STUDENTS' BELIEFS ABOUT THE EVOLUTION AND DEVELOPMENT OF MATHEMATICS

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The paper is an empirical study of students' beliefs about the history of mathematics. 26 students in an upper secondary mathematics class were exposed to a line of questions concerning the evolution and development of mathematics in the form of a questionnaire and follow-up interviews. In the paper it is argued that the existing literature on students' beliefs, in general, lacks a discussion of goals dealing with, for instance, desirable beliefs among students in order to provide them with a more coherent image of mathematics as a discipline. A couple of descriptions from the Danish literature and upper secondary regulations are provided as an example of such a dimension. The concrete student beliefs from the research study are evaluated against these descriptions.

KEYWORDS: History and epistemology of mathematics; students' beliefs and images; a goal-oriented dimension for students' beliefs.

INTRODUCTION

Beliefs about the history of mathematics is a topic which is touched upon from time to time in the literature on history in mathematics education, e.g. in Furinghetti (2007) and Philippou and Christou (1998). However, when scanning these samples, one soon finds that these concern the beliefs of in-service or pre-service teachers. Studies on students' beliefs about the history of mathematics seem to be rather poorly represented in the literature, if not altogether absent.[1] One reason for this that I can think of is that, in general, studies of beliefs in mathematics education are conducted with the purpose of improving mathematical thinking, learning, and instruction.[2] Beliefs, both cognitive and affective ones,[3] are investigated in order to identify the 'ingredients' which do or do not make students capable of solving mathematical tasks or teachers capable of teaching differently and/or more effectively. Certain beliefs are identified as advantageous in the learning of certain mathematical contents, the solving of related tasks, etc., and educational studies are then conducted on how to change already existing beliefs into these more favorable ones. In this sense beliefs are regarded as means – or *tools* – to achieve understanding in the individuals' constructive learning process. Only rarely is providing students or teachers with certain beliefs, e.g. by changing existing ones, about mathematics or mathematics as a discipline considered as a *goal* in itself. And when this is done, the term 'beliefs' is usually not used. Instead mathematical appreciation, mathematical awareness, or providing students with a more profound *image* of what mathematics is, are the words or phrases more commonly used (e.g. Furinghetti, 1993; Niss, 1994; Ernest, 1998).

It seems to me that the beliefs discussion in mathematics education lacks a goal-oriented dimension. A dimension which addresses students' mathematical world view and proposes and evaluates some desirable beliefs in order to turn students into more critical citizens by providing them with intelligent and concerned citizenship and with some *Allgemeinbildung* in general (Niss, 1994). That is to say, to provide students with a more coherent image of mathematics as a discipline, the influence of mathematics in society and culture, the impact of society and culture on mathematics, and the historical evolution and development of mathematics as a product of time and space, to mention a few of the more 'pressing' ones. Occasionally researchers will touch upon these issues in the form of personal opinions, e.g. in curriculum development. However, a dimension about 'beliefs about desirable beliefs' – meta-beliefs we may call them – can only be addressed properly if the meta-beliefs are articulated as such, i.e. as goals in themselves.

In this paper I shall first present some extracts from the 2007-regulations for the Danish upper secondary mathematics program and the Danish report on competencies and learning of mathematics, the so-called KOM-report, which may serve as such a goal-oriented dimension for students' beliefs. Especially I shall focus on students' beliefs concerning the history of mathematics. Secondly, I shall report on a piece of empirical research in which a number of students were asked about their beliefs concerning the evolution and development of mathematics.[4] Thirdly, these students' beliefs are analyzed and evaluated against the goal-oriented descriptions. The paper is ended with some final remarks and reflections on the presented empirical data and the larger research study which they are part of.

