CERME 6 – WORKING GROUP 13
COMPARATIVE STUDIES IN MATHEMATICS EDUCATION

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INTRODUCTION

COMPARATIVE STUDIES IN MATHEMATICS EDUCATION

Organisers

Eva Jablonka (chair), Luleå University of Technology, Sweden

Paul Andrews, University of Cambridge, United Kingdom

Birgit Pepin, Sør-Trøndelag University College, Norway

AIMS AND SCOPE OF THE WORKING GROUP

The call for papers for the 2009 meeting of the working group set out with a description of the scope and aims of comparative studies in mathematics education. These include studies that document, analyse, contrast or juxtapose similarities and differences in mathematics education at different levels, such as:

- cross-cultural or cross-national comparison;
- comparison between sectors of school-systems;
- comparison between groups that share specific characteristics (for example, gender, language, social and economic background, cultural affiliation or other demographic features);
- comparing mathematics education with other school subjects.

There were no restrictions in the aspects of mathematics education that can be usefully addressed in a comparative study. These might, for example, include: Intended curricula; tools, teaching materials and resources; specific mathematical activities or the enactment of distinct mathematical topics; learning environments; teachers', student teachers' and students' aspirations, goals and values; student achievement and participation; features of classroom practices or features of teacher preparation programs.

The aims of the working group included to:

- share findings and outcomes of empirical studies that adopt a comparative approach;
- further develop research methodologies that are specific to comparative studies;
- identify ways in which macro-level survey studies and micro-level case studies can productively interact;
- develop a better understanding of how various theoretical approaches and conceptual frameworks shape the goals and the design of comparative research;
- consider how comparative studies can inform teaching and learning practices.

The group invited contributions with an empirical, methodological or theoretical focus. Papers with a methodological or theoretical focus could, for example, address issues of comparability of culturally-grounded practices, challenges of interpreting outcomes of large-scale international achievement studies, methods of data aggregation in quantitative studies, technicalities of classroom-video studies, issues of cultural bias in coding or any other problématique that is specific to comparative studies.

PAPERS AND POSTERS

As the working group brings together researchers who share an overall approach rather than a focus on a set of topics, we find an interesting range of aspects of practices in mathematics education that were subjected to comparison in the research reports and posters. The participants' studies, some of which are ongoing projects, addressed mathematics education in different places of the world. The countries and regions include Australia, China, the Czech Republic, Finland, France, Germany, the Hong Kong Special Administrative Region, Hungary, Israel, Italy, Norway, the Slovak Republic, Syria, the United Kingdom of Great Britain and the United States of America. The titles of the papers and posters indicate the variety of aspects of mathematics education that were subjected to a comparison (presenting authors are underlined):

Paul Andrews, United Kingdom: *Comparing Hungarian and English mathematics teachers' professional motivations*

David Clarke and Xu Li Hua, Australia: *Spoken mathematics as a distinguishing characteristic of mathematics classrooms in different countries*

Tiruwork Mulat and Abraham Arcavi, Israel: *Mathematical behaviours of successful students from a challenged ethnic minority*

Giancarlo Navarra, Nicolina A. Malara, Italy; András Ambrus, Hungary: *A problem posed by J. Mason as a starting point for a Hungarian-Italian Teaching Experiment within a European project*

Hans Kristian Nilsen, Norway: *A comparison of teachers' beliefs and practices in mathematics teaching at lower secondary and upper secondary school*

Birgit Pepin, United Kingdom/ Norway: *Mathematical tasks and learner dispositions: A comparative perspective*

Jennifer von Reis Saari, United Kingdom: *Elite mathematics students in Finland and the Washington: Access, collaboration, and hierarchy*

Constantinos Xenofontos, United Kingdom: *International comparative research on mathematical problem solving: A framework for new directions*

As the posters are not included in the proceedings, short summaries are given in the following:

Maha Majaj, France: *Comparative study of the place of elementary number theory in the programs and the textbooks in the middle school between France and Syria*

The teaching of elementary number theory has undergone changes in the French and Syrian education systems. In Syria, its place changed with the evolution of the textbooks about five years ago and in France it was reintroduced, after fifteen years of absence, in 1998 (grade 12), 1999 (grade 9) and 2001 (grade 10). The study compares elementary number theory in the programs and textbooks, topic by topic, by taking into account a distinction between tool and object and identifies the didactical transposition choices and their effects on the design of textbooks. An initial study indicated that the choices of the Syrian educational system can be seen as corresponding to the French program since the beginning of the 20th century. This observation led to including an analysis of the evolution of the French program and textbooks from the reform in 1902 onwards.

Jan Sunderlik, Slovak Republic: *Intrinsic motivation and student teaching practice at universities from Great Britain, the Czech Republic and the Slovak Republic*

The study in progress sets out to investigate pre-service teachers' teaching practice in Great Britain, the Czech Republic and the Slovak Republic with a focus on their strategies for motivating students. It is to understand how the accumulated body of research on students' motivation may be useful for classroom teachers struggling with the issue. The notion of motivation is complex and, for example, described as linked to social needs, beliefs, behaviour and affect. One challenge of the research is to describe motivation in observational terms.

SNAPSHOTS AND CLOSEUPS FROM THE DISCUSSION

The groups at the CERME adopt a mode of working that assumes that all papers have been read before the start of the conference. The presenters in our group were invited to draw our attention to specifics and to expand on one or two points in order to provide us with 'an experience' for entering the discussion. The productive work and stimulating discussion lived on the continuous engagement of all participants, which made it possible to allude to a wide range of topics. In the following, a summary of some issues, which were not specific to a particular research report, is given.

Agendas and modes of comparison

The group agreed that although comparative studies serve to achieve a variety of goals, comparison does not itself constitute the goal of a comparative study. Comparison was seen as being always of interest because looking at practices from another culture (see below "units of comparison") provides a new 'lens' for looking at our own; it helps to make the familiar look unfamiliar. For the activity of describing similarities and differences in the empirical findings, the metaphor of "collecting stamps" was introduced. Synthesis was seen as a more far reaching goal of a comparative study than a mere description of similar and different aspects, and comparison was described as "the fuel of synthesis". A comparative approach can also aim at assisting theory construction. It is useful for this purpose especially because the emergence of differences supports cultural explanations, while similarities suggest structural (sociological) interpretations. While the improvement of “home” teaching practice was seen as an important goal for a cross-national or cross-cultural comparative study, the members of the group agreed that not all research in mathematics education has to be advocatory.

“Units of comparison”

Acknowledging that all empirical research has a comparative aspect, one recurring point in the discussion concerned the question, are there ‘units’ for comparison that are too small or too big for allowing a study to be described as comparative. Agreement was reached that comparison has to be between aspects of “social conglomerates”, between two cultures (with shared discourse and identities). Just the fact that members of a group share an attribute does not mean that their membership of the group is related to that attribute, neither as a condition for or a consequence of that membership.

Examples of “units for comparison” discussed in relation to the research reports were: curriculum, ideologies in education, schools, processes of change, students’ productions, lesson structure, lesson events, groups of students in different institutional cultures, groups of successful and unsuccessful students from the same culture.

Methodology and Methods

Many problems identified in the discussion are not specific to comparative research, but the challenge of working across cultures makes them more visible. The research designs in the comparative studies presented in the group comprise a variety of approaches for creating accounts of the practices to be compared. The discussion focused on three approaches: documentation, cross-national intervention study (a “perturbation of practices”) and on the comparison with a different teaching practice (with a different pedagogy) as a quasi-experimental design.

Interpreting “silence in the data”

This discussion emerged out of an example of interview transcripts with students from two different cultures. The participants did not say anything after a prompt from an interviewer. In the group we created several interpretations of this fact: Silence is a normal part in any conversation – it is a thinking pause; silence is a sign of cultural or social alienation; silence is a general cultural behaviour; silence is an individual’s preference.

In the course of the discussion, “silence” was used metaphorically for missing aspects of a practice. These silences go unrecognized from within the practice and thus comparison can fill the gap left by silence.

To what extent are the outcomes comparable and can be synthesised?

Group members observed that the cultural differences sometimes are so fundamental that comparison is impossible. The results can then only be juxtaposed. The question was also asked to what extent psychological frameworks could be useful in comparing groups from different cultural contexts.

Cultural affiliation of research personnel (interviewers, transcribers)

Group member were aware that inter-researcher reliability is a problem in all studies, but it is likely to be exacerbated in a cross-cultural comparative study or a study of different institutional cultures or any other social conglomerates with a shared discourse. Some methods were suggested and discussed. “Member checking” includes exchanging the accounts between the different communities (both the “researched’” or the researchers’) and letting them check from their lens. One interesting example was provided in a study in which teachers in one country had been asked to read the accounts from teachers in other countries of what they do and why they do it.

How to avoid a culturally biased interpretation?

Group members shared the observation that interpretations are loaded with values from our own teaching tradition as well as research tradition. Researchers may project their home-grown categories into the other culture’s data, which amounts to a culturally biased gaze. Researchers might as well be at risk to produce an ‘idealistic’ description of their own practice, or alternatively (depending on the culture!), provide an account that is too critical of the home practice and celebrates the other.

The group found that exploiting different conceptual frameworks might help to identify the blind spots of each. The French “praxeology” served as an example. Some found that ‘contextualised tasks’ were not given attention as a category because the French curriculum does not include those as a characteristic element. In an approach that is more focused on the empirical material and does not set out with theoretical categories, the interpretative accounts for one set of data from one site maybe considered as the framework for interpreting the other (and vice versa). This approach is reminiscent of constant comparison as a standard method in qualitative data analysis.

All agreed that language matters, also within a culture, e.g. as a sociolect, as difference between formal and informal language use. This point draws attention to how to deal with translated transcripts; the choice of language into which protocols are translation is already a source for a cultural bias. The group pointed to the need of defining the cultural frame of each report.

Eva Jablonka, Paul Andrews, Birgit Pepin

COMPARING HUNGARIAN AND ENGLISH MATHEMATICS TEACHERS' PROFESSIONAL MOTIVATIONS

Paul Andrews

University of Cambridge, UK

In this paper I present qualitative analyses of interviews undertaken with English and Hungarian teachers of mathematics. One aim of the interviews was to elicit teachers' professional motivations – what were their subject-specific reasons for teaching mathematics? I frame the analyses against the altruistic, intrinsic and extrinsic motivational framework found widely in the literature before discussing its limitations and proposing refinements to highlight substantial differences within superficially similar sets of culturally located espoused motivations.

INTRODUCTION

A frequently cited reason for undertaking comparative education research is that "studying teaching practices different from one's own can reveal taken-for-granted and hidden aspects of teaching" (Hiebert et al., 2003, 3). In part this is because:

> teaching and learning are cultural activities (which)... often have a routineness about them that ensures a degree of consistency and predictability. Lessons are the daily routine of teaching and learning and are often organized in a certain way that is commonly accepted in each culture (Kawanaka 1999, p. 91).

Explanations for such routines draw on beliefs that cultures "shape the classroom processes and teaching practices within countries, as well as how students, parents and teachers perceive them" (Knipping 2003, 282), to the extent that many of the processes of teaching are so "deep in the background of the schooling process ... so taken-for-granted… as to be beneath mention" (Hufton and Elliott 2000, 117). Thus, it is probably not surprising that a substantial proportion of comparative mathematics teacher research has focused on explicating the mathematics teaching script (Andrews, 2007a; Hiebert et al., 2003; Stigler et al., 1999), with a number of other studies having investigated particular contributory factors. For example, text books have been scrutinised (Haggarty and Pepin, 2002; Pepin and Haggarty, 2001; Valverde et al., 2002), teachers' mathematical content knowledge has been analysed (An et al., 2004; Delaney et al., 2008; Ma 1999); as have teachers' mathematics-related beliefs (Andrews and Hatch, 2000; Andrews, 2007b; Barkatsas and Malone, 2005; Cai, 2004; Correa et al., 2008). However, a largely ignored field in comparative teacher research concerns teachers' motivations for their professional activity: what stories do they tell to warrant their roles as teachers of mathematics? This paper is a first explicitly comparative examination of mathematics teachers' professional motivation.

According to available evidence, teachers' professional motives fall into three categories: altruistic, intrinsic or extrinsic (Kyriacou and Newson, 1998; Kyriacou and Coulthard, 2000; Moran et al, 2001; Andrews and Hatch, 2002), although recently Cooman et al (2007) have highlighted a fourth - interpersonal. An altruistic motive presents teaching as a socially worthwhile act related to a desire to facilitate the development of both the individual and society at large. An intrinsic motive includes, inter alia, a person's desire to work with children or their subject specialism, while an extrinsic motive pertains, for example, to salary, conditions of service, holidays or status. Lastly, interpersonal motives refer to “social interactions commonly present in a teaching job” (Cooman et al, 2007, p. 127). An individual's personal motivation to teach is likely to be an amalgam, in varying proportions, of these factors (Moran et al, 2000) although there is evidence that preservice teachers in developed countries are motivated by both intrinsic and altruistic factors, while in developing countries extrinsic motivations (or mercenary) appeared more prominent (Bastick, 2000). Indeed, in respect of the former, intrinsic motives appeared dominant for preservice teachers in the US (Serow and Forrest, 1994) Greece (Doliopoulou, 1995), England (Reid and Caudwell, 1997; Priyadharshini and Robinson-Pant, 2003; Whitehead et al, 1999), Northern Ireland (Moran et al., 2000) and Australia (Manuel and Hughes, 2006).

In respect of the professional motivations of mathematics teachers there is little research (Reid and Caudwell, 1997). In respect of the UK, students following a post-graduate mathematics teacher education programme were less intrinsically motivated than those of other subjects, showing, in their greater enthusiasm for teaching as a good career, a more extrinsic perspective (Reid and Caudwell, 1997). Also, in contrast with mathematics undergraduates, who privileged intrinsic factors such as being able to use their subject knowledge or working with children (Kyriacou & Newson, 1998), post graduate teacher education students were less enthusiastic about sharing their knowledge, continuing their subject interest, improving children’s life chances than their non mathematical colleagues (Reid and Caudwell). In terms of serving teachers, Andrews and Hatch’s (2002) study showed that few people espoused either altruistic or extrinsic reasons, with most citing motivations intrinsic to either mathematics itself or teaching as a profession.

In sum, the totality of the above highlights the extent to which the tripartite framework has been used in different research contexts. However, with so little comparative work, and with most studies drawing on different instruments, we know little about the extent to which it adequately represents the motivations and beliefs of teachers in different contexts. In this paper we examine this issue by means of an initial comparative examination of mathematics teachers’ professional motivations.

METHOD

Many of the studies cited above used survey approaches to explore teachers’ motivations. Of these, many exploited factor analytic techniques to identify or confirm, depending on the type of analysis, motivational constructs. However, such approaches rely, essentially, on predetermined categorisations of motivation and may miss not only subtle variations within the three dimensions but, importantly, components hitherto unconsidered. In this paper we attempt to let teachers tell their own stories and, to do so, look to narrative research. Narrative research is of interest due to its “potential to access the research subjects’ voices and to offer deeper, sensitive and accurate portrayals of experience that have escaped positivist quantitative research and less sensitive, objectivist qualitative research” (Swidler, 2000, p. 553). It “is probably the only authentic means of understanding how motives and practices reflect the intimate intersection of institutional and individual experience in the postmodern world” (Dhunpath, 2000, p. 544). Narrative researchers believe that teachers construct stories to make sense of their professional world (Swidler, 2000; Drake, 2006). That is, stories, “as lived and told by teachers, serve as the lens through which they understand themselves personally and professionally and through which they view the content and context of their work” (Drake et al. 2001, p. 2). Moreover, “these stories are subject-matter-specific and may differ greatly from subject to subject” (ibid).

With this in mind, 45 teachers from two regions of England, and 10 from Budapest, Hungary, were interviewed in the months following a questionnaire study of their conceptions of mathematics and its teaching. In both countries colleagues were drawn from a variety of institutions, which, as shown by various indicators, were representative of state schools in the different regions. The interviews, which were intended to elicit details about informants' professional life histories, were semi-structured and invited colleagues to describe how their careers had developed and to discuss the key episodes, “critical events” (Woods, 1993) or “critical incidents” (Measor, 1985) that had informed or transformed their professional lives. In order to frame their stories, colleagues were invited, fairly early in their interviews, to explain why they had decided to become teachers before being asked to consider the place of mathematics in the curriculum and their personal justification for both its curricular inclusion and their teaching it. Interviews, which were conducted in colleagues' schools, were tape-recorded and transcribed. Transcripts were posted to them for agreement as to their content although not one was queried. The method of constant comparison (Glaser and Strauss 1967, Strauss and Corbin 1998) necessitated that transcripts were read and re-read to identify categories of response. As new categories were identified, previously read transcripts were re-read to see whether or not the new category applied. The two sets of data, English and Hungarian, were analysed separately to ensure that culturally located differences were not obscured.

RESULTS

The reader is reminded that this paper draws on, in many cases, informants’ recollections of events of many years earlier. Thus, it is not improbable, particularly acknowledging the temporal shift between events, that for some teachers, recollections concerning decisions about career choice may have been vague and romanticised. In particular, it is not improbable that recollections drew on colleagues’ affective responses to the profession which had dominated their lives. In some cases, but clearly not all, these would have been positive and, possibly, a little heroic. Consequently, some caution should be exercised when interpreting informants' utterances. In the following, all names are pseudonyms. Due to constraints of space, only a partial analysis is reported, which draws on the same three substantial categories of response that emerged from the data of each country. These focused on personal pleasure, the extrinsic properties of mathematics and the intrinsic properties of mathematics. These were not exclusive categories with most teachers alluding to at least two of them.

English teachers: Personal pleasure

Twenty seven English teachers indicated that their professional motives were located in the pleasure they gained from working with students. Jane, typical of most, described an enjoyment located explicitly in their students' mathematical success. She said:

> I just enjoy teaching it (mathematics)… I can't explain it. I enjoy teaching it. I enjoy watching children who can't do maths suddenly discover they can add up. You know, children for whom it's not made sense all of a sudden this…"Oh that's why it works", "Oh now I understand". And I think it's that, and it doesn't matter what level that is. Whether it's down at the bottom end or it's up at the top end, it's that discovery that it works. That's what I enjoy doing. I enjoy seeing children make that leap. Sometimes it happens more often than others; with some children it's very slow, you know, the understanding, but when it comes it's like light dawning and they're so pleased and I think that's what it is.

For the others, like Hazel, their pleasure seemed less altruistically focused. She said:

> I think I would always have ended up as a teacher. I loved being around little kids when I was a child… I'm a maths teacher because that's what I was good at and if I'd been good… at… French then I think I would have been a French teacher.

English teachers: Extrinsic properties of mathematics

Forty-two teachers commented that they were teaching mathematics to prepare students to manage successfully a world beyond school. The explicit foci of these comments varied but the underlying message was essentially the same; a child who cannot understand mathematics would struggle to make sense of the *real world* or *everyday life*. James commented:

> I feel that maths is a tool and that if students… are to be fully prepared for what the modern world is to throw at them... I think that it's very important that they are... able to handle all the things that can be thrown at them.

For others this was explicitly linked to employment. Jack, who had previous work experience in cotton mills and council offices, suggested that:

> It’s there all throughout isn’t it? I mean, at basic levels, the practical jobs, measurement and things like that through to, yes, obviously people want to be well qualified to go on and do, you know, industrial engineers or civil engineers, work that involves high powered mathematics as well.

Susan indicated yet another utilitarian perspective. She said that:

> I think my main reason for supporting maths is because I think it's a support subject for other subjects as in you can't take your science or computers nowadays or anything further, if you, if students want to, without a basic knowledge of maths. So you can't do a lot of things further and develop knowledge that way. So I see it being a support subject for other subjects.

English teachers: Intrinsic properties of mathematics

Fifteen teachers offered statements indicative of their justifying their teaching of mathematics as a consequence of its intrinsic properties. Jean commented that “I always got a… buzz out of solving particular problems…especially when you've worked on them for quite some time. And so it's that enjoyment of the subject that I like to try and put across to children”. Judy, in addition, discussed wanting her students to become critical thinkers:

> I want children to feel the need to solve a problem. I give them the skills and help them to think through how to achieve that, even if it's a very, very simple idea, I always give them a reason why… I always say, don't ever be satisfied with well that is how it is, always ask and if I can't give you a reason then I should go away and find you a reason because I won't expect you to believe it just because I say so.

