FACTORS INFLUENCING TEACHER'S DESIGN OF ASSESSMENT MATERIAL AT TERTIARY LEVEL

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We study the process of design of examination papers in the first year of French university and identify some institutional constraints and some teachers' beliefs that influence this process.

Keywords: university expectations, teacher's collective work, documentary genesis, assessment material

INTRODUCTION

Numerous research works considered the difficulties met by the universities' first-year students. These works identify various reasons for those difficulties, offer various interpretations and develop various means of didactic action. The attention of researchers was initially centred on the new knowledge met and was then devoted to the new reference consisting in the practices of the expert mathematicians. It eventually moved upon new institutional expectations (see for a synthesis Gueudet on 2008). It led in particular to observe that students' private work is focused on learning how to mimic techniques, whereas teachers expect that students develop a real mathematical autonomy (Lithner 2003, Castela 2004).

The researchers who made those reports highlighted a difference between teachers' expectations and institutional expectations, the latter being particularly visible through the exam subjects. Those would in fact be organized around the mimicking of methods studied during the tutorials. As teachers of the tutorial write the examination texts, the latter would choose to question students on simple contents, such as exercises similar to those studied and corrected in class, notably to avoid a too important failure. Yet, the impact of evaluations on the work of students is very important (Romainville 2002). Besides many innovative teaching designs propose new assessment modes, such as group projects with oral examinations (Grønbæk and Winsløw 2006).

Here we do not wish to suggest an innovation, but simply to investigate whether examinations are really related to the mimic of methods. In the case of a positive response, we try to understand why university teachers propose such evaluations. This preliminary study will allow us to propose other modes of assessments.

This paper is directly related to the themes of CERME 6 group 12, adopting a mathematics-centered perspective about the teaching at tertiary level, and considering the important part of effective teaching settings constituted by assessments.

The development of an examination text is a documentary work, implying various resources, generally carried out in a collaborative way by a team of teachers. The documentary approach of didactics (Gueudet and Trouche in press) showed that such a work was influenced by beliefs, expectations, etc., of teachers, and that documents resulting from this work influenced in return these beliefs (Cooney 1999). This process of both development of documents (here of examination texts) and evolution of teachers' beliefs depends strongly on the institutional context. The institution indeed influences its actors through a system of conditions and constraints which can be very general or related to precise contents (Chevallard 2002) and which shape the knowledge within the institution.

Considering this point of view, we chose to study a first-year mathematics course in a French university, for which we followed the development processes of the examination texts. In section II, we present this tutorial and our methodology. We noted that the assessment relates only to the mimics of techniques. Thus, our central question here is the following one:

Which institutional conditions and constraints and which beliefs of the teachers control the choices carried out during the development of the examination papers?

We give some elements of answer by analyzing in section III the institutional constraints, conditions, and beliefs of teachers who lead to the choice of a specific exercise. In section IV, we illustrate the consequences of these constraints through the successive evolutions of the statement of a given exercise, and also show the phenomena of inertia related to the manner in which the examination papers are developed.

Finally, we conclude by evoking possible clues for an improvement of the assessment practices that could foster the students' mathematical activity.

CONTEXT AND METHODOLOGY OF THE STUDY

We study more particularly a mathematics course from the first semester in a French university. This course is devoted to students graduating in physics.

During the first semester, students follow six courses, only one being in mathematics. Our choice came from the author's involvement in the course. We initially thought that the context (teaching mathematics to Physics students) could lead to exercises coming from physics situations in the examination papers. We quickly noted that it occurred neither in the tests, nor in the sheets of exercises. We will not improve this question here.

To help with the secondary-tertiary transition, this course - like all those of the first semester - is organized in small groups of about thirty students (five groups), each group having a unique mathematics teacher. The course is 4 hours a week over 12 weeks. To ensure coherence between the various groups, a blow-by-blow program (the topics studied are specified, as well as the time that should be devoted to them) is given to each teacher and the sheets of exercises are the same for every group. Both program and sheets of exercises come from the background of the teachers involved in this course during the two previous years.

