WHAT ROLES CAN MODELLING PLAY IN MULTIDISCIPLINARY TEACHING

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This paper presents a research- and development project about mathematics in multidisciplinary teaching, running as a pilot in 2008-2009 and planned to run in full scale in 2009-2010. Its aim is to inquire how learning potentials in mathematics are realised in a number of cases of good practice and, besides, to prepare materials for such teaching. The issue of this paper is to report on potentials and drawbacks experienced so far in the project and to discuss how to avoid the major drawbacks. The discussion takes as its starting point one example of modelling from the project, which invites critical discussions in the classroom about the use of mathematical models in statistics.

NEW CHALLENGES TO THE SCHOOL SUBJECT MATHEMATICS

As one consequence of a reform in 2006 of upper secondary school in Denmark, there is a need for examples of good teaching throwing light on and demonstrating what works for the learning of mathematics in multidisciplinary contexts. Furthermore, the reform's introduction of multi disciplinarity draws attention to the role of mathematics in different types of collaborations: It is not uncommon that multidisciplinary projects involve cultural, historical or philosophical aspects which are important but not at the heart of mathematics taught in schools. To balance this tendency, there is a need for advice and ideas about how to empower the learning of what one might call 'core mathematics' within a multidisciplinary teaching context.

THE DASG – NAVIMAT COLLABORATION PROJECT

This paper presents a research- and development project, which is running as a pilot (15 teachers in 4 schools) in 2008-2009 and planned to run in full scale (about 20 classes) in 2009-2010. The project deals with mathematics in multidisciplinary teaching projects. Its aim is to inquire how learning potentials in mathematics are realised in a number of cases of good practice and, besides, to prepare materials for such teaching.

The project is conducted in collaboration between Danish Science Gymnasiums (DASG)[1] and Nat. Knowledge Centre for Math. Ed. (NAVIMAT)[2]. DASG is a network[3], incorporating about 36 Danish Upper Secondary Schools (out of 200[4]). Membership implies an obligation for the school to spend resources, in the form of teachers' working hours, on participation in at least one of the 5 – 8 sub-projects, which are formulated and announced every year. The sub-projects run for two or three years and each one involves about 25 teachers. The collaboration between DASG and NAVIMAT encompasses a two-stage project. During the first year, three different types of teaching materials will be produced and tried out in a pilot; each of these materials represents the interplay between mathematics and one of the three participating faculties Human Sciences, Social Sciences and Natural Sciences. During the next stage, the following year, trials and evaluations of revised teaching materials from the pilot will be offered to the DASG schools as sub-projects. The revised versions of these materials hopefully will be published by NAVIMAT to provide inspiration for teachers at the conclusion of the trials. Teams of two to four teachers, a researcher in mathematics education and a professional specialist prepare the materials. The teams autonomously plan and make arrangements for their work during the first year of the project. DASG organises joint seminars for all the teams during this stage, for the exchange of ideas and experiences so far.

The professional specialist's are picked out depending of the mathematics teachers' choice of subject. The professional specialist's role in the team is to provide inspiration and expertise with regard to the content of the teaching materials. The mathematics education researcher provides inspiration and expertise with regard to the design of the materials and observes and evaluates the teaching experiment. The researcher is responsible for development and formulation of guidelines for good practice in multidisciplinary teaching. The team's teachers design and produce the teaching materials and carry out the teaching sequences. The teachers participate in the evaluation and discuss the results with the researchers.

