesis and explanation of methods used throughout the project. The latter is provided as a way of suggesting the model of continuing professional development for teacher groups and networks wishing to introduce the historical element into the teaching of mathematics through collaborative practice.

The project began in September 2006 and was completed in September 2008 with a national conference held at the London Mathematical Society at which experiences of the teachers involved were disseminated among the mathematics education community. Over the course of the project three secondary schools, with a total of fifteen teachers (two of whom were science specialists but taught mathematics to lower ability groups), and three primary schools with a total of three teachers have been involved. More than 450 pupils have been involved in the project at various times, spanning the age range between ten and fourteen (English Key Stages 2 and 3) and covering all ability ranges.

The project has been conceived and led by the author of this paper, and, as already mentioned, was supported by the National Centre for Excellence in the Teaching of Mathematics (UK). In the second year of the project the British Society for the History of Mathematics provided financial and organisational support; the University of Plymouth Centre for Innovation in Mathematics Teaching provided the training for all involved teachers in the principles of collaborative teaching practice, and the British Society for the History of Science provided extra funds for the final conference celebrating the project. An additional private consultant has been involved in the project in the second year, offering support in the matters of teacher training and the uses of the history of mathematics in development of mathematical pedagogy.

The new curriculum for England and Wales

The recent changes in the National Curriculum, and the new approach taken by the Qualifications and Curriculum Authority (QCA) introduced a certain amount of freedom for teachers, teacher teams, and consortia of schools to develop their own syllabus in all subjects. The modernising of the curriculum is driven by the need to take into account local needs and needs for different types of vocational training. One of the more positive aspects of this development may be seen in the fact that the local provision of education will have a degree of freedom (not yet defined), and that personalised learning, project based work and mentoring will all have a big role to play in this new vision of education. This opens a valuable opportunity for teachers to demonstrate that mathematics, like any other creative pursuit, is an area where exciting and useful contributions can still be made – both by teachers and by pupils. As such, the introduction of the historical element in the mathematics syllabus, although not sufficiently developed in the quote that follows, offers the ***possibility*** of developing teaching strategies which do not necessarily provide only historical context, but use the history of mathematics as a tool for discovering facts and exploring mathematical techniques. The new curriculum states that the students should recognise the 'rich historical and cultural roots of mathematics':

> Mathematics has a rich and fascinating history and has been developed across the world to solve problems and for its own sake. Students should learn about problems from the past that led to the development of particular areas of mathematics, appreciate that pure mathematical findings sometimes precede practical applications, and understand that mathematics continues to develop and evolve.[1]

Since the completion of the project, and based on the recommendations following from the project report, measures are being taken by the Joint Mathematical Council (UK) to define the ways in which history of mathematics can and should be deployed to help shape the future development of the curriculum, and the teacher pre-, and in-service training development and provision.

The current challenge now facing English teacher-training institutions will be to address the imbalance between the desire to introduce the historical element to the teaching of mathematics and a lack of the formal teaching in the subject area for the serving teachers. The project described can therefore, give a valuable insight into the types of issues facing teachers in this situation, with a view of defining some benchmarks on which it would be possible to base a programme of in-service training in the history of mathematics.[2]

METHODOLOGY, ACTIVITIES, DATA

Collaborative Teaching Practice and the History of Mathematics

The project has been pursued by practicing teachers with various degrees of experience in the teaching of mathematics (not all of whom are subject specialists), and therefore the question arose of how to create a professional learning environment which would be able to contain all levels of experience and mathematical ability in order to support their participation. Of major interest was the possibility of introducing a model of continuing professional development based on a set of principles which could be replicated elsewhere and which would help teachers develop a range of techniques, and introduce a new element which could help them structure their own learning at the same time as structuring their teaching programme.