In similar vein Frank, discussed his belief in the importance of mathematical reasoning. He said that “the one area of maths that I really enjoy working with students is, is trying to get them to explain things, I suppose, explain, justify, prove along some sort of continuum there”.

Hungarian Teachers: Personal Pleasure

Eight of the ten Hungarian teachers talked about pleasure gained from their professional activity. For the most part, this drew on students’ mathematical successes. Vera commented that, “It feels good to teach the children to think”, although most indicated that their pleasure derived from their students understanding of mathematics. Emese, for example, said that when “I tell them something new…and although they would probably have learnt about it without me, not only do they know it but they also understand it”.

Two teachers located their comments on student understanding within the domain of problem solving. Ilona commented that:

> I would like my students to understand and think about smaller or bigger problems in mathematics with joy... And I think it’s the greatest thing in the world that I can teach mathematics because it’s a fantastic way for educating children... when I see twenty kids

> sit down and think and wrinkle their foreheads, and they put their heads in their hands and they turn the small wheels around until they get to some solution independently.

Hungarian Teachers: Intrinsic properties of mathematics

Every Hungarian teacher commented in ways indicating that, for them, mathematics possessed important and, essentially, intrinsic qualities. Emese noted that “students have to see that in mathematics you have to think logically”. Robert, expressing a similar theme, commented that “it is important that a child learns a particular thinking scheme and can solve problems with this method… how you can make a child to become a thinking child”.

At an explicitly philosophical level, Eva commented that “I like to quote an aphorism which more or less determines my life. Leonardo said *mathematics is the most important tool for understanding the truth everywhere and in everything* and this is my philosophy” while Robert added that mathematics “was a spiritual adventure and this was what attracted me so much (to the teaching of the subject)”.

Unlike the English data, three intrinsic subthemes emerged from the analysis. These concerned mathematics as problem solving, mathematics as a connected body of knowledge and mathematics as experientially learned.

Mathematics as problem solving

Nine Hungarian teachers discussed the importance of problem solving in their conceptualisation of mathematics and its teaching. Vera, outlined a view that teachers should alert students to

> ... certain types of problem which come up again and again …, they should know the typical problems that they have to go through. And then it’s also good if there are problems, we give them problems, which don’t have completely unique solution so they should find them in other ways.

Emese, in addition, acknowledged the affective domain as part of the problem solving experience. She commented that:

> We should teach them how to recognise the problem, develop ideas for the solution, put them into a logical order, and this way you reach the solution… The most beautiful and simple thing in the world is when you solve a problem and you realise that you were able to solve it…It can help you with a little more self confidence too.

Mathematics as a connected body of knowledge

Five teachers commented explicitly on mathematics as a connected body of knowledge. Rita, talking about number theory and geometry, commented that:

> Within number theory, for example... you can take the numbers apart. Think of numbers and how they are built up. This building up is very important. And with other topics too, in geometry it's important to be able to build up things… This taking apart, building up, and often the building up is at least as important as taking apart.

Robert offered a more abstract perspective, commenting that “mathematics is built in such a way that it states certain things and it calls them *axioms* or *statements* which are considered as true and then I start to build up something and I wonder how far you can get from it”.

Mathematics as an intellectual challenge

Five alluded to mathematics as an intellectual challenge, something for which learners should expect to struggle. Eva commented that students:

> shouldn’t get everything ready-made but should have to look for the truth, to search for it. I mean it’s more the research than the experience. I, for example, like geometry very much when they have scissors in their hands and they’re folding and cutting papers and getting experiences… Still you can research to look for different solutions. We get to the same truth in different ways.

Kati, commented in similar vein, that children should experience the “joy of research… I think that one of the most important things is that children should be brave and should be able to get close to an unknown problem. And it's also very important that this love of adventure shouldn't be spoilt by me”. Zsolt, commented that “they have to get experiences. No matter what topic of mathematics they’re learning, they should get as much experience as possible”.

Hungarian Teachers: Extrinsic properties of mathematics

Five teachers commented explicitly that mathematics provided key skills for a world beyond school. Vera noted, briefly, that it “has an influence on their whole life; the rational way of thinking”, while Ilona said:

> I think we teach mathematics to help children find their way in life more confidently. Whatever they become, a cleaning lady, a banker, a doctor or anything... mathematics is a logical skill. Facts and things thought over a logical way will help them make their way more confidently.

In similar vein Emese commented that children “should be able to calculate the change in the shops and I want them to understand all it's good for in everyday life… I think they have to see that mathematics is about life”, while Rita said that it’s “good if they can count. If they can look through how much is how much. Estimating is very important... I always say, you cannot read a book if you have to think about each letter”.

DISCUSSION

The above, albeit limited, results show that when located alongside their subject specialism, teachers of mathematics in England and Hungary report intrinsic motivations, although the three categories of response comprise embedded altruistic, intrinsic and extrinsic characteristics respectively. Thus, on the one hand, it could be argued that English and Hungarian teachers of mathematics present similar subject-related professional motivations. On the other hand, the widely differing proportions of teachers reporting these categories indicate something profoundly different. Of course, the differences can be explained against a variety of cultural frameworks. For example, the dominance of the mathematically intrinsic motivations of Hungarian teachers and mathematically extrinsic motivations of English teachers reflect the underlying rational encyclopaedist and classical humanist traditions of Hungary and England respectively (Andrews and Hatch, 2000). But such explanations offer little by way of highlighting differences other than in the frequencies of the three dimensions. Therefore, the following is a tentative revision of the framework drawing on notions of rhetorical and warranted motivations.

Firstly, in respect of mathematically altruistic motivations, English teachers talked, in an unspecified manner, of motivations linked to mathematical understanding, while their Hungarian colleagues spoke of understanding-informed mathematical thinking and problem solving. Thus, on the one hand, around half the English sample presented rhetorical altruistic mathematical motivations, while, on the other, almost all the Hungarian teachers articulated a warranted altruistic mathematical motivation. Secondly, in terms of mathematically intrinsic motivations, English teachers tended to articulate a perspective concerning problem solving and the logical skills necessary to solve them, while the Hungarian teachers presented a variety of perspectives concerning not only problem solving but also the structural properties of mathematics and the intellectually challenging nature of the subject. Thus, the English teachers presented a weakly warranted, almost rhetorical, intrinsic mathematical motivation when compared with the Hungarian teachers' robustly warranted intrinsic mathematical motivation. Thirdly, in respect of mathematically extrinsic motivations, almost every English teacher and half the Hungarian teachers discussed mathematical success as a necessary prerequisite for employment or the learning of other subjects. In this regard, both groups of teachers presented a moderately warranted extrinsic mathematical motivation.

In summary, the qualifiers of rhetoric and warrant allow us to distinguish between the two sets of motivations and understand more fully the ways in which mathematics teachers' professional motivations are products of the cultures in which they live and work. A speculative conclusion would be that while both sets of teachers present a moderately warranted wider-world (extrinsic) justification for the teaching of mathematics, the English tend towards rhetorically-based motivations while the Hungarian tend towards warranted motivations.

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SPOKEN MATHEMATICS AS A DISTINGUISHING CHARACTERISTIC OF MATHEMATICS CLASSROOMS IN DIFFERENT COUNTRIES

David Clarke and Li Hua Xu

University of Melbourne, Australia

This paper reports research into the occurrence of spoken mathematics in some well-taught classrooms in Australia, China (both Shanghai and Hong Kong), Japan, Korea and the USA. The analysis distinguished one classroom from another on the basis of public "oral interactivity" (the number of utterances in whole class and teacher-student interactions in each lesson) and "mathematical orality" (the frequency of occurrence of key mathematical terms in each lesson). Our concern in this analysis was to document the opportunity provided to students for the oral articulation of the relatively sophisticated mathematical terms that formed the conceptual content of the lesson. Classrooms characterized by high public oral interactivity were not necessarily sites of high mathematical orality. The contribution of student-student conversations also varied significantly. Of particular interest are the different learning theories implicit in the role accorded to spoken mathematics in each classroom.

Key words: Spoken mathematics, classroom research, international comparisons

INTRODUCTION

The Learner's Perspective Study (LPS) sought to investigate the practices of well-taught mathematics classrooms internationally. Data generation focused on sequences of ten lessons, documented using three video cameras, and interpreted through the reconstructive accounts of classroom participants obtained in post-lesson video-stimulated interviews (Clarke, 2006). The post-lesson interviews address the challenge of inferring student conceptions from video data (Cobb & Bauersfeld, 1994). The LPS approach of conducting case studies of classroom practices over sequences of at least ten lessons in the classes of several competent eighth grade teachers in each of the participating countries offers an informative complement to the survey-style approach of the two video studies carried out by the Third International Mathematics and Science Study (TIMSS) (Hiebert et al., 2003; Stigler & Hiebert, 1999). The criteria for the identification of the competent teachers studied in the LPS were specific to each country, in order to reflect the priorities and values of the school system in that country. In this paper, we report analyses of lessons documented in classrooms in Australia, China (Hong Kong and Shanghai), Japan, Korea, and the USA.

The complete research design has been detailed elsewhere (Clarke, 2006). For the analysis reported here, the essential details relate to the standardization of transcription and translation procedures. Since three video records were generated for each lesson (teacher camera, student camera, and whole class camera), it was possible to transcribe three different types of oral interactions: (i) whole class interactions, involving utterances for which the audience was all or most of the class, including the teacher; (ii) teacher-student interactions, involving utterances exchanged between the teacher and any student or student group, not intended to be audible to the whole class; and (iii) student-student interactions, involving utterances between students, not intended to be audible to the whole class. All three types of oral interactions were transcribed, although type (iii) interactions could only be documented for the selected focus students in each lesson. Where necessary, all transcripts were then translated into English. All participating research groups were provided with technical guidelines specifying the format to be used for all transcripts and setting out conventions for translation (particularly of colloquial expressions).

In this paper, our unit of analysis is the utterance and we distinguish *private* spoken student-student interactions from whole class or teacher-student interactions, both of which we consider to be *public* from the point of view of the student. Our major concern in this analysis was to document the opportunity provided to students for the oral articulation of the relatively sophisticated mathematical terms that formed the conceptual content of the lesson and to distinguish one classroom from another according to the manner in which such student mathematical orality was afforded, promoted, constrained or discouraged in both public and private arenas.

STUDYING SPOKEN MATHEMATICS IN THE CLASSROOM

This paper reports four stages of a layered attempt to progressively focus on the significance of the situated use of mathematical language in the classroom. In our first analytical pass, an utterance is taken to be a continuous spoken turn, which may be both long and complex. We restricted our second-pass analysis to those mathematical terms and phrases that referred to the substantive content of the lesson (usually designated as such in the teacher’s lesson plan and post-lesson interview). The third and fourth passes repeated the focus on utterances and then mathematical terms, but in the context of student-student (private) conversation.

We take the orchestrated use of mathematical language by the participants in a mathematics classroom to be a strategic instructional activity by the teacher. In this paper, we invoke theory in two senses: (i) the (researchers’) theories by which the actions of the classroom participants might be accommodated and explained, and (ii) the (participants’) theories implicit in the classroom practices of the teacher and the students. A particular focus is the role of the spoken word in both. The instructional value of the spoken public rehearsal of mathematical terms and phrases central to a lesson’s content could be justified by reference to several theoretical perspectives. Interpretation of this public rehearsal as incremental initiation into mathematics as a discursive practice could be justified by reference to Walkerdine (1988), Lave and Wenger (1991), or Bauersfeld (1994). The instructional techniques employed by the teacher in facilitating this progression could be seen as “scaffolding” (Bruner, 1983) and/or as “acculturation via guided participation” (Cobb, 1994).

The oral articulation of mathematical terms and phrases by students could be accorded value in itself, even where this consisted of no more than the choral repetition of a term initially spoken by the teacher. Teachers and students in some of the classrooms we studied clearly attached value to this type of recitation. In other classrooms, the emphasis was on the students’ capacity to produce a mathematically correct term or phrase in response to a very specific request (question/task) by the teacher. In such classrooms, both of these activities accorded very limited agency to the learner and the responsibility for the public generation of mathematical knowledge seemed to reside with the teacher. By contrast, in other classrooms, the instructional approach provided opportunities for students to “brainstorm” or to generate their own verbal (written or spoken) mathematics, with very little (if any) explicit cuing from the teacher (e.g. the classrooms in Tokyo).

The role of student-student spoken interactions also varied widely among the classrooms studied. The teacher’s posing of particular mathematical tasks (Mesiti & Clarke, in press) could prompt (and even promote) certain forms of individual, dyadic or small group mathematical behaviour and even monitor and guide that behaviour during classroom activities such as Kikan-Shido (Between-desks-instruction) (O’Keefe, Xu, & Clarke, 2006). However, within these constraints, students have significant latitude and agency in their use of spoken mathematics. The frequency of occurrence of student-student utterances varied from zero in some lessons (eg. Seoul) to as many as 100 distinct student-student utterances per lesson by individual students in classrooms in Australia and the USA. In each classroom, the activity of speaking mathematics was performed differently.

The results that are reported in this paper certainly suggest that the teachers in this study differed widely in the opportunities they provided for student spoken articulation of mathematical terms, whether in public or in private, and in the extent to which they devolved agency for knowledge generation to the students. The demonstration of such differences (and we would like to argue that these differences are profound and reflect fundamental differences in basic beliefs about effective instruction and the nature of learning) in the practices of classrooms situated in school systems and countries that would all be described as “Asian” suggests that any treatment of educational practice that makes reference to the “Asian classroom” confuses several quite distinct pedagogies. This observation is not to deny cultural similarity in the way in which education is privileged and encountered in communities that might be described as “Confucian-heritage.” But, the identification of a one-to-one correspondence between membership of a Confucian-heritage culture and a single pedagogy leading to high student achievement is clearly mistaken, and cultural similarity is not a sufficient indicator of those instructional practices that might be associated with the educational outcomes that we value.

THE USE OF MATHEMATICAL TERMS

In this paper, "utterance" and "mathematical term or phrase" require clear specification (below). Our analysis of public and private classroom interactions has restricted its attention to key and related (primary and secondary) terms, however the analysis of the post-lesson student interviews also considered 'other' terms used by students in interview to explicate the lesson's content or in reflecting on the nature of mathematical activity in general. This paper focuses on analysis of public and private classroom interactions. Consideration of student use of spoken mathematics in the post-lesson interviews will be reported in another paper.

Figure 1 shows the number of utterances occurring in whole class and teacher-student interactions in each of the first five lessons from each of the classrooms studied in Shanghai, Hong Kong, Seoul, Tokyo, Melbourne and San Diego. An utterance is a single, continuous oral communication of any length by an individual or group (choral). Used in this way, the frequency (and origins) of public utterances constitute a construct we have designated as public oral interactivity. This does not take into account either the length of time occupied by an utterance or the number of words used in an utterance (problematic in a multi-lingual study like this one). Figure 1 distinguishes utterances by the teacher (white), individual students (black) and choral responses by the class (e.g. in Seoul) or a group of students (e.g. in San Diego) (grey). Any teacher-elicited, public utterance spoken simultaneously by a group of students (most commonly by a majority of the class) was designated a "choral response." Lesson length varied between 40 and 45 minutes and the number of utterances has been standardized to 45 minutes.

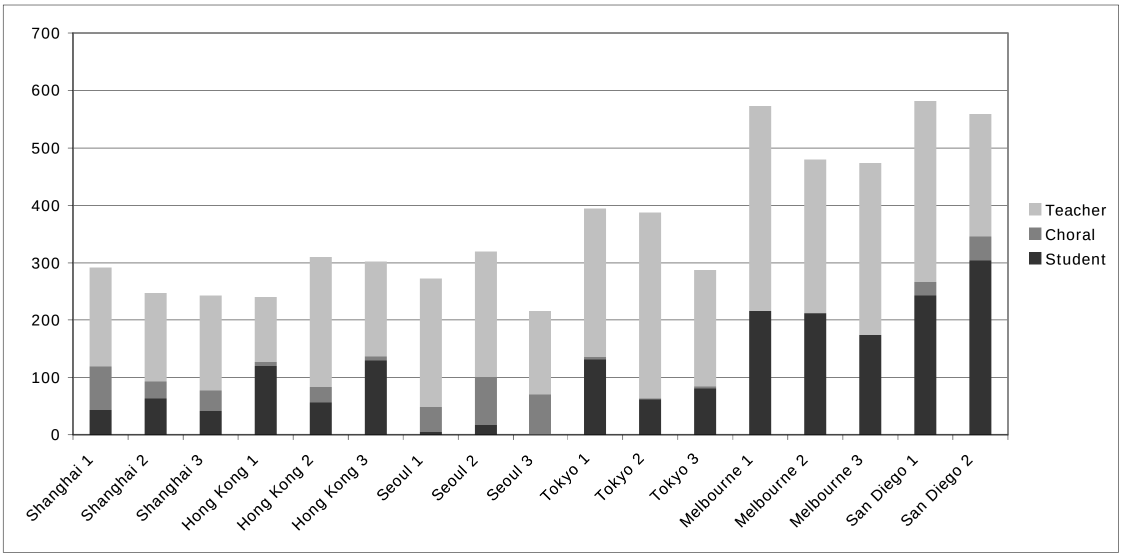

Figure 1: Number of Public Utterances in Whole Class and Teacher-Student Interactions (Public Oral Interactivity)

Figure 1 suggests that lessons in Melbourne and San Diego demonstrated a much higher level of public oral interactivity than lessons in Shanghai, Hong Kong, Seoul, or Tokyo. There were also substantial differences in the relative frequency of teacher, student and choral utterances. It is worth noting that both teacher and student utterances in Shanghai tended to be of longer duration and greater linguistic complexity than elsewhere.

The classrooms studied can be also distinguished by the relative level of public mathematical orality of the classroom (that is, the frequency of spoken mathematical terms or phrases by either teacher or students in whole class discussion or teacher-student interactions) and by the use made of the choral recitation of mathematical terms or phrases by the class. This recitation included both choral response to a teacher question and the reading aloud of text presented on the board or in the textbook. For the purposes of this paper, those mathematical terms were coded that comprised the main focus of the lesson's content.

Figure 2 shows how the frequency of public statement of mathematical terms varied among the classrooms studied. In classifying the occurrence of spoken mathematical terms, we focused on those terms that could be related to the main lesson content (e.g. terms such as "equation" or "co-ordinate"). This meant that our analysis did not include utterances that constituted no more than agreement with a teacher's mathematical statement or utterances that only contained numbers or basic operations that were not the main focus of the lesson.