THE POTENTIAL OF MULTI DISCIPLINARITY

Some of the potentials of multidisciplinary mathematics teaching were discussed in (Andresen and Lindenskov 2008). We see potentials achieving a number of different goals.

i) Students' motivation and interest. Multi-disciplinary projects can stimulate the students' interest and engagement in mathematics because the usefulness of the mathematics taught, and its links with the students' own, experienced world are in constant request in Danish school. Multi-disciplinary projects set the stage for the teaching of useful applications of mathematics in authentic, daily life settings. Hence, such projects can serve to meet the students' requests and to improve their desired understanding of connections between subjects and the world outside school. This is in accordance with Michelsen, Glargaard and Dejgaard (2005 p 33) who point to an alternative approach that stresses the importance of modelling activities in an inter-disciplinary context between the two school subjects physics and mathematics. Similarly, R. Filo and M. Yarkoni (2005) reported on a project, which integrated geometry and art, aiming at inter-disciplinary learning of parallel concepts. Filo and Yarkoni's assumption in this case was that an enriched concept formation was supplied by an advanced status of both subjects in the students' minds.

ii) Transfer. The authors report on their observations of the classroom that indicated

- Students' awareness of the possibilities to transfer concepts and results between subjects

- Students' consciousness about benefits, traps and misunderstandings caused by such transfer
- Students' reflections upon the relations between the project's subjects

The observations were interpreted in accordance with an interactionist's perspective like Heinrich Bauersfeld presents it in (Bauersfeld 1994 p 137-139). Hence, we looked for indications of a classroom culture where, for example, arguments from one subject (mathematics) were used and accepted in discussions within another subject (chemistry or physics) or used to convince other members of the group in discussions of problem-solving strategies etc. Besides, we evaluated signs of the students' formation of conceptions. The students seemed to build relations between the subjects in parallel with their formation of concepts and new skills belonging to the single subject.

iii) Implementation. Multi-disciplinarity can be seen as a means to revise the role of school mathematics and, thereby, to embed students' mathematical competence into a broad and reflected view of math and science. Compared to cross (or inter)-disciplinarity or to trans-disciplinarity, multi-disciplinarity has better odds for successful implementation because it resonates with the following four main elements of Fullan and Hargreaves's (1992 p 5) framework for understanding teacher development; 1) the teachers' purpose, 2) the teachers as a person, 3) the real world context for the teacher's work and 4) the culture of teaching

Hence, we argue (Andresen and Lindenskov 2008) that multidisciplinary teaching has important potentials for improving students' motivation and interest and for an enhanced transfer between subjects. We expect multidisciplinarity to be successfully implemented, and we expect it to serve as a means in the future to support the embedding of the students' competencies into broad and reflected view on mathematics.

MODELLING FOR CONCEPT FORMATION

In addition to this, the didactical potentials of a multi-disciplinary project rest on the role of mathematical modelling and reflections for concept formation. Mathematical models in multidisciplinary projects play a double role: on the one hand, the model can serve as the link between subjects and daily life, authentic problems etc., dealt with above. On the other hand, modelling plays an important role for concept formation. The role of modelling for concept formation in learning mathematics is described in the domain-specific instruction theory for realistic mathematics education, RME. (Gravemeijer and Stephan 2002 p 147ff). From this point of view, all mathematical activity concerns modelling, and it gives little meaning to try to discern theoretically between to learn, to apply or to develop new mathematics. Strict borderlines between the three are not to be drawn. In general, the use of the term 'modelling', therefore, has to be specified, since it depends on the context. (In this paper, though, we still use the terms in the 'common way' sense unless stated otherwise.)

POTENTIAL DRAWBACKS AND HOW TO AVOID THEM

Teaching multi-disciplinary projects in accordance with the Danish 2006 reform, hence, is a promising prospect. We also see some potential drawbacks. In some aspects, the impact of multi-disciplinarity on the students' view on mathematics is comparable to the impact of use of computers. The 2006-reform also imposed the introduction of compulsory use of computer algebra systems (CAS) in mathematics. Obviously, CAS has the potential for a huge extension and development of the teaching of models and technical modelling in the sense of comparing a number of models and fitting them with a set of data (Andresen 2007a p5). It also has potentials to support students' model recognition and capability to understand and criticize authentic use of ready-made models in different contexts.

Results from our previous research, however, show that in general, the use of CAS tends to change focus of attention to the technical and practical aspects of upper secondary school mathematics. In general, teaching with a computer is centred upon solving tasks, whereas the reading of proofs and theoretical treatments in general are carried out without use of computer (Andresen 2006 p 28).