THE DANISH CONTEXT

Since 1987 history of mathematics has been part of the formal regulations for the Danish upper secondary mathematics program (see e.g. Fauvel and van Maanen, 2000, pp. 5-7), and with the newest reform and the present regulations of 2007 this part has become more dominant. Students are now expected to be able to "demonstrate knowledge about the evolution of mathematics and its interaction with the historical, the scientific, and the cultural evolution", knowledge acquired through teaching modules on history of mathematics (Undervisningsministeriet, 2007, my translation from Danish).[5] The official regulations for the Danish upper secondary mathematics program of 2007 are to some extent based on the Danish report *Competencies and Learning of Mathematics*, the so-called *KOM-report*, (Niss and Jensen, 2002, title translated from Danish) where it says the following about history:

> In the teaching of mathematics at the upper secondary level the students must acquire knowledge about the historical evolution within selected areas of the mathematics which is part of the level in question. The central forces in the historical evolution must be discussed including the influence from different areas of application. Through this the students must develop a knowledge and an understanding of mathematics as being created by human beings and, in fact, having undergone an historical evolution – and not

> just being something which has always been or suddenly arisen out of thin air. (Niss and Jensen, 2002, p. 268, my translation from Danish).

In the report, the focus of integrating history of mathematics is discussed in terms of a certain kind of overview and judgment which the students should acquire as part of their mathematics education.

> The form of overview and judgment should not be confused with knowledge of 'the history of mathematics' viewed as an independent subject. The focus is on the actual fact that mathematics has developed in culturally and socially determined environments, and subject to the motivations and mechanisms which are responsible for this development. On the other hand it is obvious that if overview and judgment regarding this development is to have solidness, it must rest on concrete examples from the history of mathematics. (Niss and Jensen, 2002, p. 68, my translation from Danish)

The 2007-regulations describe the "identity" of mathematics in the following way:

> Mathematics builds upon abstraction and logical thinking and embraces a long line of methods for modeling and problem treatment. Mathematics is indispensable in many professions, in natural science and technology, in medicine and ecology, in economics and social sciences, and as a platform for political decision making. At the same time mathematics is vital in the everyday. The expanded use of mathematics is the result of the abstract nature of the subject and reflects the knowledge that various very different phenomena behave uniformly. When hypotheses and theories are formulated in the language of mathematics new insight is often gained hereby. Mathematics has accompanied the evolution of cultures since the earliest civilizations and human beings' first considerations about number and form. Mathematics as a scientific discipline has evolved in a continual interrelationship between application and construction of theory. (Undervisningsministeriet, 2007, my translation from Danish)

Thus, when the students are to "demonstrate knowledge about the evolution of mathematics" etc., as stated in the academic goals of the regulations, one must assume that it is within the frame of this "identity" that they are expected to do so. Another way of phrasing this is to say that one purpose of the teaching of mathematics at the Danish upper secondary level is to shape the students' beliefs about mathematics according to the above description of identity. The purpose of including elements of the history of mathematics has to do with showing the students that mathematics is dependent on time and space, culture and society, that mathematics is not 'God given', that humans play an essential role in the development of it, etc., etc.

STUDENTS' BELIEFS ABOUT THE 'IDENTITY' OF MATHEMATICS

In the beginning of 2007, I conducted a questionnaire and interview research study of second year upper secondary students' (age 17-18) beliefs about the 'identity' of mathematics. A number of these questions had to do with evolutionary and developmental perspectives of mathematics, others had to do with sociological perspectives, and others again with perspectives of a more philosophical nature. In the following I shall present the students' answers to three of these questionnaire questions, one from each aspect. All in all 26 students answered the questionnaire. The students' questionnaire answers have been indexed in the following manner: one<few<some<many<the majority<the *vast* majority, a partition which roughly corresponds to the percentage intervals: 0-5%; 6-15%; 16-35%; 36-50%; 51-85%; 86-100%. Based on the questionnaire answers 12 students were chosen as representatives for the class in general, and these 12 students were interviewed about their answers. All quotes from the questionnaires and the interviews have been translated from Danish.

1. When do you think the mathematics in your textbooks came into being?
The majority believe that the mathematics in their textbooks came into being "some time long ago". The suggestions concerning exactly when are, however, many and varied: "from even before da Vinci's time!"; "when the numbers were invented"; "when we began using Arabic numerals"; "way before it says in the books". Some points to antiquity and provide as argument that "the construction of, for instance, the pyramids must have required at least some mathematics". One of the more interesting answers goes: "Long, long ago it all began and since then it has continued. But I am confident that the development goes more and more slowly, because you eventually know quite a bit."