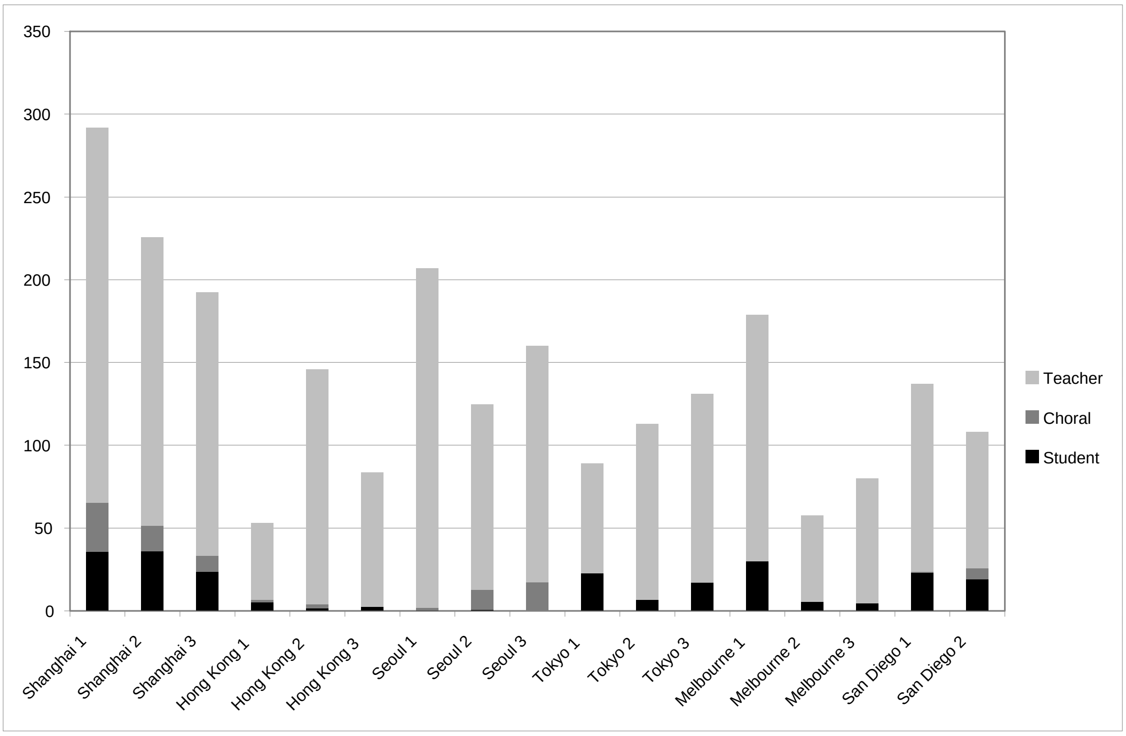

Figure 2: Frequency of Occurrence of Key Mathematical Terms in Public Utterances (Mathematical Orality)

In the case of the Korean lessons, the choral responses by students frequently took the form of agreement with a mathematical proposition stated by the teacher. For example, the teacher would use expressions such as, "When we draw the two equations, they meet at just one point, right? Yes or no?" And the class would give the choral response, “Yes.” Such student statements did not contain a mathematical term or phrase and were not included in the coding displayed in Figure 2. Similarly, a student utterance that consisted of no more than a number was not coded as use of a key mathematical term. It can be argued that responding “Three” to a question such as “Can anyone tell me the coefficient of x?” represented a significant mathematical utterance, but, as has already been stated, our concern in this analysis was to document the opportunity provided to students for the oral articulation of the relatively sophisticated mathematical terms that formed the conceptual content of the lesson. Frequencies were again adjusted for the slight variation in lesson length.

The most striking difference between Figures 1 and 2 is the reversal of the order of classrooms according to whether one considers public oral interactivity (Figure 1) or public mathematical orality (Figure 2). The highly oral classrooms in San Diego made relatively infrequent use of the mathematical terms that constituted the focus of the lesson’s content. By contrast, the less oral classrooms in Shanghai made much more frequent use of key mathematical terms and phrases. Since a single utterance might contain several such terms, and it was terms that were being counted in this analysis, Figure 2 provides a different and possibly more useful picture of the Chinese lessons, where both teacher and student utterances appeared to be longer and more complex than elsewhere.

Comparison between those classrooms that might be described as “Asian” is interesting. Key mathematical terms were spoken less frequently in the Seoul classrooms than was the case in the Shanghai classrooms. Even allowing for the relatively low public oral interactivity of the Korean lessons, the Korean students were given proportionally fewer opportunities to make oral use of key mathematical terms in whole class or teacher-student dialogue. In contrast to the teachers in Shanghai and Tokyo, the teachers in the Hong Kong and Seoul classrooms did not appear to attach the same value to the spoken rehearsal of mathematical terms and phrases, whether in individual or choral mode. It should be noted that Hong Kong 3 used English as the instructional language, while Hong Kong 1 and 2 used Cantonese, so any common features of the Hong Kong classrooms are likely to reflect dominant pedagogical practices, rather than be a specific result of the use of the Chinese or English language. The teacher in Hong Kong 2 appears similar to the three Shanghai teachers in the sense that he conducted his teaching most frequently in the form of whole class discussion. But his lessons show no signs of the pattern, evident in all three Shanghai classrooms, where the students were systematically ‘enculturated’ into the language of school mathematics. In particular, despite similarities between the public oral interactivity of Hong Kong 2 and Shanghai 1 (for example), the frequency of student use of mathematical terms in Hong Kong 2 was much lower.

While the overall level of public oral interactivity in the Tokyo classrooms was similar to those in Seoul, the Japanese classrooms resembled those in Shanghai in the consistently higher frequency of student contribution, but with little use being made of choral response. The value attached to affording student spoken mathematics in some classrooms could suggest adherence by the teacher to a theory of learning that emphasizes the significance of the spoken word in facilitating the internalisation of knowledge. The use of choral response, while consistent with such a belief, could be no more than a classroom management strategy. The Hong Kong classrooms offered students least opportunity to use spoken mathematical terms of all the classrooms studied and student spoken mathematical contribution, whether individual or choral, was extremely low, even though the student component of general public oral interactivity of the Hong Kong classrooms was at least as high as in Shanghai.

THE RELATIVE SIGNIFICANCE OF STUDENT–STUDENT INTERACTIONS

While the private conversations recorded in any one lesson were only those of the Focus Students, it was possible to compare the public oral interactivity of these students with their private oral interactivity and, similarly, their public and private mathematical orality. From the outset, it must be noted that six classrooms stood out because of the virtually complete absence of student-student interaction: those in Shanghai and Seoul. In these six classrooms, student-student conversation can be discounted as an instructional strategy (or as a subversive practice by students). For example, in Seoul classroom 1, there were no instances of student private talk in the first four recorded lessons and only two private utterances from one of the focus students in lesson five. The first utterance was “That’s yours” and the second was “No.” Obviously, neither involved any technical mathematical terms.

In reporting the results that follow, we have put both Shanghai and Seoul to one side. The role played by private student-student interactions in the remaining classrooms is particularly interesting. In Table 1, the figures quoted for both public and private Oral Interactivity and Mathematical Orality are per focus student per lesson and have therefore been averaged over the spoken contributions of around 10 students per classroom. This should minimize the effect of individual student timidity or extroversion, although awareness of being recorded will have been a common characteristic of all focus students (and of their teachers). In reading the ratio columns of Table 1, it is simplest to think of the results as indicating, for example, that focus students in Hong Kong class 1 used a mathematical term on average once every eight public utterances but only once every 48 private utterances.

It seems a reasonable hypothesis that student use of mathematical terms would be less likely in private contexts than in public teacher-orchestrated contexts. For seven of the 11 classes reported in Table 1, this was clearly the case. It is all the more interesting, therefore, that in all three Japanese classrooms and one of the Hong Kong classrooms the focus students were at least as likely to use mathematical terms in private conversation as they were to use them when participating in teacher-orchestrated public discussion. Hong Kong 2 seems anomalous in its very low number of student utterances per lesson, both private and public. With such small utterance numbers, slight variations in count may have the effect of inflating the ratio of private utterances to privately spoken mathematical terms.

Table 1: The use of spoken mathematics by students in public and private contexts

Schools	Oral Interactivity (utterances per focus student per lesson)		Mathematical Orality (mathl. terms per focus student per lesson)		Public Ratio (utts./ term)	Private Ratio (utts./ term)
	Public	Private	Public	Private		
Hong Kong 1	4.21	22.59	0.52	0.47	8.10	48.06
Hong Kong 2	2.84	7.15	0.41	1.30	6.93	5.50
Hong Kong 3	2.39	23.80	0	0.83	n.a.	27.67
Tokyo 1	6.13	14.79	0.28	2.24	21.89	6.60
Tokyo 2	2.08	33.85	0.23	9.46	9.04	3.58
Tokyo 3	6.92	11.67	0.61	0.99	11.34	11.79
Melbourne 1	16.16	99.14	2.85	5.59	5.67	17.74
Melbourne 2	14.36	83.75	0.18	0.30	79.78	279.17
Melbourne 3	15.78	73.51	0.17	5.63	92.82	13.06
San Diego 1	12.69	6.64	1.36	0	9.33	n.a.
San Diego 2	9.31	55.33	1.12	3.56	8.31	15.54

The Japanese result remains interesting; suggesting that Japanese students have a fluency in spoken mathematics that persists even across the public/private interface. It is also clear that student-student mathematical exchange was a feature of the Tokyo mathematics classrooms studied to a much greater extent than for the classrooms in Shanghai and Seoul.

CONCLUSIONS

It appears to us that the key constructs Public Oral Interactivity and Public Mathematical Orality distinguished one classroom from another very effectively. Particularly when the two constructs were juxtaposed (by comparing Figures 1 and 2). The contemporary reform agenda in the USA and Australia has placed a priority on student spoken participation in the classroom and this is reflected in the relatively high public oral interactivity of the San Diego and Melbourne classrooms (Figure 1). By contrast, the “Asian” classrooms, such as those in Shanghai, were markedly less oral. However, this difference conceals differences in the frequency of the spoken occurrence of key mathematical terms (Figure 2), from which perspective the Shanghai classrooms can be seen as the most mathematically oral. However, students in the Tokyo classrooms used spoken mathematics in both public and private situations. The relative occurrence of spoken mathematical terms is one level of analysis. We should also distinguish between repetitive oral mimicry and the public (and private) negotiation of meaning (Cobb & Bauersfeld, 1994; Clarke, 2001).

Despite the frequently assumed similarities of practice in classrooms characterised as Asian, differences in the nature of students' public spoken mathematics in classrooms in Seoul, Hong Kong, Shanghai and Tokyo are non-trivial and suggest different instructional theories underlying classroom practice. Any theory of mathematics learning must accommodate, distinguish and explain the learning outcomes of each of these classrooms. Consideration of the non-Asian classrooms is also interesting. With frequent teacher questioning and eliciting of student prior knowledge, the students in the Melbourne classrooms were given many opportunities to recall and orally rehearse the mathematical terms used in prior lessons. In terms of overall public mathematical orality and level of student contribution, Melbourne 1 resembles Shanghai 1 (without the use of choral response). In Melbourne 1, this public orality was clearly augmented by small group discussions, in which students drew upon their mathematical knowledge to complete tasks at hand. Such student-student conversations occurred much more frequently in the Melbourne classrooms. Student use of mathematical terms in situations not directly orchestrated by the teacher can be taken as a reasonable indicator of both the perceived need and the capacity for the purposeful employment of the technical language of mathematics. The relative infrequency of mathematical terms in student-student interactions in Melbourne 2 compared with the other two Melbourne classrooms suggests that these indicators are reflective of teacher influence.

To summarise: Students in the mathematics classrooms in Seoul have few opportunities to speak in class (either privately or publicly) and seldom employ spoken mathematics. Students in the Hong Kong classrooms are publicly and privately vocal, but make very little use of spoken mathematical terms in either context. Students in the mathematics classrooms in Shanghai are guided through the public orchestrated rehearsal of mathematical terms by their teachers, but seldom speak to each other in private during class time. Students in the mathematics classrooms in Tokyo participate orally in both public and private discussion and employ mathematical terms to a significant extent in both. By comparison, the students in Melbourne classroom 1 are highly vocal in both public and private contexts, and make more frequent public use of mathematical terms than any of the three Japanese classrooms, but less frequent use of mathematical terms in their private conversations. These different combinations of oral interactivity and mathematical orality represent at least five distinct pedagogies.

The next question is, of course, whether or not students are advantaged in terms of their mathematical achievement and understanding by classroom practices that afford the opportunity to develop facility with spoken mathematics. The implicit assumption in the classrooms studied in Hong Kong and Seoul seems to be that the employment of spoken mathematics is not to the students' benefit. Classrooms studied in Melbourne, Tokyo and Shanghai, despite differences in implementation, seem to make the opposite assumption. The post-lesson interviews may provide evidence of a connection between classroom mathematical orality and student learning outcomes.

This analysis is currently underway. We suggest that the empirical investigation of mathematical orality (in both public and private domains) and its likely connection to the distribution of the responsibility for knowledge generation are central to the development of any theory of mathematics instruction.

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MATHEMATICAL BEHAVIORS OF SUCCESSFUL STUDENTS FROM A CHALLENGED ETHNIC MINORITY

Tiruwork Mulat and Abraham Arcavi

The Weizmann Institute of Science, Rehovot, Israel

This study explored the mathematical behavior of resilient students of Ethiopian origin (SEO), members of an underrepresented and challenged ethnic group in Israel. Using qualitative methodologies, we examined six SEO, three in an advanced secondary school mathematics track and three in a pre-academic course while working on non-routine mathematical tasks. The mathematical behaviours and views of these students were found to be highly consistent with their professed beliefs and behaviors, which we explored in a previous study. Success was attributed to beliefs enacted during problem solving and was accounted for by neither giftedness nor special ethnic characteristics, but rather by high motivation, self-regulation, and persistence driven by positive identities, personal agency and ethnic identification.

Key words: Mathematical behavior, beliefs, self-regulation, resilient, ethnic identity.

INTRODUCTION

In many countries all over the world, immigrants and ethnic minorities often face barriers at school resulting from various factors. Many researchers and educators believe that differential student learning, achievement, and persistence along ethnic and racial lines is one of the most troubling issues in mathematics education and in education in general (e.g. Martin 2000, 2003). In the case of Israel, educators and researchers have done much to describe and classify social, cultural, educational, and other societal difficulties encountered by different groups of immigrant Jews and in particular, those students of Ethiopian origin (SEO, more than half of whom are second generation). A range of studies have documented the overall academic underachievement, the relatively high dropout rates, and the high representation of SEO in special education programs (e.g. Lifshitz, Noam, & Habib, 1998; BenEzer, 2002; Levin, Shohami, & Spolsky, 2003; Wolde Tsadik, 2007). In mathematics, SEO are significantly underrepresented in the advanced tracks towards Matriculation. For example, during the years 1999-2003, among all SEO who were eligible for the 'Bagrut', the Matriculation exam taken at the end of grade twelve in different subjects, only 2% studied mathematics in the advanced track [1], compared with 17% of the entire student population.

In different countries, some groups of immigrants and ethnic minorities achieve well academically; sometimes they even outperform mainstream students. Several studies have focused on explaining differential achievements between various minority groups and within certain minority groups (e.g. Ogbu, 1991; Martin, 2000, 2003; OECD, 2006). Most findings challenge the belief that the disadvantages and difficulties created by being an immigrant or a member of a minority prevent students from excelling in education.

Researchers are increasingly linking motivational, cognitive, and social environmental aspects of learning. Many studies have provided new insights into why individuals choose to engage in different learning activities, and how their identities, beliefs, values, and goals relate to their engagement and mathematics achievements (Steele, 1997; Nasir, 2002; Martin, 2000, 2003; Sfard & Prusak, 2005). It is argued that students' problem-solving processes are influenced by beliefs about the self, about the nature of mathematics knowledge, the task at hand, and its context (e.g. Schoenfeld, 1983). Moreover, implementing self-regulation during problem solving is regarded as an important variable affecting the quality of the solving process: self-regulated learners analyze tasks and set appropriate goals to accomplish these tasks, monitor and control their behaviors during performance, make judgments of their progress and alter their behaviors according to these judgments (Zimmermann, 1989). Social cognitive theorists, assume that self-efficacy is a key variable affecting self-regulated learning and performance (Bandura, 1986); self-regulated learning is believed to occur to the degree that a student can use personal (i.e. self) processes to strategically control and direct both his/her behavior and the immediate learning environment (Bandura, 1986; Zimmermann, 1986).

Based on the personal and environmental factors identified by research in mathematics education and especially based on the findings related to the success of individuals from populations at risk of academic failure, we sought to understand the success factors of SEO, students enrolled in the advanced mathematics track towards Matriculation. We focused on these students' views about their personal experiences in learning mathematics and the perceived impact of the personal and environmental variables on their persistence and success [2]. The conceptual framework used to guide our inquiry is based on the assumption that there are certain malleable personal and environmental factors that play significant roles in these students' academic resilience, defiance of the odds and their ultimate academic achievement. We adhere to the claim that, as opposed to studies of failure (regardless of their academic depth), studies of success constitute a more promising way of understanding and eventually increasing the circle of successful students (Garmezy, 1991; Martin, 2003). In our studies we sought to understand what enables some SEO to succeed despite the potential obstacles they face. We attempt to answer the following questions:

1. To what perceived personal/environmental variables do SEO in Israel attribute their success in mathematics?
2. What are the salient mathematical behaviors of SEO when working on mathematical tasks? How do they view, and reflect upon, their own behaviors?
3. How do the perceived variables, the enacted mathematical behaviors, and the students' views of these behaviors relate to each other?

In a previous study we explored the first question, through students' self-reports obtained using semi-structured interviews (see below a summary of this study). In the present study we present findings concerning the second and third questions.

FINDINGS FROM THE PREVIOUS STUDY: STUDENTS' SELF-REPORTS

A diverse group of SEO enrolled in the advanced mathematics track towards Matriculation were interviewed and followed up. The group consisted of fourteen students aged 17-19 (seven males and seven females), of which nine were high school students from four different cities and the other five were students enrolled in a special pre-academic program in a prestigious technological university in Israel (each from a different city). All were 'solos', i.e., the only SEO in the advanced mathematics track in their cohort at their schools, which is the optimal situation in most high schools. Our goal was to better understand how these students interpret their experiences and academic achievements within the advanced track in mathematics, in high school and in the university preparatory program, where the presence of students of Ethiopian origin is scarce. Using the qualitative methodology of a collective case study (Yin, 1984; Shkedi, 2005), we analyzed interview transcripts using a grounded approach and employing open coding techniques (Strauss & Corbin, 1990). Data were also triangulated with other sources such as classroom observations and interviews with other students, teachers, and parents. The key elements of success we identified were organized under three major categories (Mulat & Arcavi, submitted):

(1) Motivational variables related to mathematics (e.g., mathematics identity, personal agency, productive attribution beliefs, academic goals, ethnic identification, and social goals activated by a positive cultural model)

(2) Actions and strategies – perceived behavior (e.g., fostered use of academic self-regulation and coping strategies)

(3) Immediate environmental variables (mathematics classrooms, teachers, and parental support)

The central finding of the study was that the synergy among students' motivational variables, their academic self-regulation and coping strategies, shaped and supported by their interaction with the environment, appeared as the key to their success in mathematics.

THE PRESENT STUDY

The aim of the study reported here is to explore the mathematical behaviors and the task-related views of a subgroup of the participants in the previous study, and to examine how the findings of the two studies relate to each other.

METHODOLOGY

Subjects: Six SEO from the previous study participated in this study. Three of them (Eden, Melka, and Jacob) were high school students, and the other three (Selam, Ronnie, and Danny; all pseudonyms) were students in the pre-academic program. The selection of these participants depended upon the availability of extensive data relevant to this study.

Tasks: The students worked on five mathematical tasks, selected especially for this study according to the following criteria: The tasks had alternative solutions; they varied in their level of difficulty; their content level was rather basic and accessible to high school students, yet they were non-routine, challenging, and required some planning strategies. The problems were previewed by mathematics educators who agreed on the mathematical appropriateness for high school students.

Data collection and analysis: The data consisted of students' written work, the interviewer's recorded observations, the protocols of the dialogues, questions and reflections that emerged during task completion, and the transcripts from follow-up interviews. In the interviews, all students were asked to describe their solution approaches and their thinking processes in completing the tasks and to describe their perspectives. These tasks were also given to students' peers in the lower mathematics tracks of the secondary schools. A qualitative descriptive methodology was used to analyze the combined data (Shkedi, 2005).

FINDINGS

A description of students' solution processes, along with the observed behaviors and views for three of the tasks are given, followed by a summary of the significant findings.

Problem 1.

> Find the equation of the line parallel to the given pair of parallel lines and that lies exactly midway between them: (1) $3x-2y-1=0$
>
> (2) $3x-2y-13=0$

Task completion: All participants efficiently completed this problem. The task was characterized by all of them as non-routine since its formulation was seen as different from what they usually encountered at schools, yet it was perceived as easy and accessible by available tools or algorithms.

All subjects showed confidence in their ability to complete this task, and had completed it easily; appearing to be satisfied with their ability (two had minor computational errors). However, despite the existence of alternative ways to solve the problem, both the high school and the pre-academic students applied the 'slope-point formula' procedure they learned at school. Accordingly, the common stages in the students' solution procedures were in this order:

- Transformation of the equations to an explicit form
- Identification of the common slope
- Identification of the y-intercepts (some solved for the x-intercepts)

- Finding the midpoint between the intercepts (using formula or graphs)
- Writing the answer - equation of the line

The participants attributed their success to their rich experience and mastery of similar school tasks. Yet, this task was found to be difficult to many students in the lower tracks, who blindly tried to solve the pair of simultaneous equations in search of a point, after they found the common slope, the first two stages above.