The contents were chosen according to the mathematical tools necessary in the other courses: complex numbers, study of functions, Riemann integrals, first and second order linear differential equations. It thus contains secondary level knowledge in each of the first three topics, with each time a deepening and new knowledge: n^{th} roots of a complex number, inverse of trigonometrical functions, change of variables in an integral... All these topics are introduced to solve some kinds of differential equations.

The assessment consists of two one hour-long exams at the end of week 5 and of week 9 and of a two hours-long final one at week 12 (just after the end of teaching).

The mark of a student is the maximum mark between the final exam and a weighted average of the three tests (1/4 for each one hour exam, 1/2 for the last one). Indeed this topic should deserve a specific study and we will not study it in this article. Students who don’t succeed have a resit, but we focused on the three tests that gave the first final mark.

The development's work of examination texts is shared out at the beginning of the course among the teachers: the first exam (CC1) was entrusted to Omar and Georges, the second one (CC2) to Omar and Thierry while the final examination paper (E) was prepared by Marc (responsible for this course), Thierry, Georges and Marie-Pierre (author of this paper). In the three cases, the appointed teachers initially worked together before proposing an almost finished text to the other ones.

The data were gathered through interviews (appendix A) of teachers involved in a same exam, initially before the development work to question them about their intentions, then to discuss their choices afterwards. We paid attention on the following points: coordination between the teachers and supports used for the development of the text, choices for the contents of this one and objectives that guided these choices.

We now will present the analysis of the gathered elements.

CONSTRAINTS AND BELIEFS: REASONS FOR IMPLEMENTATION OF METHODS

The examination texts given since September 2004 (i.e. during 4 academic years) are mainly composed of exercises aiming to the use of methods learned during this course. In this section we detail various aspects of this choice, and the reasons for it, by illustrating our point with an exercise, which seemed to us emblematic.

Texts of assessment: agglomerates of short exercises

Each examination paper is made up of a list of short exercises: it never relates to one or two long problems. Various reasons lead to this choice. First, the duration of exams (1 h or 2 h) is limited (the mathematics exam of the French end of secondary school certificate for scientific students, "Baccalauréat S", lasts 4 hours). This duration is an institutional constraint of general level; in particular, the 3 hours examinations were gradually removed at the University of Rennes 1 in order to make possible two examinations in the same half-day: it optimizes the occupancy of the rooms of examination and the working time of the university porters. This optimization is crucial because of the increase in the number of exams. Indeed, it is observed "the bursting of the academic year in semesters and the courses in units of teaching involved an increase of the number of evaluations" (Gauthier & al 2007)

Beyond this *time constraint*, a big factor emerges from our interviews, factor which deals with the objectives that the teachers assign to assessment, and thus of what we name under the generic term of belief: an evaluation must include all the parts of the previous program, particularity that we will name the *belief of exhaustiveness*. Omar stresses that an assessment must make it possible for the student to have a diagnosis of his knowledge: any gap could then be filled before the following tutorial. This diagnosis must thus be complete. This argument is not valid any more for the final examination; however, Marc regards as very important the fact that the examination paper covers all the contents, on the one hand to force the students to revise everything, and on the other hand "to draw a distinction between those who have been working enough and those who have not". However, the content of this course is divided in five chapters: this is also an institutional constraint, which relates more directly to the mathematical contents and which we name *constraint of the knowledge organization*. Now, the final examination paper generally consists of five exercises (or four exercises, with one in two sections)

Moreover, assessment never consists in long problems because of the importance attached to the success rate: teachers fear a "snowball effect" (Omar) of a mistake because of linked questions. We will return now to this fundamental factor.