There is a potential danger that the same trend might direct the multi-disciplinary teaching into a skills based view of mathematics by the students, at the expense of giving the students a more profound insight into mathematical activities, theory and knowledge. To avoid this, I suggest that the students' more technical and practical view on models and modelling, should be balanced by explicit reflections upon the use of models and upon the modelling process, that is, upon horizontal and vertical mathematizing.

MODELLING AND MATHEMATICAL REFLECTIONS

Reflection is the driving force for the process of mathematical modelling in the sense of progressive mathematizing (Gravemeijer and Stephan 2002 p 147 ff). Hence, Andresen and Froelund (2008) discuss how to make the students' philosophical reflections explicit, as a tool for mathematical reasoning and, thereby, to strengthening the students' consciousness of the art of reflection and of the relationship between reflection and learning. In line with the idea that awareness and consciousness about one's own learning support learning outcome, Andresen and Froelund suggest the explication of mathematical reflections as a tool for learning. The use of philosophical reflections as a tool for mathematical reasoning was recently discussed (Prediger 2007). Prediger's discussion was based on the stratification (Neubrand, 2000) of reflective practice in mathematics into four levels:

1. <u>Questions at the level of the mathematician</u> concern isolated, mathematical details. The questions are meant to deepen the students' understanding of the rise from a situational to a referential model which means that a preliminary or

emergent model is to be constructed. At this level objects still are marked by the context and, for example, referred to as ‘people’, ‘number of heart attacks’ etc.

2. Questions at the level of the deliberately working mathematician concern conscious use of mathematical objects and processes. The questions set focus of attention on generalisation of entities and their relations and, thereby, on the construction of a model for the case based on the model of the contextualised problem. The same type of questions could start discussion after the rise from referential level to general level; in the actual case by introduction of several distributions etc. The later discussion could lead to the next level of questions:
3. Questions at the level of the philosopher of mathematics concern mathematical methods and applications. Rise from general to formal level tends to happen over time, sometimes in a somehow subtle way. In the actual case discussions about the range of applicability and validity of hypothesis-test methods etc. serves to support the rise and make it more explicit to the students.
4. Questions at the level of the epistemologist concern the characteristics of mathematics compared to and delineated from other sciences. These questions relate to activities at the formal level which may be widened by further reflections. In the actual case, the multidisciplinary setting itself may lead to questions and discussions of the intended type.

Andresen and Froelund (2008) argue for the teaching of mathematics based on the use of a reflection guide containing thought-provoking questions at these four levels. A short analysis of the modelling process is prerequisite for the design of a reflection guide. The aim of this analysis is to identify potential levels of mathematical activity, referring to Gravemeijer’s model which includes four levels: situational, referential, general and formal. (Gravemeijer, K. & Stephan, M. (2002). p 159)

Teaching in a multi-disciplinary setting like in the example, provides a design that particularly favours explication of mathematical reflections. The didactical potential of such multi-disciplinary teaching, though, depends on its design: the design has to ensure that the project’s modelling processes are visible to the students as well as providing the opportunity to make students’ mathematical reflections explicit during classroom discussion etc.

ONE EXAMPLE OF THE PILOT’S TOPICS: THE VIOXX CASE

The materials presented in the following example takes the Vioxx case, described below, as its starting point and concentrate on probability theory and statistics in mathematics. Preparation of the materials is still ongoing (autumn 2008), based on experiences and notes from a pre-pilot teaching experiment carried out in 2007-2008. In the pre-pilot, all the project’s lessons were spent in mathematics, although the envisioned partner subject was the school subject social science. Philosophical ethics or chemistry might also have been appropriate. The teacher with his teaching experiences referred to below are from this pre-pilot.

The VIOXX case

Vioxx was a pain-reducing drug produced by Merck, and the case was about the statistical estimation of its long-term effects. In such cases it is impossible to carry out large-scale trials to determine the serious or long-term effects of drugs such as Vioxx. Therefore, when the drug is approved, such trials may be substituted by statistical inquiry of the population of users. For such inquiries, though, statistical models suitable for large-scale trials have to be modified and in particular, the criteria for the acceptance or rejection of hypotheses must be changed. Hence, the Vioxx case served as a context for the students in mathematics to study probability value (p-value), statistical significance and confidence intervals.