Although both the high school students and the pre-academic students were equally successful in solving this task, we detected a difference in their use of a heuristic and the perception of its necessity. Two of the high-school students drew the graphs of the lines to find the midpoint of the intercepts, whereas all of the pre-academic students did not, claiming that they do not need the graphs to solve this problem and if they do, they can imagine them. The following quotations exemplify these differences among students:

> Instead of visualizing in your head, it is already in your notebook and it is hard to get confused that way. (Jacob)

> Here I do it in my head. You see that they have the same slope…when I can't see things with my imagination, I use sketches. But here you know the question leads you to the solution. (Selam)

Problem 2.

ABC is a right-angled triangle, $\angle$ ABC=90°.

AB=16; BC=12 and BE=9; BD is the median to AC, and

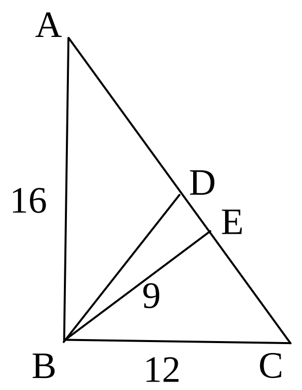
BE is the altitude to AC.

There is an error in one of the given numbers.

(a) Show that there is an error (report all your processes).

(b) Change only one of the numbers (9, 16, or 12)

to correct the error.



Task completion: Students showed different performance levels on this task. All students started by marking the given numbers on a triangle they drew and by calculating the length of the hypotenuse AC=20 (one made a computational error). Five of the students also marked AD=BD=DC=10, referring to the theorem about the median to the hypotenuse in a right triangle, but only three used this information to produce their solutions later. Only three of the students completed both parts of the task independently showing ease and confidence (but one had computational as well as other major errors and thus got a wrong answer). The other three students had difficulties in devising a plan and an effective strategy to proceed with the task; they were stuck for a long time; two of them said that they checked whether there is a side with a length greater than the sum of the other two sides. These students were confused and disturbed since they did not know how to plan their solution procedure and were uncertain about their understanding of the question. After some unsuccessful trials, they quit and proceeded with the other questions and returned to complete the task after receiving supporting clues and prompts from peers and from the interviewer.

In the first part of the task, students used different strategies to show that a triangle having the given sizes is not possible. Two showed that they got two different areas for the same triangle, three showed two different sizes for a side of the triangle; another student showed that the corresponding sides of similar triangles are not proportional. Five of these students used the same strategies they used for the first part to answer the second part of the question. One chose to use a trial and error method. Half of the students mentioned the possibility that the error could be corrected by changing any one of the three numbers. Since there were different ways to show that there is an error, the error could be corrected by changing any one of the three numbers, implying different ways and possibilities to answer the second part of the question. Yet all participants decided to change 9 (which was a good choice); four students (two of them with support) completed the problem successfully. The other two students, one who used a trial and error method and another who made a major error in her computations to change 9 got wrong results.

All students characterized this task as non-routine, saying that it is not like school tasks that they usually solve with great ease and success, and that here they could not just apply known algorithms to obtain a solution. Danny, who completed all the tasks successfully, characterized this task as 'a deceptive question'. Jacob said:

> This is a question in geometry, but never, at least I never encountered questions like this, saying that there is a mistake, correct a mistake. Usually they give you exercises that have solutions at the very beginning, and if you work by the book, you succeed, but here you have to think more.

Melka also referred to her school experiences:

> We are not used to such kind of questions; they never tell us to correct mistakes; they always provide us with given objects and ask to do other things and not to correct mistakes.

In sum all the students (some with probing), completed the first part of this task successfully by using different strategies. While four of them also succeeded with the second part, the other two students used ineffective strategies and got wrong answers.

Problem 3.

Given is an array of natural numbers arranged under four columns, A, B, C, and D, as shown here.

(a) Under which letter does the number 101 appear?

(b) Under which letter does the number 1001 appear?

A	B	C	D
1	2	3	4
8	7	6	5
9	10	11	12
	…	14	13

(c) Answer questions (a) and (b) above, for a five-column array of numbers with the same pattern

Task completion: This task seemed to be more difficult than the other tasks especially for the high school students. It also took most students more time than each of the other tasks. Only two students (both pre-academic) found effective rules and gave correct solutions with clear explanations.

As a first step towards finding a solution to this problem, all students added more numbers to the list following the given pattern. All of them tried for quite a long time to find a possible pattern or rule to solve this task (unlike students in the lower tracks, who tried to answer it by listing numbers to reach 101 without looking for a rule). As stated above, only two students (both pre-academic) proposed a similar rule: even and odd multiples of 4 can be found in alternate lines of the outer left and outer right columns (A and D), respectively. The other pre-academic student, however, did not recognize the sequences' pattern on the extreme columns and added to the list in a wrong order. Consequently, she did not succeed in completing the task, but she refused to hear a solution method and asked to complete the task at home by herself. Three of the high school students did not write their rule clearly, and their answers were mostly wrong or not justified, although two were certain they had obtained a working rule. The other student, who seemed less confident, said that she solved it logically, using her common sense, and that she did not know how to communicate her method.

This task was characterized as difficult by all participants. One of the students even commented that it is not a mathematical question; the other said it is a 'thinking' question that challenges the mind, and that schools do not offer such questions.

SUMMARY

As stated above, this study explored the mathematical behaviors displayed by successful SEO, and analyzed the relationships between these behaviors and the professed beliefs and reflections found in a previous study in which the students also participated. Some of the findings from the previous study (e.g., ethnic identification, social goals, and parental support) were not salient in the present study due to their very nature; these categories are rarely captured while students work on mathematical tasks. However, in other findings we found consistency between the 'professed' beliefs and behaviours and the 'enacted' mathematical behaviors, as described in the following.

Motivational beliefs: Students showed a variety of behaviors and performances. Although some students lacked confidence when they had no handy effective strategies, their behaviors were consistent with their professed efficacy beliefs and their confidence in their ability to solve the problems. They said that they have the mathematical knowledge necessary for completing the tasks and shared their enjoyment and satisfaction of being engaged in questions that demand thinking. They attributed their difficulties in solving these problems to a lack of previous experience with non-routine questions. They expressed their expectations that schools should provide opportunities to encounter and practice such tasks that require 'thinking'.

Self-regulation strategies: Self-regulation is one of the characteristics that we had identified in the previous study as playing a prominent role in these students' success in school mathematics. The students expressed their belief that what it takes to succeed in school is planning and evaluating their own actions and strategies by investing time and effort to study what is taught at school. When these students failed to solve some of the non-routine tasks of this study, they attributed it to not having the right tools, since their learning efforts were directed to what school had taught them. Thus, cognitive regulation and retrieval of the appropriate knowledge and the strategic tools needed for the tasks in this study were difficult for them. Many of the students quit after some unsuccessful trials, moved on to other questions but still returned to the unsolved tasks later. We took this willingness not to give up as yet another manifestation of these students' good self-regulation strategies applied to difficult situations for which they were unprepared. This strategy was found to pay off for some students, since with some probing they succeeded to complete the tasks.

Solo learning: Though students were told that they can work with their peers (three of them had opportunities to do so) and also that they can ask for support from the interviewer at any time while working on the tasks, they did not use these opportunities productively. Suggestions to support the students when they were stuck at certain stages were all initiated by the interviewer. The preference of students to work alone was also in line with these students' professed 'solo learning' characteristics.

Perceptions about the tasks: The tasks were characterized as non-routine, including the first question that all could easily solve, yet they expressed their enjoyment and satisfaction in performing such tasks. The students were very critical about mathematics lessons at schools that do not offer students opportunities to face challenging tasks.

Differences within groups: Though the tasks are appropriate for any high school student, overall, the pre-academic students showed (a) greater confidence in completing the tasks (even when they were not always successful), and (b) better communication skills to write and explain clearly their solution processes. These differences could be attributed to the pre-academic students' self-reports that in contrast to high school teachers, the teachers in the program have exposed them to meaningful mathematics learning, which also developed their confidence, intrinsic interest in mathematics and mathematics identity.

DISCUSSION AND CONCLUSION

Whereas these SEO's success in school was, to some extent, due to learning by playing well the school rules, which are mostly rehearsing and following algorithms, completing the tasks of this study engaged these students with a quite different experience. Thus, since these students were not especially gifted and their knowledge resources come only from school, their success can be attributed to their mathematics identity, motivation, and self-regulation skills; all these were supported by their other professed beliefs and views in relation to the tasks. Moreover, the heterogeneity of solution approaches and strategies observed in this study is proposed as a further confirmation how resilient and minded to success these students are, each of whom mustered resources and alternatives from his/her own to solve the tasks.

In sum, neither exceptional cognitive ability nor common cognitive characteristics of a certain "ethnic" group are variables that play significant roles in analyzing success (or failure) of these SEO. It is their determination, personal identities and support that shape their self-regulation, persistence and beliefs that shape their behaviours and ultimately their success. From this and related findings, we argue that educational systems that want ethnic minorities to succeed academically have much to learn from these and related findings regarding the roles of identities, self regulation, enhancement of motivation and support of learning which can take place in collaboration with peers.

NOTES

1. In the Israeli education system not all students are eligible for Matriculation; eligibility is determined according to the students' prior achievements. In mathematics, those eligible have taken one of three levels: basic (3 units), intermediate (4 units), and advanced (5 units).

2. In this study success refers to enrolment in the advanced track towards Matriculation

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A PROBLEM POSED BY J. MASON AS A STARTING POINT FOR A HUNGARIAN-ITALIAN TEACHING EXPERIMENT WITHIN A EUROPEAN PROJECT

Giancarlo Navarra (*), Nicolina A. Malara (*), András Ambrus (°)

(*) University of Modena & Reggio E., (°) Eötvös Lóránd University of Budapest

The paper reports on a collaborative project involving Italy and Hungary, within the European Project PDTR [1], *and presents an analysis of its implementation and outcomes. The work stemmed from a problem about the exploration of regularities, proposed by John Mason, scientific advisor of the project. We start from the preliminary analysis of the problem carried out by the two teams, present re-elaborated versions, planning of the activities and modalities for implementing them in the classroom in the respective countries, discuss the outcomes of the experiment, final reflections made by experimenting teachers and general ones made by the teams about the materials elaborated during the activities.*

Key words: Arithmetical Regularities, Early Algebra, Teachers Professional Development, Teaching Experiment, Teaching-Research

INTRODUCTION

The central aim of PDTR project has been to engage teachers of mathematics in the process of systematic, research-based transformation of their classroom practice so to initiate, using teaching-research as the leading methodological agent, the transformation of mathematics education towards a system which, while respecting the standards and contents of the national curricula, would be more engaging and responsive to student's intellectual needs, promoting independence of thought, and realizing fully the intellectual capital and potential of every student and teacher.

The teachers' work, in a first phase, addressed issues and questions of the PISA test, with particular reference to the promoted competencies, some of them - such as argumentation, posing and solving problems, modelling and representations – are clear indicators of a new way of conceiving the mathematical teaching and classroom activity. In a second phase, the PDTR apprentices and IT designed teaching experiments, collected data, observed their pupils with a new investigatory eye, analyzed and discussed the data with their team members.

In this context, some teaching experiments were carried out with the aim of promoting a direct exchange between the teams on the ways of implementing common activities in the participating countries. The richest exchange occurred in the Hungarian-Italian Bilateral Teaching Experiment (HIBTE), which was developed in the field of the algebraic and pre-algebraic thinking (Malara & Navarra, 2003).

METHODOLOGY

Meaningful increasing research in mathematics education points to the renewal of its teaching through a linguistic and socio-constructive approach in the sense of early algebra with pupils of k-8th gr. In this perspective, teachers come to play a complex role in the classroom and they need to face a number of unpredicted and not easily manageable situations. Regarding this, several scholars highlight the importance of a critical reflection by teachers on their activity in the classroom (Mason, 2002; Ponte 2004) so that they can also become aware of the macro-effects on classroom activities caused by their (sometimes not appropriate) micro-decisions. To promote this attitude in teachers, within the Italian Team (IT), a complex written activity of critical analysis of classroom transcriptions, in which the teachers, their mentors, the mentor coordinator and the academic researcher cross their comments, has been enacted. It is called *Multi-Commented Transcripts Methodology* (MCTM) (Malara, 2008). The methodology of work between the two teams developed in 5 phases: 1) Adjustment by HT of the proposal made by John Mason, PDTR expert, to the Hungarian Team (HT) teachers; 2) didactical transposition of the adjusted proposal in HT classes (9th-12th gr.), evaluation of the results; 3) analysis, adjustment of HT proposals by IT and transposition in IT classes (6th-7th gr.); 4) implementation of MCTM; 5) analysis by HT of IT transcripts; 6) cross reflections.

DISCUSSION

The original proposal by John Mason to Hungarian PDTR teachers

During his lecture in Debrecen (Hungary), Mason asks the participants (about 30 pre-service mathematics teacher and about 30 secondary school mathematics teachers) to solve the following problem (Fig.1). After 10-15 minutes, it is clear that such type of problems are very uncommon to Hungarian teachers and students, most of them cannot do anything. Seeing the difficulties, Mason numbers the rows and sketches the fourth row in the shape of a 'cloud' which hides the sum (Fig.2).

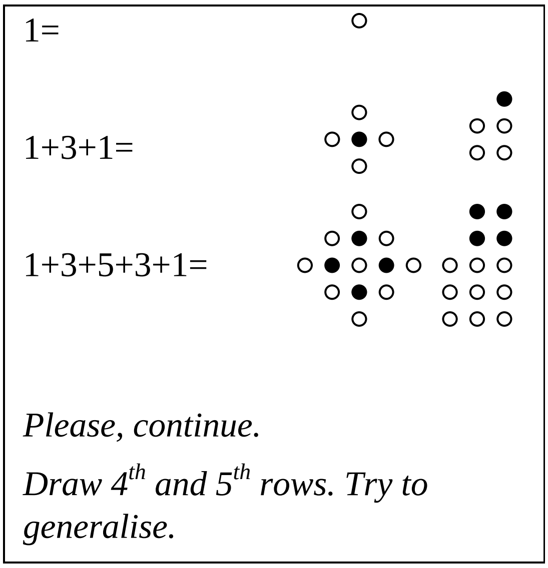


Fig.1: Mason's problem

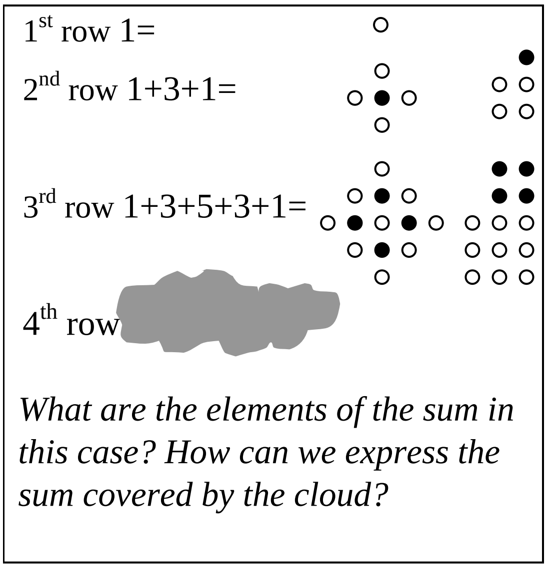
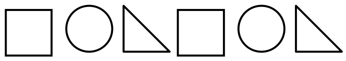


Fig.2: Mason's problem adapted

At this point, a lot of participants still have difficulties, so the generalization is led by the lecturer himself. Based on this experience, the Hungarian team (HT) decides to investigate this phenomenon and leads an a-priori analysis of the question.

Two additional preparatory problems to Mason's problem

On the base of the analysis, HT decides to employ two additional preparatory problems (Figg.3, 4, 5) in the classroom-based experiment.



Let us continue the sequence till to 17.th element! Which figure is standing on the 243-th place? What is the order number of the 25th circle? Try to find a general expression for the positions of squares, circles and triangles!

Fig.3: HP_1 - first preparatory problem

1st row 1=

2nd row 1+3=

3rd row 1+3+5=

4th row

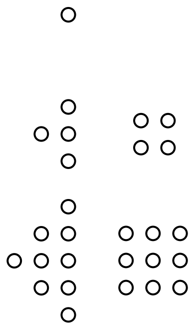
…

10th row

…

nth row

…



Prove your conjecture for the nth row! You may use algebraic and geometrical arguments (if possible, prove with both methods).

1st row 1=

2nd row 1+3+1=

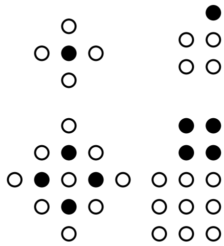
3rd row 1+3+5+3+1=

4th row

…

nth row

…



Prove your conjecture for the nth row! You may use algebraic and geometrical arguments (If possible, prove your conjecture with both methods).

Fig.4: HP_2 - second preparatory problem

Fig.5: HMP_3 - Mason's problem

The Hungarian teachers involved in the experiment report after two weeks that their 9th grade students are able to do some steps of the first problem but no one in the second and third problem. HT asks other teachers to conduct the test in higher grades (170 students of 9th, 11th, 12th), but difficulties and blocks are still detected in the students. Based on these results, the Hungarian team (HT) decides to share the experiment with other PDTR teams, by posing the question to investigate on these difficulties and particularly on the reasons underlying students' inability to generalise and represent the sequences in general terms.

Reactions by Hungarian students and teachers

In November 2007 Mason's problem, its *a priori* analysis, HP_1, HP_2, HMP_3 and the commented outline of the results obtained in Hungarian classes are sent out to the IT, together with comments like the following:

> "… The first experiences with Mason's proposal are very negative. The Hungarian students are not used to open problems, to visual representations, to induction and generalization".

The Italian team in turn analyses the problems. The coordinator writes to the Hungarians:

> "… The teachers reacted to these problems by saying that it is nonsense to bring this task into a class, independently on the plan of work, because this proposal requires a lot of time (time for the students' individual and/or small group exploration, for assessing students' results, for organizing and realizing in the class the discussions on the students' contributions)."

The teams are stuck. Both students and teachers react to the experiment with either a sense of frustration or hostility. An in-depth reflection on the HIBTE is then enacted, and the discussed themes start from the Mason-episode to widen up.

FIVE KNOTS

Five central issues emerge from the analysis:

1) <u>What are HIBTE's objectives?</u> The first answer, provided by both Hungarian and Italian teachers, was: *to look at if/how students explore/solve the three problems*. But **the main issue** is: were these Mason's objectives, or those which HT and IT attributed to Mason's proposal and consequently to HIBTE?

2) <u>Who is HIBTE's referent?</u> There are three possible answers: the students, the teacher-researchers, the researchers. The answer 'the student' was the first one and brought about problems to both Hungarian and Italian teachers: unusual problems, classes not prepared to tackle them, missing pre-requisites, activity not included in a planning which requires a lengthy time (particularly if the class has not experienced similar activities). But is it true **that students** were the main referents of the HIBTE?

3) <u>What are the needed competencies?</u> **Are the mathematical ones the only or main ones?** The question is: perhaps the needed competencies are wider and the mathematical ones are only a subset?

4) <u>How can the problem proposed by Mason be set in the class' teaching and learning context?</u> Mason's proposal may be viewed as a *virtual* proposal. He provided an input and it was up to the single countries to compare it to their own cultural reality, their school systems, their teacher training programs and their usual behaviours. In the prior analysis, HT and IT needed to give *a sense* to the proposal, with relation to their specific theoretical frameworks, for instance: in the prior analysis HT focused on

didactical-mathematical aspects and on *students*, whereas IT focused on *methodological* aspects and on *teachers*. So: actually setting the problems out in the classes, is this the *sense* of the proposal?