Exercises of detailed implementation of methods

Let us consider the following exercise, resulting from the final examination paper (December 2007):

1. Determinate the square roots of 3+4i.
2. Solve, in C, the equation $z^2 +3iz-3-i=0$.

We want to underline some important points about this exercise. It applies the method of resolution of quadratic equations with complex coefficients, method learned during the tutorial. The intermediate calculation of square roots is the subject of the first question. Thus the student can check the result in question 2), since they have to find the value given into 1) (it is a typical effect of contract didactic, Brousseau 1997). In addition, all the numerical values are whole numbers, never exceeding two digits, which allows the student to check very easily, and even allows a relatively effective method by trial and error in question 1.

However, this exercise is emblematic of such assessment. The same kind of exercise is found in each subject of the first exam and of the last one for the 4 last years.

The use of whole numbers is an institutional constraint specific to mathematics in the first year at the University of Rennes 1: the *constraint of ban on calculators*. This constraint is associated with the teachers' beliefs of the need for the students to understand calculations that a software can carry out automatically: this topic requires a specific study, which we will not undertake here.

The primary reason that explains the choice of such an exercise is the objective of a sufficient success rate. This clearly appears in the exchanges of emails, when this exercise is proposed, following remarks on the fact that “it misses complex numbers” (Georges); “one could have put a short exercise, but easy, on the complexes” (Thierry). Marc then suggests the exercise saying: “It should easily improve their marks. What do you think about it?” The other teachers approve: "this exercise seems very fine to me" writes Georges. "I agree with Georges, as that will increase the chances of the students" Thierry adds. In his interview, Marc recognizes that question 2 could have been only asked, but, according to him, question 1 ensures that the intermediate stages will be visible in the writing of the students, thus making it possible “to give points”.

The *constraint of success rate* is crucial in the choices of examination papers on all school levels, but perhaps even more in universities in scientific studies, victim of disaffection. The average mark for a given course cannot be under 10. This exercise provides any student who attended the course with 2 valuable points. The degree of freedom left to teachers for the development of the assessment is restricted by these constraints and beliefs. This, however, is not enough to explain the astonishing similarity of the examination papers year after year.

RULES IN ACTION: GENESIS OF AN EXERCISE

We saw in previous section some very strong constraints and beliefs: time constraint; belief of exhaustiveness associated with the constraint with the knowledge organization; constraint/belief of ban on calculators; constraint/belief of success rate. We will now see their influence upon the development of one of the exercises of the second exam.

Work in each group of the appointed teachers always started by the choice of the contents to evaluate. These contents are divided into exercises, and each teacher then assumes the wording of some of these exercises.

During their first meeting, Omar and Thierry identify four contents of knowledge to be evaluated in the second examination: integration with, on the one hand its definition and on the other hand calculations, then two topics on functions. The exercise that we will study was relating to the definition of the Riemann integrals, i.e. by the integral of step functions. Omar was in charge of its drafting.

A non-standard exercise is proposed

The first text proposed by Omar is given in appendix B. The announced objective was the approximation of ln(2) by integrals of step functions "In the first questions, the objective is to make them calculate the integral of step functions, then, in the last one, to see that it is convergent, therefore to make them apply what they learned". Omar is a young teacher (PhD student): he proposes a relatively non-standard exercise.

He wanted to give sense to the calculations usually requested from the students by showing that these calculations yield the approximation of ln(2).

Omar submits this exercise to Thierry thinking it is too long (*time constraint*) and that the only first three questions will be kept. The exercise looking indeed too long to Thierry, he decides, after having spoken about it with Marc, to remove the last two questions "it is a little long, it is necessary to remove the question which embarrasses more the students, therefore *n* ". One thus finds the *constraint of success rate* to which one could add a belief of the teachers that calculation with parameters are too difficult for students. We will not speak about this didactic difficulty, which does not enter within the framework of our study.