Vioxx, which was withdrawn from the U.S. market in 2004, is part of the class of drugs known as nonsteroidal anti-inflammatory drugs (NSAIDs). Vioxx was used to reduce pain, inflammation and stiffness caused by osteoarthritis; to manage acute pain in adults; to treat migraines and to treat menstrual pain. Merck, the manufacturer of Vioxx, announced a voluntary withdrawal of the drug from the U. S. and worldwide market, due to safety concerns of an increased risk of cardiovascular events (including heart attack and stroke) in patients taking Vioxx.

According to the U. S. Food and Drug Administration (FDA)'s website[5], FDA originally approved Vioxx in May 1999. The original safety database included approximately 5000 patients on Vioxx and did not show an increased risk of heart attack or stroke. A second study was primarily designed to look at the side effects of Vioxx such as stomach ulcers and bleeding and was submitted to the FDA in June 2000. The second study showed that patients taking Vioxx had fewer stomach ulcers and bleeding than patients taking naproxen, another NSAID, however, the study also showed a greater number of heart attacks in patients taking Vioxx. This second study was discussed at a February 2001 Arthritis Advisory Committee and the new safety information from this study were added to the labelling for Vioxx in April 2002. Merck then began to conduct longer-term trials to obtain more data on the risk of heart attack and stroke with long term users of Vioxx.

Merck's decision to withdraw Vioxx from the market was based on new data from this, later, trial in which Vioxx was compared to placebo (sugar-pill). The purpose of the trial was to see if Vioxx 25 mg was effective in preventing the recurrence of colon polyps. This trial was stopped early because there was an increased risk for serious cardiovascular events, such as heart attacks and strokes, first observed after 18 months of continuous treatment with Vioxx compared with placebo.

The Vioxx case attracted public attention since a large number of people had been taking Vioxx and amongst them, some had heart attacks. Heart attack victims and surviving relatives had taken legal action and were, in a number of cases, rewarded. For example, John McDarby, 77, and his wife were rewarded a $4.5 million dollar verdict and $9 million in punitive damages from a New Jersey jury in one of the first Vioxx trial cases against Merck[6]. The controversial question for judgement about

Merck's responsibility was to determine, whether data were sufficient to validate any hypothesis about correlation between Vioxx and the heart attacks.

Role and content of mathematics lessons

From the viewpoint of mathematics, Binomial distribution, Poisson distribution and Normal distribution were sufficiently strong tools to deal with these issues. Data from the original and from the later trials are available on Merck's website and then, the determination rests on decisions about level of significance and the confidence intervals. More profound model discussions may concern standards for comparison, compatibility and transfer of results etc.

In the pre-pilot, the teacher designed a sequence of about twenty lessons. This teacher had economy as his minor, so he agreed to spend some time and efforts on the inclusion of societal economics aspects in his teaching. The design was based on preceding discussions at a two-day seminar on authentic mathematics in upper secondary school and, subsequently, in a team with another mathematics teacher; a bio statistician and a researcher in mathematics education. This small group gathered twice during the semester where the experiment took place, for inspiration, exchange of ideas and evaluation.

The students had no prior experiences with probability or statistics. Consequently, the major part of the time was spent on the introduction and training of basic terms and relations within these branches. This introduction and training was based on the textbook with additional tasks collected from the web. In addition, the team prepared a spreadsheet for the students to experiment with distribution, confidence intervals and correlation coefficients.

In parallel, the students learned about the Vioxx case. Different aspects of the case were discussed in the class; in particular, the weighting between ethical and economical aspects and the role of mathematising in such cases were examined and debated. This part of the teaching might have taken place in the lessons on social science as well.