5) Why studying sequences and regularities? The answer is: Mason meant to be *provocative*. He perfectly knows that the theme is highly important (modelling, generalizing and so on) but he also knows that its underlying spirit is completely, or at least largely, stranger to the school systems of many countries. His proposal means: do not think of setting the problem in the class immediately, get really engaged with this question, and think about what might/should happen in your class, *and therefore* in your way of thinking, *and therefore* in your school system *and therefore* in your country's teacher training system, so that *these problem situations and activities may become components of the spine of a different way of conceiving mathematics teaching, as well as of implementing it.*

Let us get back to our initial questions: who is the referent of HIBTE? Which are the objectives? If we think that students are the referents and their competencies in mathematics the objectives, we would break an open door: given the premises, a negative outcome would be easily predictable. *The actual referents are trainee-teachers-researchers and researchers*. The objectives are not 'only' mathematical knowledge and the strategies to enact it, but rather *reflection* – initially individual and then shared – *on methodological issues that, appropriately set, can make this type of problems feasible and meaningful in the class*. It is in this line that IT opens up the theoretical umbrella under which the HIBTE will develop. It is decided that an initial experiment will be carried out by Navarra [2], with his class (6^{th} grade) and later by some other *trainee teacher-researcher*, in 6^{th}- 7^{th} grade classes, on the basis of HP_1 and HP_2. Mason's problem is left aside, because teachers consider it as unsuitable for the expertise of pupils of this age.

1) The teaching experiment in Italy

The transposition of Hungarian problems in two 6^{th} and 7^{th} grade classes

Navarra's class could be defined as 'expert' since pupils have in their background (K-5^{th} gr) more than five years activities on the study of regularities in an *early algebra* setting (40-50 hours with Navarra teaching together with the class teacher). The class is used to working in an ArAl environment and therefore to verbalizing, arguing and constructing knowledge socially. Navarra proposes a new version of HP_1 (**Fig.6**):

Pupils are asked to start from the drawing to imagine what questions might be proposed to another class, so that their curiosity might be stimulated, and organize both drawing and questions in a problem.

Fig.6: HNP_1 – initial problem situation, HP_1 version

Turning an input into a problem is not a new practice. Pupils, divided in groups, elaborate 36 questions and then reduce them to 13, through a large collective discussion. The first 6, out of the 13 questions, are defined ‘ice-breaking questions’ purposefully organized for a ‘non expert’ class; 4 are defined ‘opening questions’; the last 3 questions (‘difficult questions’) are, in fact, the same as in HP_1 (Fig.7).

A. Ice-breaking questions	**B. Opening questions**	**C. Difficult questions**
□○◺□○◺□○◺□○◺□○→ 1. *What does the arrow mean?* 2. *Which is the module?* 3. *How many figures is a module made of?* 4. *How does the sequence carry on?* 5. *If I repeat the module 50 times, how many times is the circle repeated? And the square? And the triangle?* 6. *When triangles will be 345 how many modules will there be?*	□○◺□○◺□○◺□○◺□○→ 7. *The squares are at places 1, 4, 7, 10, 13. What about circles and triangles?* 8. *Is every type of figure at even places? Only at odd places? Both at even and odd places?* 9. *In 23 modules how many figures are there?* 10. *Were the shapes 100, how many modules would we have?*	□○◺□○◺□○◺□○◺□○→ 11. *Explain how you can find the figure at place 34. And place 95? And 243?* 12. *Explain how you can find out in what position are the 56th triangle, the 192nd square, the 368th circle?* 13. *Can you arrange general formulae to find out at which position is any odd square, circle or triangle?*

Fig.7: Questions proposed by pupils

Pupils themselves solve the questions, during discussion, analyzing, comparing, modifying and eliminating them. Altogether, eight hours of work in class; four diaries drawn from four digital recordings. The class goes through the experience productively because they set it in a familiar context. Warning: one does not say ‘extraordinary context’, but rather ‘familiar’; one means a suitably constructed context, with an internal consistency pupils were aware of, undertaken when they were five years old.

The problem of analyzing pupils’ questions is proposed by Navarra in a 6th grade class of a colleague of his. Pupils’ reactions to the first six questions are of confusion, and make Navarra realize that, before tackling them, he needs to broach, although in a short time, with some very delicate *methodological questions coming well before the solution*, that is: pupils are scarcely used to talking about mathematics, have an initial block when they need to explore a problem situation, are not familiar enough with competencies like verbalizing, arguing, controlling and comparing different languages and translating from one language to another; focus more on ‘results’ than on strategies and thinking processes. Moreover: the approach to generalization and modelling are nearly unknown; there is a stereotype about the impossibility of a creative and functional attitude in the production of mathematical expressions; there is a weak control over mathematical contents such as: multiplicative structures, divisibility, division algorithm, properties of operations, use of letters, etc.; there is a poor use of tables to explore and compare data as well as to analyze what is constant and what varies. One could say that it is a standard class, with standard pupils, a standard teacher, standard programs.

The ‘ice-breaking’ questions allow groups to produce mathematical expressions that are reported on the blackboard, compared and selected in a search for the most correct, consistent and the clearest. The first 10 questions turn out to be effective, and the outcomes of the activity in this second class (8 hours) are globally satisfactory.

The eight hours of work in the first class on the first task produce four diaries, drawn from four digital recordings. The transcripts, commented by Navarra, are sent out to other components of the IT who comment them in turn, following the multi-commented transcripts methodology. After this, HP_2 (Fig.4) is analysed and then structured in three worksheets A, B, C [3] so that the difficulties may be diluted. The worksheets are meant to favour a representation through letters: (A) of the relation between the last addendum (*a*) and the ranking number (*n*) of the n^{th} row $(a=2n-1)$; (B) of the relation between the ranking number (*n*) and the sum (*s*) of the n^{th} row ($s=n^2$); (C) of the sum of the first n odd numbers. The protocols relating to Navarra’s experiment are analyzed and classified by IT. Based on the outcomes, the worksheets are refined with some changes and then proposed to a 7^{th} grade class, with teacher Marco Pelillo, novice trainee researcher**.**

Classification of the results is based in particular on the following aspects: (i) identification of how different perceptions of written expressions and of drawings influenced the related algebraic or ‘pseudoalgebraic’ expressions produced by pupils (i.e. many interpreted the two graphical representations, seeing the first, as representing the operations of sum of odds indicated, and the second, as representing the result of the sum; this interpretation was encouraged by the fact that a dot was missing in the first line of the second representation); (ii) strategies and consistency used by students to develop their explorations up to the identification of general forms and ways to express them in either natural or algebraic language; (iii) analysis of pupils’ verbal representations’ like “The line number is always doubled by 2 and decreased by 1”; “The difference between the line number and the last term of the sum is always equal to the number of the previous line; adding up the line number to the number of the previous line you get the last term of the sum as result”; (iv) identification and analysis of algebraic expressions that could be reduced to $a=2n-1$ like: $a=n+n-1$, $a=(n+1):2$, $a=n\cdot 2-1$, $a=n+(n-1)$ (*a* = ‘last addendum’ and *n* = ‘row number’); (v) analysis of written expressions that could be reduced to $s=n^2$ or to $s=n\times n$ (*s* = ‘sum’ and *n* = ‘row number’); (vi) analysis of written expressions to be reduced to $1+2+3+\ldots+2n-1=n^2$ or $n\times n$, to test pupils’ capacity to spot the equality between the sum of the first n odd numbers and the square of n. At the end of the experience Pelillo makes the following comment:

> “…It was very hard to make pupils represent the equality, since they were not able to express the sum of the first n odds in general terms, despite the hard work made to

represent the last term... I produced a justification of that equality in a recursive way, on the basis of geometric remarks, and representing the odd number to be added to the subsequent line of data with the gnomon of the square corresponding to this one... Many pupils immediately grasped the regularity. The identification of the result of the sum of the first n odds was easy, whereas more problematic was the representation of the sum of the first n odds... The linguistic aspects turned out to be problematic. A basic difficulty was evident in pupils' linguistic expression... We might talk about *a proximal use of the Italian language*."

In February 2008 the Italian versions of the problems, the commented transcripts by Navarra (32 pages), the classifications of protocols are sent out to HT.

2) The teaching experiment in Hungary

HT analyses materials sent by the IT and, on the basis of this, decides to carry out a teaching experiment in two classes (5^{th} and 6th grade, Béla Kallós, novice teacher researcher trainee). In July 2008 HT sends to IT the synthesis of the work carried out at Kallós on HP_1 and HP_2 together with the teachers' remarks on the Italian materials.

Comments by Béla Kallós

"… The students were divided into two groups. The groups received the task sheet. I asked the students to read the text carefully, if they did not understand something, they could ask me. I have planned 25-30 minutes for the pair work. In the last 10-15 minutes we discussed the solutions with the whole class... The students did not understand the problems in all cases... We have seen that at this age some students can express their solution using formal language"

"Some reflections on myself as a teacher. In PDTR J.P. da Ponte formulated four main phases in the development of the teacher-researchers: teacher; good teacher; researcher; teacher researcher. I am a very young teacher yet, not with much experience. I am just on the way to be a good teacher. Most of my teaching actions are intuitive, based on my personality and some experiences as a student, teacher student and teacher. Until now my main aim was to teach mathematics and science as might as possible effectively. These two experiments are my first trials in research in mathematics education... I was socialized by the traditional Hungarian education. Mathematics has a high prestige in Hungary, the competitions, the fostering of talented students are in the centre. We in Hungary are focused on teaching mathematics and not on children."

"About my teaching style: I audio-recorded my lessons first time and it was a surprise for me to hear myself. I need to develop my articulation, my construction of sentences. I should have given more time for the students to think about the solution of the problems. I need to have more tolerance to the students' misconceptions and mistakes."

Use of open problems: "We have seen how much difficulties the most open formulated version caused for Hungarian students. In my experiment I modified the task sheet into such small concrete questions that the originally open problem became a closed one. It is clear that in such a case the students do not have too much freedom to be creative, flexible. I think I should use more time for problem posing, problem variation."

Some Hungarian teachers' reflections on Navarra's transcript

"As for the used teaching method: the students of 9th, 11th, 12th worked in groups, they got about 15 minutes to solve Mason's problem... In Hungary the group work is very rare, the teacher's leading role is very strong and is based on the ideology that everybody must achieve the same high level."

"In the Italian commented transcript the activity contains very detailed analysis of students' products. In Hungary, we usually close the discussion after some minutes, very fast with the right result!... From the point of view of handling the mistakes, for us it was interesting to observe how tolerant the teacher was with the students' mistakes. We must accept the effectiveness of the Italian style: *the students need to explain the source of the mistakes*. For example, Navarra says to the pupils: 'It is important for you to understand the mistake' and, in one case: 'What is more important for you in this moment, focusing on the tenth at the division, or on the remainder?' In Hungary the written division algorithm is taught in 4th grade, in higher classes our teachers don't consider this question necessary to handle anymore, because 'everybody must know it'."

"In developing the students' way to form arguments and explanations, it is fascinating to observe how the teacher tended to improve students' arguing: 'Please, make your thinking method understandable!'... It is typical for this age that pupils cannot express themselves: 'I can do it, but I cannot explain why!' Very often students repeat the process they used as explanation. We can only agree that to develop the PISA competence 'mathematical communication' is a long process, and we must do it consciously".

"Varying the figures of the unit is a good possibility to check the understanding of the students both of the process vs. product and of the general rule. The younger students tend to concentrate only on the product and not on the process... Simply, the Hungarian mathematics teachers do not care for this problem."

"We wondered how many children participated in the communication at this problem, changing the number of figures in one unit, changing the type of figures, using reverse problems... Navarra always summarized the results and the pupils analyzed them on the whiteboard. In our opinion for this age group the clear visual explanation is important."

CONCLUSIONS

Enacting International collaborative projects in the educational field requires great involvement by all participants. But enacting *meaningful* forms of collaboration, regarding issues with a shared value, requires the *construction of a common ground*, where conceptions (of mathematics and its teaching) and educational values might be questioned and the cultural and environmental operating conditions are made explicit. In the case of HIBTE, the will to engage in a single task and communicate methods and results, provided a basis for important in-depth analysis, far from the initially predicted one. The original proposal by Mason was lived as a stimulus to lead teachers to reflect upon many issues, very important from general points of view: the role and the way of being in the class, the capacity of anticipating the class' behaviours as a reaction to teaching proposals; the need to acquire a range of competencies to enable improvisation in the classroom. Therefore, more than carrying out an in-depth analysis of mathematical aspects, which is in the 'natural spirit' of the exploration of problem situations like the ones we proposed, in our case, exchanges occurred under a *methodological*, *before being mathematical*, theoretical umbrella. The main referents were teachers, well before students; the main questions concerned linguistic and social competencies, well before cognitive aspects. The meaningful part was the fact that teachers acknowledged how much verbalization, argumentation and dialogue with peers may be productive to promote the mathematical construction, as well as to produce conscious and meaningful learning in pupils.

NOTES

1. The European PDTR project, *Professional Development of Teacher-Researchers,* involved seven teams of mathematics teachers, apprentices in the craft of teaching-research, from: Hungary (Debrecen); Italy (Modena, Naples); Poland (Rzeszów, Siedlce); Spain (Barcelona) and Portugal (Lisbon).

2. G. Navarra is a teacher-researcher sharply involved in teachers education in early algebra. He is responsible with N.A. Malara of the teaching experiments and production of the ArAl teaching materials. In PDTR Project he has been mentor of the Italian team (leader N.A. Malara).

3. Due to space constraints, worksheets A, B, C can be found in www.aralweb.unimore.it.

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A COMPARISON OF TEACHERS' BELIEFS AND PRACTICES IN MATHEMATICS TEACHING AT LOWER SECONDARY AND UPPER SECONDARY SCHOOL

Hans Kristian Nilsen

Sør-Trøndelag University College, Norway

The focus of this paper is a comparison of lower and upper secondary teachers' beliefs regarding teaching mathematics in general. This is linked to a research project concerning the transition from lower secondary to upper secondary school and the learning and teaching of functions. In Norway the transition from the 10th to the 11th grade always involves these separate institutions. The results presented here are based upon interviews with teachers at both lower and upper secondary level of schooling and some interesting differences in their views of mathematics teaching are uncovered. Hopefully, these preliminary findings could give rise to meaningful discussions related to how a qualitative approach to the transition issue might be carried out.

Keywords: mathematics teaching, transition, lower secondary, upper secondary

INTRODUCTION

In Norway, the transition between different phases of schooling, particularly in relation to the learning and teaching of mathematics, is an area where little research has been done and the major part of the international research in this field concerns the transition from upper secondary school to university/university college (often denoted as the secondary-tertiary transition) (Gueudet, 2008; Guzmán et al., 1998). My own experiences as a student and a teacher, at both lower and upper secondary school levels have led me to believe that the traditions and beliefs in these institutions differ in ways which in turn might affect students' learning. As a PhD student (in my second year), I have chosen this transition as the focus of my research. It is important to note that in Norway, upper secondary schooling is divided in two main programmes: the vocational programmes, which are orientated towards practical professions and the general study program, which aims to prepare students for higher education. The curriculum is different in these programmes and is considered to be more 'theoretical' at the general study program. This is also the case for mathematics as a subject. Both of these programmes are included in this research. Further, I have chosen to focus on *functions* as this is an area highly relevant to both levels of schooling, and personally I find the development of students' conceptual understanding of functions to be an interesting research area. It is also possible to expand this area of research, for example by taking the universities/university colleges into the consideration, as the learning and teaching of functions is an important issue in several of these study programmes. However, in this paper I will focus on mathematics teaching in general (not only teaching related to functions).

RESEARCH QUESTIONS

I pose the following research questions, relevant for this paper:

What are the differences in the didactical approaches related to mathematics teaching, in lower secondary versus upper secondary school? How are such possible differences perceived by the teachers at both these levels of schooling?

To approach the first question, I compare the lower and upper secondary teachers' views and practices concerning the teaching of mathematics in general. Concerning the second question, I present the lower secondary teachers' statements related to how they think upper secondary teachers perceive the teaching of mathematics in lower secondary school. These statements are then being compared to the actual statements of the teachers at upper secondary school.

THEORETICAL BACKGROUND

An established and well-documented argument within educational research is that teachers' beliefs are one of the best indicators of the decisions teachers make throughout their career (Pajares, 1992). The link between beliefs and actions, therefore, motivates for many of my interview questions. As indicated by Mosvold (2006, p. 37) research shows that many of these "beliefs are shaped from the experiences of those who taught them". What often seems to be conflicting interests, or even paradoxes, experienced in teachers everyday practice, is described by Mellin-Olsen (1987; 1991) as characteristics of a 'double bind'. According to Mellin-Olsen, double bind can be recognized at many levels. One aspect of this can be that the individual is tightly connected with his environment, and consequently left with few individual choices. Often this relates to the 'didactical contract' which in its simplest form means that "the teacher is obliged to teach and the pupil is obliged to learn" (Mellin-Olsen, 1987, p. 185). Hidden (or in some countries even explicit) competition between teachers at the same time as they need to cooperate can be an example of a double bind. The confidence the teachers often express that they feel in traditional teaching, for example the early introduction of standard algorithms without giving their students 'permission' to use alternative methods, can be another example. Such 'permission' could, from the teachers' point of view, imply a break in the didactical contract. In turns this could lead some teachers into what they consider as 'safe' and effective curriculum-oriented teaching, preparing students for an oral or written exam. According to Mellin-Olsen (1987, p. 150), a double bind "is due to the handling of *metaknowledge* about the control caused by the taxonomies." Based on information found in some of my interviews, I have reasons to believe that at least some of the teachers on different levels experience what could be described as aspects of double binds. Some, especially recently educated teachers, state that their "ideals of teaching" often have to be set aside because of their obligations to the curriculum and the upcoming exam.

As my observations in the classroom concern the teaching of functions I find it relevant to include the Leinhardt et al. (1990) quote: “There is no proven optimal *entry* to functions and graphs” (p. 6). It is therefore, in my view, important to be aware of the multitude of different didactical approaches and to be conscious about the various conclusions.

METHODOLOGY

Five different classes in five different lower secondary schools participated in this research. Two of these schools are private schools while the other three are public. The private schools were included in an attempt to seek some diversity in the sample, while the public schools were somewhat randomly selected, with the only criteria being that they, due to practical reasons, were located within a ‘reasonable’ distance from my working place. As the Norwegian school system is quite homogenous I believe that these schools are representative to their area. The headmasters were contacted via telephone and their school was invited to participate. The number of students willing to participate from each class varied from three to ten. In total 33 students participated and I am currently conducting follow-up research on ten of these as they have now entered upper secondary school. I have chosen the follow-up students on the basis of three criterions: equal gender distribution, students at both vocational and general study programmes, and variations of ‘skills’ (on the basis of their marks). The purpose is to gain a rich material with some diversity. My data collection at lower secondary school mainly consisted of five “phases”: Observations of the teacher teaching, recorded conversations with the students engaging in mathematics in the classroom, interviews with the students, collection of students’ handwritten material and an interview with their teacher. This provides me with a diverse and rich data material which allows me to study mathematics education from various perspectives. The data collection at upper secondary school is done in a similar way, and I consider the fifth phase (teacher interviews) to be most valuable for this paper, as this relates to both teachers beliefs and practices. My use of research instruments did vary somewhat from school to school, primarily due to the fact that some teachers imposed restrictions for example on my use of a video camera. By the use of semi-structured interviews I aim to seek information mainly about teachers’ beliefs. However, I also try to get a broader picture of their teaching practice, by asking them to estimate the use of different teaching methods. They were interviewed for about 45 minutes, and in addition to their teaching practice they were asked about their views on ‘good teaching’ in general. They were also asked to provide some personal background information. I have aimed to design the interview questions in accordance with Kvale (1997, p. 77), suggesting that “The questions should be easy to understand, short and free for academic terminology” [1].

It was also important for me to formulate questions that would make it possible to compare teachers’ beliefs and ideas in lower and upper secondary education. These interviews were all recorded with a Dictaphone.

EXAMPLES AND ANALYSIS

Teachers at lower secondary school

I will start this section by presenting excerpts from teachers own statements regarding what they consider as good teaching in mathematics. These first three statements are excerpts from the interviews with the teachers at the 10th grade at lower secondary schools.