Out of this majority of students, some share the perception that mathematics has always existed, or at least has existed as long as human beings have been around. One says: "Mathematics in general has existed since the dawn of time, but highly developed [mathematics] has only emerged within the last 200-100 years." Only one student believes the mathematics in the textbooks to be of a more recent date, and he is not afraid to fix this to "40 years ago".

In the follow-up interviews, events in the history of mathematics were occasionally fixed within some not too unreasonable orders of magnitude, for instance, the beginning of mathematics to 4000-5000 years ago; Pythagoras to the first couple of centuries; and Fermat's last theorem to "the Middle Ages or something". But only few students were able to do this. Whether this is due to lack of knowledge about history of mathematics or lack of knowledge about history in general, or maybe both, is not to say. Finally, one of the students seemed very strong in her belief that it was impossible to practice mathematics without the Arabic numerals. When asked why, she answered: "the mathematics you do today, you wouldn't have been able to do that... [without the Arabic numerals]".

2. Do you believe that mathematics in general is something you discover or invent?
The majority of the students believe that mathematics in general is something you discover. Only a few believe that it is something you invent. Some students, though, believe that it might be a combination of the two. Examples of the discovery answers are: "Discover. I don't think you can invent mathematics – it is something 'abstract'

you find with already existing things."; "Discover. Because mathematics is already invented. What happens today is only that you discover new elements in it."; "Mathematics is all over – in our society, our surroundings and in the things we do. Therefore I do not believe mathematics to be something you invent, but on the contrary something you discover along the way. Of course, it might be difficult to say precisely, because where do we draw the line between discovery and invention?" Examples of students believing it to be a combination of discovery and invention are: "Many things might begin as an invention, but afterwards they are explored and people discover new elements in the 'invention' in question"; "Both, [I] think that you discover a problem and then solve it by inventing a solution or applying already known rules of calculation"; "You invent formulas after having discovered relationships". One student's answer touch upon the question of what mathematics 'really' is: "Good question... very philosophical. I think there are many different standpoints to this. I personally believe that it is something you discover. Numbers and all the discoveries already made are all connected. So for me it is more a world you enter into than one you make."

In the follow-up interviews the student responsible for the last remark explained further: "Well, I see it as if mathematics is just there, like all natural science is, for instance, outer space. Outer space is there and now we are just discovering it and learning what it is. That's what I think: It's the same thing with mathematics." When the remaining interviewees in favor of discovery were asked if the 'exploration' of mathematics corresponds to the exploration of the universe they all confirmed this belief. That is to say that they believed mathematics to always have existed, or as one student phrased it: "Mathematics has always been there, in the form of chemistry or something like that at the creation of Earth. And then we haven't found out about it until later." Or another one: "I think it has always been there, but I just think that the human beings are exploring mathematics more and more and are discovering new things."

3. Do you think mathematics has a greater or lesser influence in society today than 100 years ago?

The vast majority of the students believe the influence is greater. This answer is in general based on the increased amount of technology in our everyday life in society. Answers as "definitely, more computer=more mathematics" and "everything develops and everything has to be high-technology" are often given. A few of those who believe that mathematics has a greater influence today also points to economic affairs as the reason, or that "the use of mathematics has become more advanced in our time". Some think that mathematics has the same influence today as it had 100 years ago, and only very few believe that the influence today is lesser. One of the more 'sensational' answers of the latter kind is: "No, I don't believe that, because even though we use mathematics a lot more in space etc. we have modern machines to do it."

The follow-up interviews to a large degree confirm the beliefs described above. To the deepening question of why a student found the influence today to be greater, she answered:

> Student: Because today you can, for instance, get an education at... or study mathematics at the university and things like that, and that you couldn't do a hundred years ago. [...]
>
> Interviewer: *So it is something relatively new that you can study mathematics at the university?*
>
> Student: No not new, but I do believe at a higher level. That is, you didn't know as many things back then as you do today.
>
> Interviewer: *And you couldn't get an education as a mathematician in the same way, you think?*
>
> Student: No.