Change of aim

Thierry will not be satisfied with the simple shortening. He will return it strongly modified to the great distress of Omar: the idea of approximation (chosen to give sense to calculations) completely disappeared. There remains only the calculation of integrals of step functions. The values remain the same ones with two exceptions: the value of *f* on the interval] 4/3,5/3 [became negative and *f* takes a different value in point 4/3. This second change is, according to Thierry, “to see whether the students understood that integration is independent of the choice of the value in a point”. The change of sign allows the calculation of the integral of *f*, then of its absolute value. The set aim is, always according to Thierry, to evaluate a usual error: “there are people who are also mistaken, [thinking that] the absolute value of the integral is the integral of the absolute value”.

In both cases, the aim is not to check the understanding of the implementation of a method, but rather of mathematical concepts. In the first case, the question illustrates a concept, whereas, in the second one, it illustrates some properties of this concept.

This exercise is also non-standard in the choice of the numerical values. If the choice of these values had a mathematical reason at the beginning (approximation of the function $1/x$), they were kept in the final version, in spite of a relative opposition of the other teachers. Marc will ask for example: "do you really want all those 1/3...?" He will add, at the end of the module, that: “the colleagues for the second control were a little creative, which resulted in the average not being good”. One finds again the *constraint of success rate*, here joined however with the belief that to propose non-standard exercises (that is to say exercises not present in the sheets of exercises) will not answer the institutional constraint of success rate. However here, this exercise, that Marc qualifies the “creative one”, did not induce a specific failure of the students contrary to the opinion that he expresses.

There thus still exists a certain degree of freedom in the design of the subjects, but it seems to be exploited only by young teachers (Thierry has been teaching only for 4 years). It would be interesting to follow their later evolution.

The effect of documentary geneses

Our observations show that the documentary geneses constitute an important factor of inertia. All the teachers consulted past papers: either for the contents of the exercises by changing only some values, or in the structure of the evaluation with the choice of the exercises' number and of the selected topics. "The reasons for which I thought of making 4 [exercises], it is that the last time, they were 4” tells us Omar who will recognize: “I nevertheless looked at past papers” and “I looked at the exercises' sheets to give exercises which are not completely new”. Marc will be more positive on this point: “the exam is rather standard; examination papers always have 5 exercises out of the 5 topics. […] I asked people to send exercises on the 5 topics”. Past papers are distributed to students before each exam and are corrected during the course. Students interpreted thus these texts as matching to the didactic expectations of the teachers.

The teachers looked at these former subjects in their development of a new examination paper because they made it possible to obtain the average expected by the institution. “The average [with CC2] was not good and so I absolutely wanted to make again a [standard] subject” will acknowledge Marc

Which didactic actions can one consider following this study? We give hints in the conclusion below.

CONCLUSION AND PROSPECTS

Our study deals with the teachers' activity, and more precisely with a part of this activity which goes on apart from the class. It must not be forgotten that the students and their learning constitute the central objective of our work. We stressed the importance of the questions of didactic contract in the teachers' choices of assessment. However the didactic contract involves teachers as well as students, and fixes the responsibilities for each one concerning the knowledge. The past papers constitute for the student a central reference, determining the institution expectations. Exam texts are composed of short exercises, consisting most of the time of the implementation of techniques: thus the private student's work turns naturally to the mimics of techniques.

Beyond this consequence on students' work, one observes an influence of the assessment on the teaching contents, and on the evolutions of these year after year. This extract of Marc's interview seems extremely significant to us in this respect:

“The more I teach this course, the more I… for example last year [...] I defined the integral [...] This year I said: listen, it has something to see with the area [...] if I teach that still 2,3 years I do not know what will remain. I make really more and more recipes by requiring nevertheless more rigor than in the physics tutorials."

"According to you, what leads you to teach more recipes? "

"The level of the students and the expectations of the students."

Marc gives us the worrying description of a teaching emptied little by little of its contents, because of the "level of the students" (perceptible by their marks) and their expectations; however these expectations are largely determined by the didactic contract, and thus by the examination texts.

Thus to leave the present situation, to escape in particular inertia related to the documentary geneses, seems to us a real need.