In your opinion, what characterises good teaching in mathematics? [2]

Jon: Good teaching…eh…variation, organised towards the individual student…eh…, adjusted according to different teaching styles, and that you go through the given exercises with this in mind.

Interviewer: Could you please go into some details about how you organise teaching towards the individual student in your practice?

Jon: Yes, this can be done by different tools, we might use the blackboard as a medium, and we might use the computer as a medium. We can do some practical exercises, where we work in a physical way, or we can make some problem solving exercises. We can do this interdisciplinary along with other subjects.

……

Sue: Good teaching in mathematics…eh…ideally, good teaching in mathematics, the start of a lesson…eh…it should be some repetition from the last time, in terms of "what did you learn?" Eh…maybe about five minutes, "what did you learn the last time?" Then a period in which you go through new content on the blackboard. And maybe a longer period, where the students can do some exercises.

……

Ann: In general, I think it is important that the individual student is making progress from his or her own starting point, within the subject that we are dealing with. Of course this has to be done in accordance with the curriculum, and so forth. But you have to achieve this. That is what I think.

Interviewer: Do you have any concrete ideas related to how this might be carried out?

Ann: Well, this has to do with differentiation. You know…eh…it is a very big gap, and you have to motivate students to make progress from where they stand, actually. But this is difficult to achieve. This can be done by giving different levels in the tasks given at the students' working plans. We also try to differentiate in the tasks given in the folder. [3]

We notice that their answers are not quite univocal, and the three teachers' views on "good teaching in mathematics" seem to differ in some ways. Jon seems to give an account of some general aspects of good teaching, and Sue seems to relate the question to a concrete situation, like a recipe of a good lesson. Common for both Jon and Ann is the importance of differentiation. The tables below show the teachers' suggestions of how frequent different teaching methods are used. The time measured in minutes estimates the time used in each lesson. The three schools all have 4 lessons a week, each 45 minutes. These numbers are only based upon what they have done related to the class participating in this research.

Teacher	**Lectures-blackboard**	**ICT**	**Homework Discussions**	**Individual Exercises**	**Pair/group-work**	**Problem solving**
Jon	1-3 lessons a week 15-20 min	1-3 lessons a week 30 min	1 lesson a week 10 min	Almost each lesson 30 min	2 lessons a month Whole lessons	Sometimes (hard to establish)
Sue	Each lesson 30 min	6 lessons (this year) Whole lessons	Each lesson 5-10 min	2-3 lessons a week 15 min	Not organised[4]	Never
Ann	2 lessons a week 30 min	Sometimes (hard to establish)	2 lessons a month 5-10 min	2 lessons 15 min + 2 whole lessons	2 lessons a month Whole lessons	A few Times (hard to establish)

Table 1: The frequencies of different teaching methods (assumed used most frequently)

Teacher	**Interdisciplinary Projects**	**Excursions**	**Outdoor Activities**
Jon	2-3 weeks a year	Never	Never
Sue	2 weeks a year (together with Art and Design)	Never	Never
Ann	Never	Never	1 day a year

Table 2: The frequencies of different teaching methods (assumed used less frequently)

The tables show for example that Sue states that she never uses 'problem solving' as a method of teaching and seldom uses ICT. She also seems to use the blackboard and discussions related to homework more frequent than the others. It is also interesting to notice Jon's relatively frequent use of ICT. The pre-assumed more rarely used methods, as interdisciplinary projects, outdoor activities, and excursions appear with quite similar frequencies.

The idea behind the next question is to grasp one aspect of the teachers' beliefs concerning upper secondary school.

How do you think that the teachers at upper secondary school conceive of the teaching in mathematics at lower secondary school?

Jon: I do not really know – maybe they shake their heads and think "what in the world have we done at lower secondary school?" But I also think they have completely different pre-conditions for their activity.

Interviewer: In what way?

Jon: Well, you do not have “the herd” in an ordinary class at upper secondary school – they come there because they have applied for going there – but we have the average of the whole Norwegian population in one class!

……

Sue: I am very convinced that the teachers at upper secondary school feel frustrated about the students at lower secondary school and their total lack of knowledge.

Interviewer: Ok…?

Sue: Well, maybe, and here they come at upper secondary school, and they can not add two fractions!

Interviewer: Mm…?

Sue: Here they come at upper secondary school and do not manage this! They have not learned anything…

……

Ann: I do not really have any strong opinions here, but my impression was, when I worked there myself, that the teachers there were very different. I also think that there was a big difference among the students, related to which lower secondary school they attended before they started.

Although this question could be regarded being a bit speculative, since most of the answers are hypothetical, I was surprised by the level of consensus. As we can see, both Jon and Sue indicated some negative assumptions, while Ann was more neutral. Both of them seemed to share the worries that the teachers at upper secondary school, to some extent, are frustrated by the limitations of their students’ starting point. The negative assumptions were also shared by the two other teachers, not presented here.

Teachers at upper secondary school

I will now consider four of the teachers at upper secondary school answering the corresponding questions. The first two excerpts are from teachers at the general study programme.

In your opinion, what characterises good teaching in mathematics?

Tony: Well, maybe the most important aspect in such a subject dealing with systematics, is clarity. Clarity in the presentations and that one manages to simplify complicated issues. The teacher’s job, in a way, is to simplify the textbook for the students, because we observe that this is a subject that is very hard to study on your own and you are very dependent on going through the content.

……

Mary: It must be teaching…eh…in such a way that the students understand what they are doing. Eh…and that they are motivated to continue to work with mathematics

Interviewer: Do you have any thoughts of how this can be done?

Mary: I think on this level, if they are mastering the mathematical content, this in itself is good enough for motivation. Helping them to master the exercises is very important, because most of the students like mathematics.

In this next excerpt, the same question is asked to a teacher at a vocational programme.

Lisa: I have some years with experience from the lower secondary school, and I think that working with concretes and go outdoors and do things is a good way of working with mathematics. Good teaching will be to organize such activities in a good way. Now at upper secondary school I almost only teach by giving lectures at the blackboard, in and old-fashioned way.

Interviewer: What is the reason for that, you think?

Lisa: It is another culture here. They are all working, determined to get the students through the textbook in an efficient way.

Interviewer: Why do you think it is difficult to teach the way you would like?

Lisa: Well, I am new here and I do not want to go against my colleagues.

It is interesting to notice Lisa's reflections on her own situation, probably much due to her background from lower secondary school. The two other teachers at the general study programmes do not express the same kind of worries. They both seem to share the value of good explanations and the importance of doing exercises from the textbook. Jon stresses the importance of clarity and Mary the importance of mastering the textbook content.

In the same manner as for the teachers at lower secondary, the teachers at upper secondary school were asked about their use of different teaching methods. The results are presented in the tables below. Tony and Mary's classes have five lessons a week and Lisa's has three.

Teacher	**Lectures-Blackboard**	**ICT**	**Homework Discussions**	**Individual Exercises**	**Pair/group-work**	**Problem solving**
Tony	Each lesson 15 min	2 lessons a month 30 min	1 lesson a week 10 min	Each lesson 30 min	Not organized	Never
Mary	Each lesson 15 min	5-10 lessons this year Whole lessons	1 lesson a week 10 min	Each lesson 30 min	Not organized	Never
Lisa	Each lesson 20-25 min	Never	A few times (hard to establish)	Each lesson 20-25 min	Not organized	Never

Table 3: The frequencies of different teaching methods (assumed used most frequently)

Teacher	Interdisciplinary Projects	Excursions	Outdoor activities
Tony	Never	Never	Never
Mary	Never	Never	Never
Lisa	Sometimes at the mechanical working rooms	Never	Never

Table 4: The frequencies of different teaching methods (assumed used less frequently)

As illustrated the use of methods assumed less frequently, are rarely/never used. The more common methods appear in quite similar frequencies, and the 'typical' lesson seems to be divided in two, with the first part consisting of a lecture at the blackboard and the second part consisting of individual exercises from the textbook. In general it seems like there are only small variations between these teachers and their use of methods.

The next question was posed with the intention to compare the upper secondary teachers' statements with the lower secondary teachers assumptions.

Which thoughts do you have concerning mathematics teaching at lower secondary school?

Tony: It is always easy to blame the teacher responsible for the class, the previous year, but they have whole classes with enormous gaps between the students. Probably much time is used just to keep them quiet. So the students coming to us may not have got the follow-up which they should, from the lower secondary school. They take to easy on it [the students] and their efforts are not as they should have been.

……

Mary: I think teaching at lower secondary school is very dependent on the personality of the teacher…eh…and this is of course also the case at upper secondary school. But in general I will assume that it is quite similar. Maybe it is more group work at lower secondary school.

……

Lisa: I think the students get to work on their own to much, and they do not take that responsibility, they are not keeping up and they end up here. That being said I think the teachers vary their methods more, as I said before. I also think that much of the differences are due to the teachers' background. At lower secondary school they are educated at general teacher education institutions, but here they are educated at universities.

By the exception of Mary being more neutral to the question, the other two seem to express some kind of worries. Common for these are the suspicions that the students do not get the required follow-up from their teachers. It is also interesting to notice how Lisa is pointing to the teachers' background as a possible reason for different ways of teaching.

The comparison of these interview excerpts and the tables from the lower and upper secondary level of schooling, gives rise to some reflections. While at least two of the teachers at lower secondary emphasized differentiation and the importance of reaching the individual student, the teachers at upper secondary school tend to emphasize the importance of good explanations, techniques and individual task solving, mainly from the textbook. The exception here is Lisa, who expresses some frustration of being 'forced' into a teaching tradition which seems to go against her own principles. The tendencies expressed by these teachers are also to some extent reflected in the tables, and the overview of the teachers' use of methods in the classroom.

The lower secondary teachers' beliefs concerning the upper secondary teachers' perception of teaching in the lower secondary level showed some consensus. These were at most negative assumptions, and to some extent they were in accordance with what the teachers in upper secondary actually stated. Although their suspicion of the insufficient follow-up of the student was not actually stated among the lower secondary teachers, they shared the worries concerning their students' 'insufficient' mathematical knowledge. Despite these remarks, it is important to notice that the statements within the group of teachers at both lower and upper secondary school are far from univocal. This is also the situation if we study the interviews in a more holistic manner.

CONCLUSION AND FURTHER DISCUSSION

So what can we infer from the examples above? The teachers at lower secondary school related some of the challenges in teaching to their students' abilities, and the diversity within their group of students. This was also mentioned by some of the teachers at upper secondary school. I think that common to these, and similar statements, are the relation to what Mellin-Olsen (1987; 1991) denotes as a double bind. This is because the concerns of most of these teachers relate to what in their view are conceived of as conflicting issues. The obligations of getting through a given curriculum, and at the same time being able to teach in a fruitful way, for some, seemed to cause a dilemma. Apparently the teachers at upper secondary school feel that the most 'safe' way of coping with the demands of the curriculum is in terms of traditional teaching methods. One reason might be that there usually is a higher probability for the students in upper secondary school having to take an exam. Another reason, also indicated among both group of teachers, could be that there exists a view that students at upper secondary level have made a more specific choice related to their career, and the mathematics is in a way a part of that choice. Therefore it becomes important for the teachers that nothing is 'omitted', and hence few 'risks' are taken. Being aware that these are only speculations, I still think these could be important hypotheses to investigate further upon. In Lisa's case, being loyal to her colleagues and at the same time manage to teach in a way that she considered as appropriate obviously constituted a dilemma.

As Lisa further mentioned, cultural issues such as the fact that teachers at upper secondary level tend to have a university background while teachers at lower secondary tends to come from general teachers education should also be considered, in an attempt to understand possible differences in their beliefs and practices.

NOTES

1. Translated from Norwegian by the author.

2. All the transcriptions are translated form Norwegian, with an attempt to preserve the teachers' original statements as authentic as possible.

3. This teacher regularly gave her students exercises which they were supposed to put into a folder. The folder was evaluated by the teacher. In total the folder counted as one third of their final marks in mathematics.

4. This means that the students were allowed to cooperate at their individual tasks, but no group work was organized by the teacher.

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MATHEMATICAL TASKS AND LEARNER DISPOSITIONS: A COMPARATIVE PERSPECTIVE

Birgit Pepin

University of Manchester, UK

Mathematical tasks in textbooks, their 'mediation' by teachers and the classroom environments in England, France and Germany are the focus of this study. The author claims that the different mathematical tasks in textbooks (in connection with their mediation by teachers) influence, to a large extent, the differences in activities and practices that are going on in mathematics classrooms, and that these in turn mediate different kinds of learner dispositions. The classroom culture, with its differing dimensions, is likely to set the scene for pupil development as 'learners of mathematics'. The web of these connections is studied in this report.

Keywords: Mathematical tasks; learner identity; comparative education; socio-cultural; culturally figured worlds.

INTRODUCTION

Mathematical tasks in textbooks, learning opportunities and pupil dispositions

Students spend much of their time in classrooms working on mathematical tasks chosen from textbooks. In recognition of the central importance of textbooks, the framework of the Third International Mathematics and Science Study (TIMSS) included large-scale cross-national analyses of mathematics curricula and textbooks as part of its examination of mathematics education and attainment in almost 50 nations (Valverde et al, 2002). They claim that

> Textbooks are the print resources most consistently used by teachers and their students in the course of their common work (ibid., p. viii).

Moreover, they comment on different learning opportunities being offered to students in different mathematics classrooms.

> Clearly, one issue of pervading importance to the nations that participated in TIMSS was the quality of educational opportunities afforded to students to learn mathematics and science - and the instruments that optimise such quality (ibid, p. viii).

Textbooks are a major source of provision of these educational opportunities. Romberg and Carpenter (1986), for example, noted that the textbook was consistently seen (in the US) as "the authority on knowledge and the guide to learning". (p. 25)

It appears that tasks in textbooks influence, to a large extent, how students experience mathematics. Textbooks provide children with opportunities to learn, and learn those things which are regarded as important by their government. Teachers mediate textbooks by choosing and affecting tasks, and in that sense student learning, by devising and structuring student work from textbooks.

It can also be argued that tasks, most likely chosen from textbooks, influence to a large extent how students think about mathematics and come to understand its meaning. Indeed, Henningsen and Stein (1997) assert that

> the tasks in which students engage provide the contexts in which they learn to think about subject matter, and different tasks may place different cognitive demands on students …. Thus, the nature of tasks can potentially influence and structure the way students think and can serve to limit or to broaden their views of their subject matter with which they are engaged. Students develop their sense of what it means to "do mathematics" from their actual experiences with mathematics, and their primary opportunities to experience mathematics as a discipline are seated in the classroom activities in which they engage … (p. 525)

Hiebert et al (1997) similarly argue that students

> also form their perceptions of what a subject is all about from the kinds of tasks they do. … Students' perceptions of the subject are built from the kind of work they do, not from the exhortations of the teacher. … The tasks are critical. (p. 17-18)

Moreover, they assert that

> the nature of the tasks that students complete define for them the nature of the subject and contribute significantly to the nature of classroom life …. The kinds of tasks that students are asked to perform set the foundation for the system of instruction that is created. Different kinds of tasks lead to different systems of instruction. (p. 7)

It appears that mathematical tasks are central to student learning, their developing perceptions of what the mathematics is and what doing mathematics entails.

CLASSROOM ENVIRONMENT, MATHEMATICAL TASKS AND LEARNER IDENTITY

According to Lave and Wenger (1991), tools (and artefacts) constitute the resources, and students learn by participating in social practice using the tools. This also relates to 'conceptual tools', most likely reflected and used in tasks. If students use a conceptual tool, as perhaps advised by a worked example, or teacher's exhortations, or an exercise, and if they use the tool actively, they are likely to build an increasingly rich understanding of the 'usefulness' of this tool in their mathematical world, and of the tool itself. Learning how to use a conceptual tool involves much more than the set of explicit rules it may describe. The occasions and conditions for the use arise out of the contexts of tasks and activities that students are expected to do, and they are framed by the ways the members of the community (e.g. textbook authors) see the world of mathematics.

Different practices in mathematics classrooms are likely to influence the development of different learner identities. For example, Boaler et al (2000) investigated the practices of secondary school teaching from a student's perspective "in order to understand how they construct a sense of themselves in relation to mathematics" (p.

4). They argue that in the US and UK classrooms they studied there exists an "unambiguous vision of what it means to be successful at mathematics, and of what it means to be a mathematician" (p. 8).

According to Henningsen and Stein (1997) what it means to 'do mathematics', or to 'behave mathematically', for students, is largely dependent on the nature of the tasks and activities students are engaged in, and these in turn 'colour' their perceptions of the subject. Thus, doing mathematics, and developing certain perceptions of the subject, is likely to 'produce' particular 'mathematical dispositions' or a 'mathematical point of view' (Schoenfeld, 1988), as well as acquiring mathematical knowledge.

As Boaler (2000) emphasises, students do not just learn methods, or how to carry out a task or to apply algorithms, in mathematics classrooms, but they learn 'to be mathematics learners'. Different classroom cultures, different constraints and affordances, provided by different settings and opportunities for engagement in mathematical practices, are likely to influence their perceptions of what it means to learn and do mathematics. Learning how to engage successfully with the mathematics means learning how to and identifying with the norms of the classroom community. Particular tasks in textbooks may reinforce practices initiated and propagate by the teacher, or vice versa.

Furthermore, Boaler and Greeno (2000) use the notion of identity formation in "figured worlds" (Holland et al., 1998) to explore pupil learning and the influence of pedagogies on their learning. Figured worlds are perceived here as places "where agents come together to construct joint meanings and activities" (p. 173). Mathematics classrooms can be regarded as such figured worlds, because students and teachers work together in these environments and construct meanings of the mathematics, and within that of themselves as learners of the mathematics. Holland et al (1998) is cited to draw attention to actors, and to interpretations by actors when asserting that figured words are socially and culturally constructed realms "of interpretation in which particular characters and actors are recognised, significance is assigned to certain acts, and particular outcomes are valued over others" (p. 52).

This is particularly interesting in terms of comparing "figured worlds" in different countries' classrooms. Questions such as the following may arise: What is similar, or different, in mathematics classrooms in England, France and Germany? What are the rituals of practice? What kinds of tasks are pupils expected to perform, what kinds of activities do pupils, and teachers, engage in? What kinds of interpretations are made, what kinds of acts are respected, what kinds of outcomes are valued?

RESEARCH DESIGN

In a previous study (e.g. Pepin, 1999; Pepin, 2002) the author developed an understanding of practices in lower secondary mathematics classrooms in England, France and Germany, concluding that national educational traditions were a large determinant and influence on what was going on in these classrooms. In a more recent study, Pepin and Haggarty have investigated mathematics textbooks in the three countries, and connected to that, the ways they were used, by teachers (e.g. Pepin & Haggarty, 2003). This not only supported some of the earlier findings, but also suggested that the use of curricular materials (such as textbooks), together with the selection of (mathematical) tasks, impacts to a large extent on the mathematical ‘diet’ offered to students.

The author thus re-analysed the amount of data collected over the years, in particular mathematical tasks in selected textbooks, in terms of potential pupil disposition and identity formation. Particularly relevant, and useful, was the work of Boaler and Greeno (2000) and the notion of pupil identity formation in ‘figured worlds’ (Holland et al, 1998). In terms of analysis a procedure involving the analysis of themes similar to that described by Burgess (1984) was adopted, which had already proved useful in other cross-national studies (e.g. Broadfoot & Osborn, 1993). However, due to the additional cross-cultural dimension, it was important to address the potential difficulties with cross-national research, in particular issues related to conceptual equivalence, equivalence of measurement, and linguistic equivalence (Warwick & Osherson, 1973; Pepin, 2002). In order to locate and understand teacher pedagogic practices and the classroom cultures in England, France and Germany, it was useful to draw on knowledge gained from earlier research (see above) which highlighted the complex nature of practices in mathematics classroom environments, and the value of comparing.

The main questions asked was: How may mathematical tasks in textbooks, teacher practices and classroom environment influence pupil identity construction as learners of mathematics in England, France and Germany?