We chose the model of collaborative teaching practice as one which would offer opportunities for teachers to develop their subject knowledge through research into the history of mathematics. Collaborative teaching practice was developed in different countries as far back as the 19th century (most prominently Japan, but recently also in the United States and England) and is sometimes also closely linked and/or referred to as 'lesson study'.[3] The collaborative teaching practice that was part of the described project as a way of peer-discussion and collective teaching tool was based on the simple cycle of planning - researching - sharing resources - teaching collaboratively - and finally assessing the outcomes of a lesson.

At the core of this envisaged professional learning model stood a belief that the interest and personal development can only be achieved in those situations and environments where the professionals themselves find an area of research they would like to pursue further.

Various mathematics educators have seen the different roles the history of mathematics can take through its introduction into the education of mathematics teachers - Freudenthal (1981) for example conceived it as giving a background to the teachers' mathematical knowledge, while others concentrated on offering a

possible pathway to the deepening of teachers' reflection capabilities through an in-depth study of the development of mathematical concepts through history (see Arcavi, Bruckheimer, & Ben-Zvi, 1982, 1987; Swetz, 1995). One of the approaches, developed by Hsieh and Hsieh (2000), and Philippou and Christou (1998a, b) dealt with using the history of mathematics as a particular tool and context to develop beliefs and attitudes in mathematics.

The benefit of the use of history of mathematics however, in the context of the described project, can be best seen on the influence in which it created an opportunity for a ***focus*** of cooperation and collaboration as well as an impetus for the creation of a ***conceptual landscape*** which offered opportunities to teachers to develop their individual interests.

This highly individualist approach to the continual professional development of teachers can increase their subject knowledge and enable them, through the modern technologies, to share their experiences and knowledge with mathematics teachers and students from around the world. Our agreed aim was to adopt a creative and individualistic ethos in teaching, providing ample opportunity for bringing the history of mathematics alive to the present generation of school children. Eventually, in practical terms, the defined foci were enlarged to include, apart from the collaborative teaching practice and the individual research, the creation of a networking platform in the form of web-quests[4].

Teachers' learning in an inquiry-led learning environment, and the collaborative teaching practice

The inquiry-led learning as developed throug[5]h this project grew organically from the collaboration with similary-minded colleagues. The successful outcomes were produced in those instances in which a few necessary prerequsites were fulfiled - existence of full professional trust and exchange of information and knowledge had to be devoid of all performance management in participating groups of teachers. Collaborative teaching practice was described in the teacher reflections thus:

> The students appreciated the teachers cooperating between themselves and being more relaxed and focused on learning rather than discipline.
>
> It (this project) has certainly been a huge milestone in my professional development. Firstly, it has shown me the true value of collaborative teaching and the focus on the 'learning' rather than the 'teaching'. Secondly, it has made me question why I am teaching what I am teaching, and how to help the children answer the 'why' do we do this questions by giving them relevance and meaning to the maths. My next milestone experience will be to embed this into my teaching and more crucially into the teaching of my colleagues.

History of mathematics and the development of the curriculum

In the description of the other aspects of this project it is described how the history of mathematics helped shape the building of the professional learning environment

which then spilt over into the classroom. Historical dimension, apart from earlier mentioned benefits (see pages 1-4) was also important for teachers in terms of their involvement with the whole-school issues:

> The maths becomes 'embedded' in the culture and life and is not seen as something totally dry and devoid of meaning. This also changed the perception of mathematics in my department… (by a science teacher)

> There is a large scope in my school to bring about change in the mathematics curriculum and I am hoping to introduce an element of the History of Maths into the curriculum. 'Using and Applying Mathematics' is the common strand that is across the whole maths curriculum, and my experience on the project is that practical maths (in and out of the classroom) is a powerful medium by putting the children in the shoes of mathematicians from history so they can appreciate the 'why' and not just the 'how'.