To master methods is important in mathematics. Part of the assessment could be officially turned towards this objective. It would even be possible to make pass such an exam on computer by using e-exercise bases (such as WIMS, Cazes et al. 2007). Indeed, the implementation of methods is hardly the requirement object of wording: assignments were not corrected.

An exam on computer, directly providing a mark, could make it possible to free up time for another mode of assessment, based on a real problem solving, and to give place to a written work. Must this work have a time limit; must it be completed by an oral examination? The precise organization has to be specified.

In addition, in particular for a course involved in the mathematical tools for physics, the use of a calculator seems absolutely necessary to us. Indeed, the use of whole numerical values is clearly out of touch with the physical situations. Our study shows that a change of assessment, and even a joint change of the pedagogic resources and practices, are essential if the mathematics teaching at University must contribute to the increasing of students' mathematical autonomy.

The context of our work was a course for Physics students: what about assessment in the case of Mathematics students? We conjecture a similar development - testing rather methods - but a precise study has to be done.

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APPENDIX A

Questionnaire before the development of the examination paper (written answers)

1. What coordination is planed between the teachers dealing with the conception of the examination paper (meetings, mail exchange...)?
2. What coordination is planed with the other teachers of the tutorials (contents of the assessment, proof reading...)?
3. Which resources do you expect to use (exercises books, past papers of this tutorial or of another one...)?
4. Which a priori shapes do you think to give to this exam (exercises, problems, multiple-choice questionnaire)? Why?
5. What do you want to assess in this exam?

Questionnaire after the test (interview guide)

1. Presentation of the teacher and his teaching experiences.
2. Looking back on the first questionnaire: Has the conception of the examination paper happened as expected? Otherwise, what have been the changes, and why?

3. Analysis of the examination paper, exercise by exercise. Details of choices and expectations. As far as the intermediate exams are concerned: which exploitation during the next tutorials?
4. In general about the reasons for the choices made in the conception of an examination paper in this course:
   - To give something close to exercises made in the tutorial
   - To give something which allows to adapt the teaching according to the results of the test
   - To test all the studied contents
   - To test the most important points (which one ?)
   - To test what will be useful for the following tutorial
   - To respect the time-frame
   - To give a subject quick to correct

APPENDIX B

First version

Let I be the value of the integral $\int_1^2 \frac{1}{x}\,dx$ and f the step function defined by:

$$f\ :\ x \mapsto \begin{cases} f(x) = 1 \text{ si } x \in [1, \frac{4}{3}[; \\ f(x) = \frac{3}{4} \text{ si } x \in [\frac{4}{3}, \frac{5}{3}[; \\ f(x) = \frac{3}{5} \text{ si } x \in [\frac{5}{3}, 2]. \end{cases}$$

1) Plot on the same graph f and the mapping $x \mapsto \frac{1}{x}$.

2) Calculate $\int_1^2 f(x)\,dx$, and deduce an estimation of I obtained by the left rectangle method with a regular subdivision into 3 intervals.

3) Prove that the estimation of I obtained by the left rectangle method with a regular subdivision into 4 intervals is equal to $\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$.

4) Prove that the estimation of I obtained by the left rectangle method with a regular subdivision into n intervals is equal to: $\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n-1}$.

5) Calculate I. Deduce the approximation

$$\ln(2) \sim \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n-1}$$

Final version

Let f be the mapping defined by:

$$f(x) = \begin{cases} 1 \text{ si } x \in [1, \frac{4}{3}[; \\ -2 \text{ si } x = \frac{4}{3}; \\ -\frac{3}{4} \text{ si } x \in]\frac{4}{3}, \frac{5}{3}[; \\ \frac{3}{5} \text{ si } x \in [\frac{5}{3}, 2]. \end{cases}$$

1) Calculate $\int_1^2 f(x)\,dx$.

2) Calculate $\int_1^2 |f(x)|\,dx$.