DISCUSSION AND CONCLUSIONS

To connect tasks in textbooks to students’ developing identities as learners of mathematics is not a common link made. Textbooks are often frowned upon, and teachers do not wish to be seen to teach ‘according to the book’. However, for better or for worse, and as research indicates, textbooks are the main resources used in mathematics classroom all over the world (Valverde et al, 2002).

This is also true for England, France and Germany. Moreover, teachers choose tasks and exercises from those books, for pupils to complete, students learn from the kinds of work they do during class, and the tasks they are asked to carry out shape to a large extent the kind of work they do. Pupils learn the conceptual tools provided by the tasks in textbooks, by ‘legitimate peripheral participation’ (Lave & Wenger, 1991) in the practice of school mathematics. However, there are particular school mathematics practices in different countries, and within those countries differing practices in different school ‘streams’ and ‘sets’ that are supported by different textbooks for those groupings. Moreover, the types of tasks, the mathematical connectivity between tasks, the conceptual tools suggested for solutions, amongst others, reflect and support a particular school mathematical culture. Pupils are socialised into these cultures, and as members of the cultures, develop dispositions and form identities as learners of mathematics. However, it would be difficult to claim that in each country there exists a homogeneous mathematical culture supported by textbooks. Instead, the developing 'identities' here are seen as those potentially emerging from the analysis of mathematical tasks in textbooks, and the mediation of those tasks by teachers, thus the tools used by teachers in their classroom practice.

What would pupils learn from the tasks provided by the textbooks analysed, and what kinds of work/activities would they do related to the tasks? In order to engage in the mathematics, pupils must find the task intriguing, something they would like to resolve. This assumes that students relate to the task in the sense that the contexts and situations make it real for them. On the basis of results from this study it is argued that in all three countries pupils are likely to be asked to do exercises and to complete tasks (from textbooks) that are presented in context- context embeddedness seems to be important- and these contexts are similar. Whether the contexts are relevant to pupils, whether they connect to their life experiences is beyond the scope of this study. What is different in the three countries is how the mathematics is linked to the contexts and what pupils are asked to do in those tasks. Whereas in German textbooks it appeared that context and mathematical concepts are connected in the tasks analysed, and links are forged between them, in the English textbook chapter pupils are asked to do contextualised tasks where context are chosen seemingly for their own sake, and with little logical progression or connection to the underpinning mathematical ideas. Most exercises could be done without knowing about concepts of the topic area. To what extent students may deduce concepts, by simply doing the exercises, is not clear. Interestingly, French textbook exercises studied appeared to use contexts as a pretence for introducing the mathematics, a Trojan horse to lead students to the 'essential' section, the 'cours', the mathematical concepts.

To ask what students would learn from these tasks also needs a more nuanced perspective. By addressing the mathematics at the conceptual level (e.g. 'oppositeness' in negative Numbers) one could argue that in France and Germany students would get more insights into the conceptual nature of mathematics, and perhaps its structure, than through English textbook tasks. A second type of 'residue' (Hiebert et al, 1997), it can be argued, may be given through the strategies or methods, for solving problems, provided. French textbooks are explicitly addressing this in a separate section ('apprendre a resoudre') and exercises are organised accordingly. Putting the three country's textbooks on a continuum, it is argued that English textbooks leave it to pupils, or their teachers, to devise or identify strategies to solve problems, and this is likely to be with common sense, whereas in particular French textbooks are explicit about how to solve particular problems.

The message that students may therefore get is that (1) mathematics is simply there to be done (e.g. English *KM 7*[2]), and that contexts and concepts do not necessarily 'talk to each other'; that (2) it is not the contexts that matter (e.g. French *Cinq sur Cinq*), but the underlying mathematical concepts, and that there are strategies to ‘reduce’ the contextualised problems to ‘simple’ mathematical tasks; or that (3) concepts and contexts may be connected, and that the formally structured mathematics, including its strategies for solving problems, may be useful in real life problems (e.g. German Grammar school *LS7*).

In terms of teacher mediation of tasks it appears that one of the most important responsibilities for a teacher is to set appropriate tasks. Teachers in all three countries chose those tasks predominantly from textbooks. What was different were to what extent teachers initiated pupils into those tasks and the ways they chose to introduce the mathematical ideas necessary to do the exercises selected from textbooks. The picture that was painted was that whereas in one country (Germany) teachers introduced the mathematical notion in whole-class discussions and chose particular tasks to ‘consolidate’ the concept, in another (England) teachers gave relatively brief introductions or rules, and wanted a large number of straight forward exercises to practice. In another (France) teachers were provided with activity type tasks, from textbooks, to initiate pupils into the concept, and after explaining the ‘essentials’ (*cours*) teachers wanted differentiated exercises to attend to the perceived heterogeneous class.

To what extent teachers selected appropriate and related tasks, so that pupils could see the same mathematical idea from a different angle, or to chain tasks in such a way that opportunities are created to gradually increase pupil understanding was not clear. The literature (e.g. Hiebert et al, 1997) claims that tasks that are related in such away increase the coherence of students’ mathematical experiences. Coherence here means that students would perceive the sequence of activities and exercises to fit together and make sense. This goes beyond the scope of this study, but it could be argued that, from the analysis of textbook tasks in selected English textbooks, and looking at the sequence of tasks in selected chapters, students are likely to be asked to do a series of individual, nearly random, tasks that are relatively disconnected and appear not to be leading anywhere. French textbooks provide exercises, graded with respect to the level of perceived difficulty and for particular areas within the topic.

In addition, results from a previous study (e.g. Pepin, 1999) show that French, and in particular German Gymnasium teachers chose exercises, that were perceived to exemplify the idea well and to be ‘difficult’, for solving in class, and sometimes in whole-class discussion, whereas ‘easy’ routine exercises were assigned for homework. English teachers said that most of their students needed ‘much of the same’ to practice.

In terms of classroom environment and culture teachers have a great influence, and this was true for England, France and Germany. Within the limits of the system, whether students were taught in mixed classes (*collège* France), whether they were setted (England) or streamed (Germany), teachers had some freedom to select tasks that could potentially guide their instruction and they could mediate those tasks in ways they thought best. To what extent teachers created cognitive conflict, in order to challenge pupils' ideas, is beyond the scope of this investigation, but in terms of tasks in textbooks this may potentially be provided by selected cognitive activities (*activitées*) in French textbooks (Pepin & Haggarty, 2003). Moreover, allowing mistakes, perhaps even inviting them for pupil learning, or asking open questions would be another way of influencing the mathematics classroom culture. Looking at tasks in textbooks, there were no open questions in the English textbook chapter analysed, and hardly any in the French and German textbooks. Teacher pedagogic practices, however, may be interpreted as going some way towards that goal: all teachers, but particularly German teachers, used mistakes in homework exercises as a site for deepening pupil understanding (Pepin, 1998). These were discussed in detail and at times over an extended period of time.

In summary, it can be argued then, albeit from this limited research, that the dispositions that pupils are likely to develop as learners of mathematics, are linked to the textbook tasks provided by teachers, the practices that pupils are engaged in when doing those tasks, and the environment they work in and experience in class during engagement- and these are different in the three countries. Whereas in all three countries one could argue that pupils are 'conditioned' to become 'conformists'- hardly any negotiation about the mathematics and its learning is provided-, in England the mathematical diet in textbooks may also offer learners to become 'common sensers'. Can one say that in France the 'instrumentalist' identity may be favoured, and in Germany the 'connector', in addition to the 'conformist'? If this link was seen to be strong, one would need to consider to what extent pupils are 'trapped' in these identities, for better or worse, according to what they are offered by their teachers. What kinds of opportunities would need to be provided for change to be possible?

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ELITE MATHEMATICS STUDENTS IN FINLAND AND WASHINGTON: ACCESS, COLLABORATION, AND HIERARCHY

Jennifer von Reis Saari

Faculty of Education, University of Cambridge

This paper draws from a small scale study of elite mathematics students' beliefs, motivations and access in Finland and Washington State. In particular, students' experiences with extracurricular mathematics, collaborative learning, and their elite peer groups are examined.

INTRODUCTION: FINLAND, WASHINGTON AND ELITE MATHEMATICS

Large scale international comparisons exert seemingly unavoidable influence on educational systems. Such numerical comparisons of performance are often read as competitions; the results become lists of winners and losers, focusing attention on the high-scoring educational systems. However, even if large scale international comparisons can tell us where to look, they cannot tell us what to look for.

Within Mathematics and Science education, Finland has recently drawn such attention for its success in the PISA studies. One of the most striking features of the Finnish educational system is the lack of tracking, or separating students according to perceived ability, until the end of lower secondary (*yläaste*), at roughly age fifteen or sixteen. This has drawn the attention of de-tracking reformers (see e.g. Oakes, 2008).

While the efficacy of tracking has been questioned (e.g. Rothenberg, McDermott & Martin, 1998 or Boaler, 2002), de-tracking may have negative consequences for high -achieving students (Terwell, 2005, p. 663). In this paper, I focus on those students who would be expected to benefit most from tracking: students enrolled in the highest possible track available, whom I call *elite mathematics students*. In Finland, these students have enrolled in an academic upper secondary, and then in Long Mathematics (*pitkä matematiikka*) instead of Short (*lyhyt matematiikka*). In Washington, where tracking may begin as early as third grade (age 8 or 9) these students are taking courses classified as Honours, Advanced Placement (AP), or International Baccalaureate (IB). All would reach at least Calculus by graduation.

Participation in elite tracks has been shown to have lasting negative effects on students' mathematical self-concepts (e.g. Marsh, Trautwein, & Lüdtke, 2007), best known as big-fish-little-pond effect. Structure, then, seems to effect the development of students' beliefs and identities as mathematics learners, influencing students' academic decisions. It seems worthwhile, then, to ask how elite mathematics students' identities and beliefs, as well as opportunities to learn within a partially de-tracked system, Finland, compare to those of students in a heavily tracked system, Washington State [1]. Osborn (2004, p. 265) cautions against the "...growing tendency to `borrow' educational policies and practices from one national setting where they appear to be effective and to attempt to transplant these into another, with little regard for the potential significance of the cultural context..." The object of this study, however, is not to set policy, but to illuminate, through the juxtaposition of two systems, features of each.

RESEARCH DESIGN

The research described in this paper was a small scale study designed to explore elite mathematics students' identities, beliefs, and access to learning in Finland and Washington State conducted with the help of Jasu Markkanen from the University of Turku. The study consisted of 13 student interviews conducted in Spring 2008 in Finland and Washington State. Markkanen conducted four interviews (at Päijänne and Keitele). While many themes emerged from these interviews, in this paper, I will briefly focus on three specific questions:

What extracurricular mathematic experiences have these students had, or had access to?

What are the students' experiences with and views on cooperative learning?

What are students' characterisations of their peer groups, which were cited by participants from both countries as a key benefit of elite mathematics tracks?

These are a combination of prefigured themes and themes that emerged during the interviews. While questionnaires already exist regarding students' beliefs and motivations, (Malmivuori & Pehkonen, 1996), and are being refined to function internationally (Diego-Manecón, Andrews & Op't Eynde, 2007), they are not focused on the particular population of elite mathematic students I wished to examine, hence the need for an exploratory study.

Semi-structured interviews were chosen to allow opportunity for participants to impact the research, while considering the need for some comparability across interviews. Students were intended to be interviewed in pairs, but sometimes were interviewed in groups of three; extra students who turned up for the interviews were not turned away. Paired interviewing was inspired by its use in other studies (Boaler 2008, Evens & Housartt, 2007). The interview schedule was piloted with two Finns and one Washingtonian, all who had studied mathematics at the tertiary level.

When analysing the data I have attempted to consider that '...there are clear dangers in saying that the interviews simply tell us more about the answers of the individual, as this ignores the presence of their interview partner.'(Evens & Houssart, 2007, p. 22). I see the students' words as public statements, at times inspired, supported, or edited by the presence of peers in the interview setting. I also acknowledge that the interviews may also have served as much in constructing or clarifying certain beliefs as in recording them.

The Selection of Cases and Participants

Eisenhardt (1989, p. 537) writes that while "...cases may be chosen randomly, random selection is neither necessary, nor even preferable." Here, I have chosen cases with an eye towards both comparability and capturing a diverse population. These highlighted characteristics of the schools make them more identifiable, and so to assure anonymity, Finnish and American English pseudonyms have are used for the cities as well as the schools and student-participants. The cities I shall call Jokimaa and Riverview are small metropolitan areas with a similar population (roughly 170 000 people), with higher than average immigration when compared with Finland or Washington at large, and containing at least one university.

From each community I chose one IB school with higher immigrant enrolment, and two schools considered strong in mathematics or mathematics related fields. A fourth school was added in Riverview as described later. In Finland, these schools were:

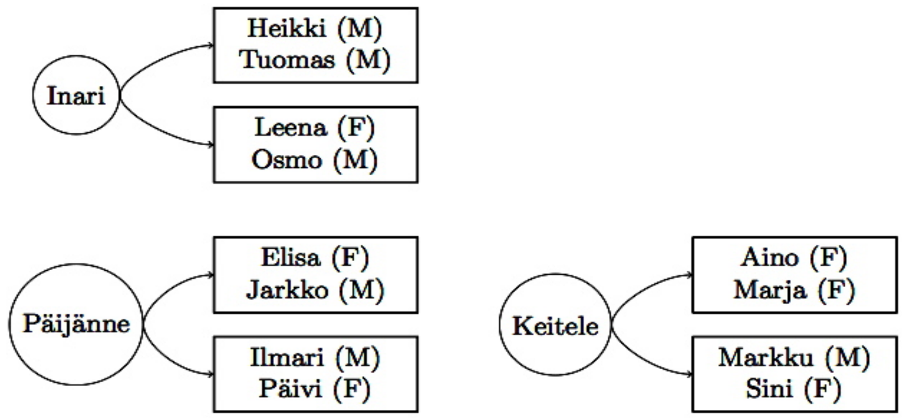

Figure 1: Interview Map for Jokimaa, Finland

Keitele Lukio, known for having a strong and extended mathematics programme

Inari Lukio, an IB programme in an area of high immigration for the Jokimaa area

Päijänne Lukio, offering a special IT line including university level courses

In Washington these schools were:

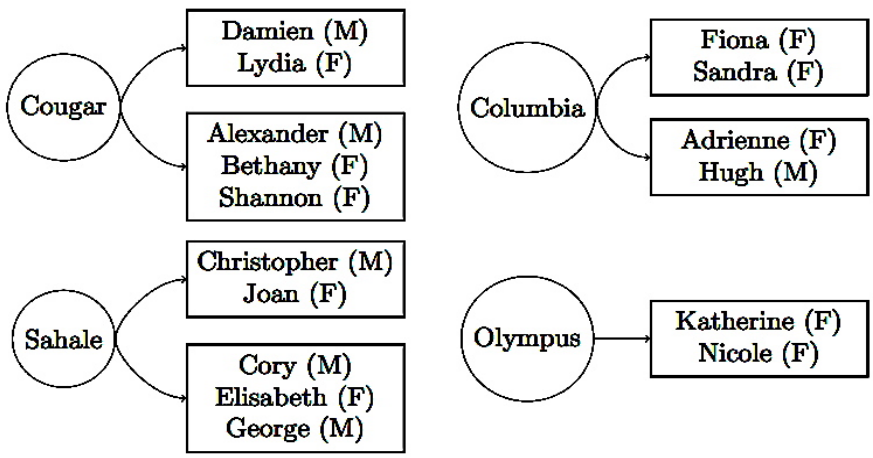

Figure 2: Interview Map for Riverview, Washington

Columbia High, known for strong performance in academic competitions and state exams and offering the most advanced AP mathematics course

Sahale High School, an IB programme with a higher minority enrolment rate

Cougar High is the most affluent high school in Riverview

Students from a fourth school approached me to be included in the study:

Olympus High has the lowest state tests scores, and is majority Latino/Hispanic.

RESULTS FROM THE INTERVIEWS

In this section I will discuss the development of three themes: extracurricular involvement, collaborative learning, and the conceptions of the elite mathematics peer group, first in Finland, then in Washington. The quotes below are selected to illustrate general themes (or exceptions) throughout the interviews.

Jokimaa, Finland: Extracurricular mathematics

Students interviewed from Jokimaa had no experience with extracurricular mathematics besides sitting for an optional national exam. Neither did they seem to be aware of any opportunities such as mathematics clubs. However, when explicitly asked, students did not seem to regret the lack of opportunity:

Saari(JS): Do you think you would have used the opportunity if there'd been some kind of extra-curricular mathematics?

Tuomas: Well, maybe not. [Laughing]

Heikki: [Laughing] To be honest no!

JS: And why, why not?

Heikki: Well, I, uh, value my other leisure activities more, perhaps.

Jokimaa, Finland: Collaborative Learning

Similarly, most Jokimaa students seemed to have little experience with collaborative learning, either formally or informally. For example Äinö said "...usually I've just done things by myself, and haven't cooperated with anyone."

While collaborative learning was described as mostly positive, when there was a mismatch in the level of achievement, it becomes. For example, while Leena enjoys the group work assigned in her IB mathematics course where collaborative work '...benefits, because if you know something and the other one knows something else then you can combine those and maybe understand it better.', she found it frustrating in other contexts, for instance in lower secondary prior to tracking:

Leena: Well, not in that case cause they were the easy problems that I had already solved and other ones asked me all the time that ``how can you do this?'' and stuff and... yeah I didn't like it. [laughs a tiny bit]

JS: Okay, so you didn't really feel like you were getting any academic benefit?

Leena: Yeah, I was just telling them how to do it.

Neither informal nor formal collaborative learning seemed to play a large role in the students' experiences, and perceptions of collaborative learning were ambivalent.

Jokimaa, Finland: Elite Mathematics Peer Groups

Among the students interviewed, the community of peers within elite mathematics courses in Finland was considered a key benefit of the course. Students believed their peers to be more interested and focused on mathematics, and that this enriched the course. For example, from Marja:

> On the Short, there are many people there who study it because they have to, because maths is obligatory, and there is an atmosphere that maths isn't fun, even though there may also be people there who have just wanted to choose short maths [...] it's my experience that on the Long Maths, there are many who really want to invest in the subject and are able to listen during the lesson and all.

Students considered that the nature of the peer group allowed for deeper and more worthwhile content:

> Jarkko: Yeah, I think I sort of feel, like, in principle, when the study group in long mathematics consists of the people who are interested in mathematics, at least, then the environment is easily more pleasing than the short mathematics study group where you can have many people who simply aren't interested in anything mathematical. So it is more encouraging as a study environment, and also in that you get deeper into all the things, you don't- it's like- you can see things as wholes and not only get small bits.
>
> Elisa: Yeah, I actually agree... that at least is an advantage- that those who only take the courses and aren't at all interested, those people aren't there. And that when you have interested people you get to go deeper.

Jokimaa students seemed to emphasise that peers' interest and willingness to learn mathematics was a key asset for their own learning, and a mechanism of selection into elite courses. Students did not portray peer groups as a reason for retention in mathematics. This coincides with Jokimaa students' choice of elite tracks in accordance with future plans, as well as a greater independence from peer and family influence in school and track choice when compared with Riverview.

Riverview, Washington: Extracurricular Mathematics

All of the Riverview students had ample access to mathematics related extracurricular activities and most participated. However, they did not seem to consider involvement as an influential factor in their mathematical careers. One exception was Cory, who had an intention of pursuing mathematics at the tertiary level:

> I feel like I'm almost entirely developed on the outside. Cause like, I have my classes which I kind of just do...like not just do it like C's but I mean, I do and I do good and I um- But like usually I find- cause I don't- I don't know, sometimes I don't feel challenged in a lot of my classes anyways.

Elsewhere, students revealed a lack of real enrichment in these activities, such as when I questioned two of the most accomplished students about a mathematics competition they had been involved in for several years, Math is Cool:

JS: Okay, so, hmm... did you do anything related to number theory?

Sandra: Um.

JS: Have you- have you guys seen-

Sandra: What is number theory?

JS: Well have you seen like modular arithmetic? I'm just curious.

Fiona: Oh! Modu- okay like

Sandra: Yeah

Fiona: Modular arithmetic

JS: I'm not asking you what it is I'm just- just wondering if-

Sandra: Like mod, like that thing, with the dividing?