OUTCOMES - STRUCTURING THE SELF-REGULATORY CONTINUING PROFESSIONAL DEVELOPMENT THROUGH COLLABORATION AND RESEARCH

The project showed how the history of mathematics can set the 'scene' and act as a catalyst in creating a professional learning environment as well as giving a structure to endorse inquiry both in the student and in the teacher. In mathematics, this dimension is or can be, added to any such particular conceptual landscape.

The history of mathematics and the process of reorientation

As Furinghetti has shown (2007) some teachers tend to believe that the style of mathematics teaching they were affected by or exposed to must be reproduced in their own practice. In the case of the described project, this was most evident in the attitudes of teachers who were non-specialists in the subject. Furinghetti showed that the history of mathematics context allows for an exploration of topics in a new light and hence helps teachers construction of teaching sequences. While this was one of the added benefits of introducing the history of mathematics into the collaborative practice, we were also aware of the uses of history of mathematics in teaching, therefore allowing us to explore the various roles the history of mathematics can take in the classroom practice.

Whilst the history of mathematics in teacher education programmes has been described at some length by Furinghetti (2007), Schubring (Schubring et al., 2000), and Heiede (1996), little has been so far written about the in-service training of practicing teachers in this regard. This project aimed to begin the task by making a sketch of the possible influence the history of mathematics can have on in-service specialist and non-specialist mathematics teachers.

Therefore one of the project's aims became to try to introduce what Furinghetti (2007) calls 'reorientation':

> …the learners involved in the process … are forced to find their own path towards the appropriation of meaning of mathematical objects.[6]

In this context, the acquisition of meaning was attempted through exposing beliefs about, and the partial understanding of, the concept in question with the new, 'foreign' meaning:

> A meaning only reveals its depth once it has encountered and come into contact with another, foreign meaning: they engage in a kind of dialogue, which surmounts the closedness and one-sidedness of these particular meanings.[7]

In short, one of the teacher testimonies illustrates these described process thus:

> … I was… astounded (by)… the depth there is in so many topics we have covered through this project. It has rekindled interest in mathematics in me; students find it interesting as well.

Scaffolding knowledge for non-specialist mathematics teachers

An increasing body of research shows that inquiry-based-learning helps create an environment in which the teacher may be required to act in manifold ways.[8] These manifold roles of a teacher relate to the theory of 'Knowledge Manifolds', in which teachers are 'promoted' from teacher/preacher to teacher/consultant and teacher/resource type of roles. Naeve (2005) defined the 'Knowledge Manifolds' as 'linked information landscapes (contexts) where one can navigate, search for, annotate and present all kinds of electronically stored information'.[9] Such open information landscapes have developed with an exponential speed since the founding of Wikipedia (domain launched only in January 2001), and rest on fundamental principles of communal and self-governance in the same way in which Naeve suggests future 'teaching landscapes' will develop. This theory is in concordance with the network theories of knowledge as much as it is with the theory of 'mobile learning'. The described project opted to further explore in practice such approach to teaching and learning in which teachers are as much learners as their pupils by making parallels between the sets of teachers with the sets of pupils. Some teacher reflections addressing this particular aspect are:

> This project has developed my skills to be able to find resources and to try to relate things to the history.

> Research was good for subject knowledge; because of the historical content, it widened our own perspective about mathematical topics, and gave us time to find about something in more depth.

> Historical element shows you the different aspects of something in more depth; it allows for 'scaffolding' of the knowledge and easier transference to children. The historical element can also offer easier focus.

Furthermore, Naeve's (2005) approach to knowledge which he identifies as that consisting of 'efficient fantasies' and learning as that consisting of 'inspiring fantasies' has a lot to offer in the context of creating a learning environment in which both teachers and students discover new facts and exchange ideas in a more elaborate, creative, and yet mathematically sound ways. Naeve's description of fantasy has a lot to offer in terms of initiating a process of learning not only in the

here and now, but one that draws upon the initial interest in the 'fantasy' and how it (the fantasy) occupies a mind of a learner for a longer period of time, offering a prolonged urge to find ever increasingly new content about a subject matter. Teachers from the project spoke often about these 'fantasies' as most important in the initial stages of introducing a new mathematical topic or concept. The length of this paper does not, unfortunately, allow for further analysis on the subject matter in more depth.