The student who argued for lesser influence due to the use of modern machines is also given the opportunity to expand on her view in the interviews. She finds, amongst other things, that mathematics appears less present because we rely on technical aids to a great extent, and because the use of mathematics is mostly about "pushing some buttons".

EVALUATING STUDENTS' BELIEFS AGAINST THE 'GOALS'

How do the above presentation of students' beliefs about the evolution and development of mathematics correspond with the goal-oriented description of overview and judgment in the KOM-report and the 'identity' of mathematics in the 2007-regulations? For example, are students able to "demonstrate [display] knowledge about the evolution of mathematics and its interaction with the historical, the scientific, and the cultural evolution"? Overall the students' answers to some of the questions appear rather diffuse, but let us look at the questions in turn.

In the answers to question 1 there seem to be an agreement that mathematics is 'old'. One student implies that da Vinci is old and that mathematics is older than him. However, only very few are capable of providing years on the origin of mathematics as well as on concrete mathematical results. That some students believe that mathematics only could come into existence by aid of the Arabic numerals does not strengthen the interpretation that the students possess knowledge about the evolution of mathematics in interplay with historical and cultural events either.

In question 2 the majority give expression to the fact that they believe mathematics in general to be discovered. In a Danish educational context this may appear surprising since, as Hansen (2001, p. 71, my translation from Danish) puts it: "it is clear that the strong position of constructivism in school circles fertilizes the ground for a more radical constructivist perception of the entire nature of mathematics. Because of the pedagogical constructivism in schools, children and young people are likely to have difficulties believing in special existence of mathematical quantities, figures, and concepts." Of course there are students who are inclined toward a view of mathematics in general as something invented, but they are few in number. The majority give expression to a Platonic stance. With the words of one of the students, it is "a world you enter into" – a world of ideas – where you explore the already existing mathematical objects in a similar way as we are exploring the Milky Way and the rest of the universe our planet is part of.

On the other hand, the students seem to have a quite good understanding of the fact that mathematics today has a much greater influence in society than it did 100 years ago (question 3). Again it is computers and other technology that are given credit for this. The fact that students only pay scant attention to economic affairs and political decision-making, e.g. based on mathematical models, may be seen as a consequence of the invisibility of mathematics in society (Niss, 1994). One student touched upon this when she said that mathematics appears less present due to use of technology. Another example is the student who in question 1 believed that the development of mathematics was happening at a slower and slower pace and who in the interviews explained herself:

> Yes, but they just discovered more a long time ago, didn't they? It isn't very often you hear about someone who has discovered something new within mathematics, is it? Maybe it's just me who isn't enough of a mathematics geek to be told about it. But it just seems to me that nothing is really happening. Things are happening more often within natural science: now they have found a method to see the fetus at a very early stage by means of a new type of scanning or something.

This student seldom hears about new discoveries in mathematics, even though she is exposed to the subject several times a week, therefore she believes nothing is happening. Beside this, her remark also touches upon one of the differences between mathematics and the natural sciences: just because mathematics now is able to prove Fermat's last theorem or the Poincaré conjecture, then this is not something that will change our everyday or society neither tomorrow nor in 50 years (most likely), something which would be far more likely for discoveries in physics, chemistry, or biology – and to a larger extent for technology basing itself on these disciplines.

In general the fact that mathematics is driven by both outer as well as inner driving forces is not an aspect which the students seem to be very aware of. And concrete examples from the history of mathematics, in the form of the KOM-report's talk of "solidness" (cf. page 3), is not something which the students seem able to provide either.