Fiona, JS: Yeah

Sandra: That's in Math Is Cool.

Fiona: It's in Math Is Cool, like, it's a really challenging- but we don't actually know what it is, just if you give us one simple type of problem with that we'd be able to do.

Sandra: We'd be able to do it. We don't understand it, but we could do it. [Laughs]

While students were exposed to mathematics to which they would otherwise not have had access, it did not often seem to facilitate deeper understanding.

Riverview, Washington: Collaborative Learning

Many of the students interviewed in Riverview had strong collaborative networks outside the classroom. Such students considered these networks crucial in their success and persistence in elite mathematics. Students, such as George and Elizabeth, created lasting partnerships with daily mathematics collaboration.

As in Finland, however, there were students who found the idea of collaboration compelling, but frustrating in practice. For example Adrienne said:

> Well, to teach someone something you have to really understand it, so... you learn it better and you have to remember it more, because you have to figure out exactly what you are talking about before you can help them understand it.

However, her experience was dissonant with this ideal. Again from Adrienne:

> Well, sometimes it's frustrating because I'm not exactly patient, so if a person has trouble understanding something that I think is really obvious then I have to keep trying to find different ways to explain it to them and that's kind of tiresome...

While in general, collaboration was discussed positively, as in the Jokimaa case where there was a mismatch in achievement, actual encounters could be negative.

Collaborations were also limited by hierarchy, which Sandra describes legal terms: `There's like this kid John, who's like the smartest kid, and then we're like the second, legally, or third'. Hierarchy determines collaboration as Fiona says, "It's more like among the smart people we ask each other questions".

While intensive collaborations were more evident in Riverview than in Jokimaa, they did not seem to regularly extend past a tight sub-group of peers.

Riverview, Washington: Elite Mathematics Peer Groups

As in Jokimaa, elite mathematics students enjoyed their peer groups, and emphasised that such a community was a strong motivation for staying in elite mathematics tracks. Riverview students also defined themselves against other students in order to explain the benefits of their elite tracks. Here Bethany and Alexander use their experience with a 'regular' or mixed-ability class:

Bethany: And there was- half the people would not care at all, they were just- they- Some of them were just going to drop out of high school right there, but there were some people who actually cared, they wanted to learn what was, the teacher was trying to teach, and as the AP honours classes are introduced, it's the people who care about what they... get in a high school or want to go to college and need good grades and good classes, those are the people that go on to the AP classes. So instead of being held back by a group of trouble makers-

Alexander: [overlapping] Oh it's so hard to learn- [laughing]

Bethany : or potential drop outs, [Alexander: sound of disgust]- instead surrounded by people who keep on wanting to learn more who are kind of the driving force of the class, and you're all about the same level throughout it.

Throughout the interviews, the peer groups' positive characteristics were a motivation to continue in elite mathematics, and separation from struggling, ill-behaved, or unmotivated students a key benefit. Furthermore, access was believed to be mediated by character. Hard work and desire were the necessary prerequisites, even when students discussed significant parental involvement in track placement.

Nicole and Katherine were the only students who questioned the sorting mechanisms:

Katherine: [It] kind of makes you wonder. [...] It makes you wonder if-

Nicole: The racism is really gone.

Katherine: Yeah. And then you see in your class when you're a class of almost-

Nicole: Thirty

Katherine: All Caucasian people [In a majority Latino/Hispanic school] talking about Affirmative Action it's kind of like, how...

However, while questioning the visible sorting at Olympus in several instances, Nicole and Katherine also see access to elite courses as a question of character. Nicole said: “It has a lot to do with work ethic. And if they want to be pushed or if they just wanna breeze right through.”

DISCUSSION AND CONCLUSIONS

There were stark contrasts in access to extracurricular mathematics in Jokimaa and Riverview; Jokimaa students had no opportunities for sustained involvement, whereas Riverview students had diverse choices, and almost all of them had been involved in mathematics related activities. Most Riverview students downplayed the effects of such involvement. However, for at least one student, Cory, involvement was key to his interest and persistence in mathematics.

In Finland, participation in mathematics competitions such as Math Olympiad is used as a signifier of talent (see e.g. Nokelainen, Tiiri, and Merenti-Välimäki, 2002). Yet, the students I interviewed had no access to this, or other, enrichment programmes. So, while PISA finds evidence of equality in Finland's performance, it may be masking inequality of access at the top.

In neither Jokimaa nor Riverview was there evidence of the sort of collaborations described by, for example, Boaler (2008). While collaborative learning is often associated with de-tracking, the Finnish students seemed to have less experience with peer-supported learning. Students from both communities had ambivalent feelings about collaboration where there was a mismatch in achievement. There seems to be room in both communities for further exploration of modern collaborative learning.

For both Jokimaa and Riverview students, an elite group of peers was a positive aspect in mathematics tracking. However, the descriptions used by Riverview students were more hierarchical, and attributed blame to low performing students. Their characterisations seemed close to Sayer's (2005, p. 233) description of belief in the `moral well-orderdness' of the world, where:

> ...[T]he extent to which individuals' lives go well or badly is believed to be a simple reflection of their virtues and vices. It refuses to acknowledge the contingency and moral luck which disrupt such relations arbitrarily.

George said “...it kind of disgusts me to see the people that sit there and just ‘Oh- I have a D in this class and I’m taking Algebra for the fifth time because I don’t do my homework’” That such descriptions seem common among elite mathematics students in Washington, but seemingly not in Finland, is notable. They would arguably be more appropriate in Finland, where there is greater intergenerational class mobility (see Pekkarinen, Uusitalo & Pekkala 2006 or Breen & Jonsson 2005). Furthermore, these themes have resonance with Zevenbergen’s (2005) study of Australian students within a tracking system, where the discussion of classroom ethos and mathematical *habitus* using Bourdieu presents a possible way to deepen future work on this project.

The strong positive characterisations of elite peer groups in both Finland and Washington (also seen in Zevenbergen's (2005) study), and their place in improving learning and retention in elite mathematics, raises questions about how elite students might reply to the big-fish-little-pond concept or the possibility of de-tracking.

Limitations and Conclusions

There are several limitations to this study: more students were interviewed, and interviewed for slightly longer in Riverview, generating richer data from Washington, the linguistic aspects of the research are rough, and there were differences in interview styles between Jasu Markkanen and myself. The students' responses are thoroughly embedded not only in their schools, but their wider communities. However, important reforms, such as universal education and desegregation have involved changes in culture; culture is not fixed.

Regarding elite mathematics students, this study suggests a potential benefit from conducting international comparisons beyond the focus of studies such as PISA. Equality of provision may look different depending on the questions asked, and a comparative lens may clarify where to focus our attention.

NOTES

1. Education is governed mostly on the state level in the US. Washington is a better unit of comparison (than the US) with Finland in terms of population and resources and in addition, recently revised its mathematics curriculum through comparison with Finland, see Plattner (2007).

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INTERNATIONAL COMPARATIVE RESEARCH ON MATHEMATICAL PROBLEM SOLVING: SUGGESTIONS FOR NEW RESEARCH DIRECTIONS

Constantinos Xenofontos

University of Cambridge, U.K.

This paper is divided in two sections. In the first part, three problem solving views are discussed (problem solving as a process, as an instructional goal and as a teaching approach). In the second part, four research dimensions for international comparative studies on problem solving are proposed: (a) the research trends on problem solving in different countries-the researchers' perspective; (b) the curricular importance and justification of problem solving-the policy-makers' perspective; (c) teachers' beliefs, competence and practices in problem solving-the teachers' perspective; (d) students' beliefs and competence in problem solving-the students' perspective.

PROBLEM SOLVING-A MULTIDIMENSIONAL CONCEPT

Within the domain of mathematics education, the words *problem* and *problem solving* are extensively used. However, there is no consensus upon definitions, since many people use these terms to mean different things. The apparent agreement on the importance of problem solving does not say much about what problems and problem solving mean. In fact, it may mask very different views of what constitutes a problem and what kinds of problem solving abilities are desirable, teachable and evaluable (Arcavi & Friedlander, 2007). In respect of 'problems', there is evidence of polarisation, with some labelling problems as routine exercises that provide practice in newly learned mathematical techniques and others reserving the term for tasks whose difficulty or complexity makes them genuinely problematic (Schoenfeld, 1992; Goos et al., 2000). Furthermore, problem solving has been mostly viewed as a goal, process, basic skill, mode of inquiry, mathematical thinking, and teaching approach (Chapman, 1997). It appears, however, that the main perspectives on problem solving are those seeing it as a process, as an instructional goal and as a teaching approach.

Problem solving as a process

Various writers have developed frameworks for analysing problem solving as a process. Polya (1945), as the inaugurator of the research in the field, suggested four phases for the problem solving process: understanding the problem, devising a plan, carrying out the plan, and looking back. Polya's model comprised the basis on which other models were developed, for instance the six-phase one proposed by Kapa (2001): identifying and defining the problem, mental representation of the problem, planning how to proceed, executing the solution according to the plan, evaluation of what the problem solver knows about his/her performance, reaction to feedback.

However, 'Polya-style' models are often misinterpreted as a linear application of a series of steps, either because of the way they are presented in numerous textbooks (Wilson et al., 1993) or because they are perceived as such by most teachers (Kelly, 2006). In recognising the above deficiency, Mason et al. (1985) analyse three phases for the process of tackling a question; Entry, Attack and Review. It could be argued that Mason's phases are parallel to those of Polya. This is partly true, since there are obvious similarities between the *Entry* and *understanding the problem*, the *Attack* and the *devising and carrying out the plan*, the *Review* and the *looking back.* Nevertheless, Mason et al.'s (1985) *attack* phase appears not to necessitate a predetermined plan in the manner of Polya's *devising and carrying out a plan*.

Problem solving as an instructional goal

Mathematics proficiency, according to Kilpatrick et al. (2001), refers to successful mathematics learning and has five strands (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition). *Strategic competence* is defined as the ability to formulate, represent and solve mathematical problems. For many educational systems, the strategic competence in problem solving has a central role in mathematics teaching/learning and has been set as a fundamental instructional goal. For instance, problem solving has been identified as one of the five fundamental mathematical process standards along with reasoning and proof, communication, connections, and representations, by the National Council of Teachers of Mathematics (NCTM, 2000). For NCTM, mathematics teaching and learning and problem solving are synonymous terms; therefore the building of new mathematical knowledge through problem-solving should be in the centre of mathematics education. Similarly, in the context of China, Cai and Nie (2007) argue that the activity of mathematical problem solving in the classroom is viewed as an important focus of instruction that provides opportunities for students to enhance their flexible and independent mathematical thinking and reasoning abilities.

Problem solving as an instructional approach

Kilpatrick (1985), in a retrospective account of research on problem solving between 1960 and 1985, has identified five instructional approaches in *teaching mathematical problem solving* (osmosis, memorisation, imitation, cooperation, reflection). Despite the differences on how mathematical problem solving is approached in each of these categories, there is a common element: Problem solving is viewed as a cluster of skills students should acquire. From a different perspective, Nunokawa (2005) proposes four types of problem solving approaches in teaching mathematics. These approaches equate problem solving and mathematics teaching/learning. The first type refers to *emphasizing the application of mathematical knowledge students have*, through which students are expected to enrich their schemata of the targeted mathematical knowledge. This corresponds to 'teaching *for* problem solving'. The second type is about *emphasizing understanding of the problem situation*. As Nunokawa points out, "what is important in this type is deepening students' understanding of the situations that they are exploring using their mathematical knowledge" (p. 330). The third type regards *emphasizing new mathematical methods or ideas for making sense of the situation*. In other words, the teaching of mathematics occurs *via* problem solving. The teacher should select problematic situations that are appropriate to bring to light informal or naïve approaches from students, some of which can be formulated into the targeted mathematical knowledge. Finally, the fourth type is about *emphasizing management of solving processes themselves*. This corresponds to 'teaching *about* problem solving'; what students should obtain is "the wisdom concerning how to treat problematic situations, manage their solving processes, and put forward their thinking" (Nunokawa, 2005, p. 334).

THE NEED FOR INTERNATIONAL COMPARATIVE RESEARCH ON PROBLEM SOLVING

The diversity and interactivity of the international mathematics education community provides both the opportunity and motivation for comparative studies. Comparative research can claim to be a useful tool towards a better understanding of the educational process in general and in one's own system in particular (Grant, 2000); it is not necessarily meant to supply answers to questions but rather to enable planning and decision-taking to be better informed (Howson, 1999). Comparative research could be about the mutual benefits of sharing good practice and about the adaptive potential of the policies and practices of other educational systems to our own (Clarke, 2003).

> Challenges confronting the international research community require the development of test instruments that can legitimately measure the achievement of students who have participated in different mathematics curricula, research techniques by which the practices, motivations, and beliefs of all classroom participants might be studied and compared with sensitivity to cultural context, and theoretical frameworks by which the structure and content of diverse mathematics curricula, their enactment, and their consequences can be analysed and compared (Clarke, 2003, p. 144).

Comparative studies in mathematics education can be distinguished as two types: large-scale (mostly quantitative) and small-scale (mostly qualitative) studies. Large-scale studies such as TIMSS and PISA, have had much criticism. In my opinion, their biggest weakness is that they implicitly promote the idea of a *global mathematics curriculum* (a curriculum to which all school systems would subscribe), an idea based on the awareness of the world as one (Andrews, 2007b). Additionally, they are increasingly interpreted as competitions with inevitable winners and losers. Small-scale studies usually compare only two or three educational systems in relation to mathematics (Kaiser, 1999). They "share a common characteristic of seeking insight into the ways in which mathematics is systemically conceptualized and presented to learners in different countries" and generally celebrate cultural differences and identify the adaptive potential of one system's practices for another, by acknowledging culturally located traditions (Andrews, 2007b, p. 489).

During the 1980s and 1990s, problem solving has been the subject of extensive research in the U.S.A. The results of these studies have influenced the research and curricula development in many countries, such as in China (Cai & Nie, 2007), Australia (Clarke et al., 2007), Japan (Hino, 2007), Brazil (D'Ambrosio, 2007), Singapore (Fan & Zhu, 2007) and so many others. However, despite the US's influential research and curricular lines, problem solving research in many countries has evolved differently. Not only does the term *problem solving* mean different things in different countries, it has often changed dramatically in the same country (Torner et al., 2007). This has to be taken into consideration by comparative researchers in the field of problem solving, because many attempts to make international comparisons across countries fall into the trap of assuming that things with the same name must have the same function in every culture (Grant, 2000).

There is a lack of small scale studies on problem solving in the whole gamut of international comparative research. Taking all the above into account, I propose four distinct but also overlapping dimensions that comparative research on problem solving could focus on. Studies regarding these four dimensions should aim at in-depth investigation and analysis of how mathematical problem solving is being conceptualised in different educational settings. Nonetheless, studies of this kind should be approached and interpreted as efforts of the international mathematics education community towards international cooperation and national improvement. In the following pages I describe each of the four dimensions briefly.

a) The research trends on problem solving in different countries - The researchers' perspective

Comparative studies, from this point of view, should aim at comparisons between the research interests of mathematics educators and the research produced in each system. Comparing evidences from single-national studies around the world reveals that the problem solving research produced in different countries varies enormously. From the Australian perspective, for instance, Clarke et al. (2007) describe problem solving research in terms of three themes (obliteration, maturation, generalisation). Similarly, with respect to Portuguese research, Ponte (2007) states that the interest has now moved from mathematical problems to mathematical investigations and describes three research themes: the development of students' ability to do investigations, the promotion of students' mathematics learning, the influence of these activities on students' attitudes and conceptions. Other countries have not developed problem solving as a separate area of mathematics education research for various reasons. In the context of France, didactic research is influenced both by the Theory of Didactic Situations and the Anthropological Theory of Didactics (Artigue & Houdement, 2007). In both theories, problem solving has a central role; therefore the didactic research on mathematics is not separated from research on problem solving. In Brazil, however, this phenomenon appears for a different reason: problem solving is not examined as a separate area of mathematics education, but as part of the current reflection on Education and Cognition (D'Ambrosio, 2007).

b) The curricular importance and justification of problem solving - The policy-makers' perspective

Comparative research in this area should examine the explicit and/or implicit emphasis on problem solving in intended curricula and how problem solving within them is cultivated. By *intended curricula* I refer to "documents or statements of various types (often called guides, guidelines, or frameworks) prepared by the education ministry of by national or regional education departments, together with supporting material, such as instructional guides, or mandated textbooks" (Mullis et al., 2004, p. 164). In his paper, Xie (2004) compared the cultivation of problem solving between national mathematics standards issued by the National Council of Teachers of Mathematics (NCTM) in the U.S.A. and the Ministry of Education (MoE) of China. Both NCTM and MoE consider problem solving abilities to be the main goal of mathematics education. The definitions they offer of problem solving seem to be related to similar goals. However, there are certain differences between their goals. In NCTM, the term "problem-solving" is used to refer both to an end and an approach; while in MoE, problem-solving is seen mainly as a goal. Unlike the NCTM, the MoE does not mention students learning on their own but rather that they should apply the learned mathematics language to think or communicate mathematically. Differences do not only exist cross-nationally. In their single-national study in Israel, Arcavi and Friendlander (2007) interviewed the managers of different curriculum development projects. Despite the similarities on the participants views and approaches to problem solving (i.e. its importance, recognising the existence of different sorts of problems, etc) there are noticeable differences among the different theoretical and practical approaches to problem solving, even within the same community (of curriculum developers), focusing on the same target population (elementary schools) within a centralised system (in Israel) with a uniform syllabus.

c) Teachers' beliefs, competence and practices in problem solving - The teachers' perspective

International comparative studies about teachers' mathematics related beliefs (i.e. Whitman & Lai, 1990; Correa et al., 2008; Santagata, 2004; Andrews & Hatch, 2000; Andrews 2007c) and practices (i.e. Leung, 1995; Andrews, 2007a; Givvin et al., 2005) suggest that these two factors are more similar to each other within single countries than they are across countries. While there are some single-national studies about teachers' problem solving beliefs and practices, as for example in Australia (i.e. Anderson et al., 2008) and Cyprus (i.e. Xenofontos & Andrews, 2008), I am not aware of any cross-national studies in this area. From a different starting point (examining English and Hungarian teachers' beliefs about mathematics teaching), Andrews (2007c) concludes that English teachers tended to view mathematics as applicable number and the means by which learners are prepared for a world beyond school, while Hungarian teachers perceived mathematics as *problem solving* and logical thinking and independent of a world beyond school. Taking all the above into account, the similarities and differences of teachers' problem solving beliefs, competence and practices could be another dimension of the international comparative research in the field.

d) Students' beliefs and competence in problem solving - The students' perspective

Students' beliefs, competence and performance have traditionally attracted mathematics education researchers all around the world. Problem solving literature is, in my opinion, dominated by papers from students' perspective (i.e. Mason, 2003 in Italy; Nicolaidou & Philippou, 2003 in Cyprus; Op'Eynde & De Corte, 2003 in Flanders; Goos et al., 2000 in Australia, Cooper & Harries, 2002 in England and so on). International comparative studies, such as TIMSS (Mullis et al, 2004) and PISA (OECD, 2003) have examined students' problem solving performance in different countries. Particularly, PISA included *mathematical literacy* in its mandate (Clarke, 2003) and looked at mathematics in relation to its wider uses in people's lives (OECD, 2003). Mathematics literacy in PISA is measured in terms of students' capacity to recognise and interpret mathematical problems encountered in every-day life, translate these problems into a mathematical context, use mathematical knowledge and procedures to solve problems, interpret the results in terms of the original problem, reflect on the methods applied, and formulate and communicate the outcomes (Clarke, 2003). Both TIMSS and PISA were large-scale projects. What is needed in researching students' beliefs and competence in problem solving are small-scale qualitative studies that compare two or three educational systems.

CONCLUSIONS

The importance of mathematical problem solving in mathematics teaching and learning is internationally well defended. By acknowledging and investigating the cultural diversity of problem solving in different educational systems with respect to the four dimensions proposed above could be beneficial. The creation, promotion and establishment of a problem solving culture around the world is, in my opinion, important for better mathematics teaching and learning. International collaborations and comparative research could be the vehicle towards this direction.

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