What the conclusions teachers made however, agrees with Naeve's suggestion that the education process consists in

> …exposing the learner to inspiring fantasies and assisting her/him in transforming them into efficient fantasies.[10]

While Naeve somewhat exaggerated the view of the traditional 'learning architectures' being exclusively teacher-centric and consequently his concept of knowledge 'pushing' rather than knowledge 'pulling' may be lacking in subtlety, his intention to shift the focus onto the system of initiation into an interest field, whilst at the same time offering the system of skills to equip a learner with a set of tools to undertake the task of discovery and learning is at the centre of all: 'collaborative', 'flexible', and 'personalised' learning concepts.[11]

So far, as in the case of Mariotti (2000), the focus on developing strategies to initiate 'learning fantasies' has been on the pupils. In the new type of learning environment, one in which 'knowledge pulling' rather than 'knowledge pushing' is taking place, teachers and pupils are learners and communicators of insights into mathematical facts at the same time, interchanging roles at different levels. From the experience of our project it became clear however, that some of the roles of the learner and some of the roles of the teacher are interchangeable, whilst others remain strongly rooted in the

a) evolutionary roles and

b) social roles these two groups represent.

CONCLUSION

Although no external evaluation had taken place to date, the internal, self-evaluation, concluded that this was an invaluable opportunity for all teachers involved in the project in terms of re-awakening their interest in the subject and increasing their self-awareness on their abilities in terms of subject knowledge, pedagogy and ability to conduct academic research. Additionally, teachers identified acquisition of skills in terms of ability to envisage their own CPD landscapes through building 'knowledge patches' and increased ICT competencies as further valuable benefits of their involvement in the project.

The nature of learning is a constantly changing environment, in which learners are often ahead in terms of their technological competencies than their teachers. The knowledge content does not move at such a great speed, but it's presentation and availability is something that often lacks sophistication in the eyes of the learner. In

mathematics this is sometimes more often apparent than in subjects such as literature or history.

Mathematics learning has to gain an enormous amount from developing landscapes of knowledge patches that students can tap into through and because of their interests and abilities. This project began the process of enabling the teachers to be able to start developing these landscapes in collaborative environment, and having for a focus the wealth of resources that the history of mathematics has to offer.

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[1] Page 4 of the QCA Mathematics Curriculum, accessed 20th March 2008, <<http://curriculum.qca.org.uk/subjects/mathematics/keystage3/index.aspx>>.

[2] As this paper was being completed, the new module in the history of mathematics was being developed at the Open University UK, aimed at anyone interested in the history of mathematics.

[3] See Lewis (1995), Lewis and Tsuchida (1998), Stigler and Hiebert (1999), and more recently Fullan (2004), (2005).

[4] Self-contained websites offering materials for the study of particular mathematical topics. First webquest from this project is available from http://www.webquests.mathsisgoodforyou.com/.

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[6] Furinghetti (2007), 113.

[7] Bakhtin (1986), 7, as reported by Radford, Firinghetti, and Katz (2007), 108.

[8] Naeve describes these roles as that of "*knowledge cartographer* [who] constructs context maps, the *knowledge librarian* [who] fills the maps with content, the *knowledge composer* [who] combines the content into customised learning modules, the *knowledge coach* [who] cultivates questions, the *knowledge preacher* [who] provides answers, the *knowledge plumber* [who] routes questions and the *knowledge mentor* [who] provides a role model and supports learner self-reflection." Described in Naeve (1997).

[9] Naeve (2005), 6.

[10] Naeve, (2005), 4.

[11] All part of the national strategies on 'Every Child Matters', 'Personalised Learning' and 'Extended Schools'. See related sections at the <<http.//www.standards.dfes.gov.uk>>.