FINAL REMARKS AND REFLECTIONS

According to Lester, Jr. (2002, p. 352), Kath Hart at a PME conference once asked: "Do I know what I believe? Do I believe what I know?" Lester's version of this question is: "Do students know what they believe?" Furinghetti and Pehkonen (2002) argue that one should take into consideration both the beliefs that students hold consciously as well as unconsciously. But how to do this? Lester, Jr. (2002, pp. 352-353) sows doubt about some of the more usual methods for doing this: “I am simply not sure that core beliefs can be accessed via interviews [...] or written self-reports [...] because interview and self-report data are notoriously unreliable. Furthermore, I do not think most students really think much about what they believe about mathematics and as a result are not very aware of their beliefs.” Thus, the results above must perhaps be viewed in this light. However, other researchers (e.g. Presmeg 2002) argue that questionnaires, interviews, etc. are perfectly well suited to access students’ beliefs about mathematics as long as the usual precautions, for example the interviewee trying to please the interviewer, are taken into account.

In the research reported in this paper, the students knew nothing about my personal viewpoints on the evolution and development of mathematics; they were not familiar with the descriptions in the KOM-report, nor the ‘identity’-description in the regulations for that matter. So it seems reasonable to say that none of these views could have affected the students’ answers. Of course, they knew that the interviewer was a mathematician which might have led them to alter some of their views. Also, it is true that many students do not have a clear and conscious idea about their beliefs about mathematics, as Lester says. When asking the interviewees to deepen or expand their questionnaire answers some of them would have trouble remembering what they answered, some would be puzzled about their own answers, and some would take on different viewpoints in the interviews than what they had expressed in the questionnaire. Especially the question of invention and discovery was one that seemed to puzzle the students; often they would have difficulties in making up their minds. From an educational perspective, this is, however, the power of precisely this question: that there is no correct answer to it. It is a matter of conviction, whether you are a Platonist, a formalist, a constructivist, a realist, an empiricist, or something else. Thus, students will have to *reflect* about the question on their own in order to take a standpoint.

Especially reflection and the ability to perform reflection are considered to be major factors in changing beliefs (Cooney et al., 1998; Cooney, 1999). Thus, if the students who took part in the research presented above were to have their beliefs ‘molded’ or ‘shaped’ in such a fashion that they would fit the previously presented goal-oriented descriptions, then one way of doing this would be to set a scene which enabled them to perform reflections. In fact, the students’ questionnaire and interviews reported above are an initial part of a larger research study, one purpose of which was to provide the students with classroom situations in which they were expected to work actively with and reflect upon issues related to, amongst other questions 1, 2, and 3. More precisely, these situations consisted of two larger teaching modules which the upper secondary class was to engage in over a longer period of time.[6] During and after the period of implementation, the changes in students’ beliefs were attempted evaluated through more questionnaires and interviews but also by means of videos of classroom situations taking as the point of departure the 'initial' student beliefs as presented in this paper.[7] A comparison of the questionnaire and interview results presented in this paper, i.e. those from before implementing the modules, with the later research findings, those from during and after the implementations, will be presented in Jankvist (2009).

As a very final remark, I shall point to my own belief that reflections ought not only be considered as a means for changing existing beliefs, or creating new ones. A students' image of mathematics should include an awareness of mathematics as a discipline that consists of and gives rise to questions to which there are no correct answers (e.g. that of invention versus discovery), and for this reason the ability to reflect is equally important. That is to say that not only is the act of providing students with an image of, or a set of beliefs and views about, mathematics as a discipline a goal in itself, the act of making the students capable of reflecting about their images is a goal as well.

NOTES

1. An exception is a Danish study of Christensen and Rasmussen (1980).
2. A few examples are Schoenfeld, (1985) and Leder and Fortaxa, (2002).
3. I shall not here enter the discussion of defining 'beliefs'. I do, however, implicitly base my understanding of beliefs on the definition given by Philipp (2007).
4. The full questionnaire consisted of 20 questions covering the three different aspects mentioned as well as more personal, affective matters of mathematics to be used in a larger study (Jankvist, 2009).
5. The word 'demonstrate' in Danish has a dual meaning; it may be used both as the word 'prove' and as the word 'display'. Thus, students may only need to display knowledge.
6. Descriptions of and preliminary results from this research study may be found in Jankvist, (2008a) and Jankvist, (2008b).
7. E.g. beliefs on question 2 were evaluated by posing more specific questions relating to the cases of the two modules.

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