The challenge for the teacher was to combine the following three elements:

i) The mathematical content: introduction and basic training of terms and relations in probability and statistics. The content was taught in line with common practice in this class, based on the same textbook.

ii) The role of mathematics in the Vioxx case. In the Vioxx case the process of mathematising, obviously, was an issue of debate because of its implications for clients, the Merck Company etc. Thus, the case did not serve as a bare illustration of a 'neutral application' of mathematics. On the contrary, the case intended to draw attention to the modelling process itself.

iii) A look from outside at the societal role of mathematics. Development, test and application of medical treatments are based on the use of bio statistics and play an important role for healthcare at individual and societal levels. Though, it implies ethical and economical perspectives. Besides, public discussion of these issues may be seen as one element of democracy.

RESULTS AND OUTCOMES OF THE PRE - PILOT

The design of revised teaching materials and plans in the pilot will be based on the following summary of resulting outcomes related to the bullets i) – iii) above:

Mathematical Content: During the teaching experiment, the students showed large interest in the subject and in the Vioxx case. According to the teacher, the students were so eager to understand and to feel comfortable with the mathematical terms and relations and as a consequence, the class had to spend more time than expected on the technical-mathematical part of the course. For example, they spent six lessons just on working with level of significance. The teacher noted that this part of the sequence worked very well for the students.

Mathematical Modelling: The teacher indicated that the discussions stayed at the level of ready-made models. No attempts were made to modify the binomial distribution or to critically sort out the website's data. Modelling as such appeared not to be self-explanatory; on the contrary, every step had to be pointed out explicitly if the students were expected to be aware of it. For example, it was complicated for the students to make mathematical decisions, such as stating the level of significance. The teaching experiment, evidently, intended to demonstrate exactly that point to the students; so the stage was set to go deeper into the – complex - questions. The class spent time on changing the levels of significance and studied the consequences and effects. But they did not have time to follow these studies up.

Societal Role of mathematics: The teacher had the impression that the envisioned 'look from outside' on mathematics and its role could give input to very interesting and fruitful lessons on societal economy, law and on issues about democracy, public opinion and politics. It could be great, according to the teacher, to arrange a replay of one of the big hearings as a game with students arguing for and against. To complete this, teachers from both subjects should collaborate. The instructional materials for such a replay could be found on the various web sites but it should, preferably, be prepared in the – enlarged – team, including a teacher of social sciences.

GUIDELINES FOR THE PILOT

The big challenge for the pilot project will be to make the three elements, listed above as a coherent and convincing whole. In the pre- pilot part i), the mathematics content, was marked by its status at the introduction. In the pilot the sequence, consequently, will start only after the students' introduction to basic probability and statistics. They will then be able to concentrate on the role of mathematics and to work deliberately with the involved models and modelling. Connections, then, should be established more easily between the mathematical activities and the other elements of the Vioxx case. Such connections will be established, based on the teacher's guiding questions and sub tasks, focusing on specific aspects of the distribution, the level of significance or the parameters influencing the basic probabilities etc. combined with special points of interest from a social science point of view.

To sum up, the teacher recommended that part i) precede the proper multidisciplinary part which should then combine parts ii) and iii). The complete project should build up to a student role play of one of the big hearings, with arguments and a final verdict in the form of a verdict.

Further, preparation of a reflection guide should be included in the design of the pilot. The reflection guide should contain thought- provoking questions, which aim to stimulate the students' mathematical reflections and put them in focus of attention. The guide should be tailored to fit with the teaching materials, not vice versa. Preparation of one example of a guide is outlined in the following – a more detailed example may be found in (Andresen 2008).

CONCLUSION

In this actual case, the reflection guide's questions to the students can give rise to reflections upon the modelling process as a whole, as well as reflections upon the single parameters and how they are related, what they stand for etc. (level one and two) and, besides, to reflections upon smaller parts of the modelling process picked out to be studied separately. So, the scene is successfully set for reflections at all four levels in Prediger/Neubrand's model. Hence, it may be concluded that even if the teacher chooses a design where the technical mathematical part of the sequence precedes the other parts, and even if the VIOXX case in itself attracts the students' attention, it is still possible to choose a design that 1) makes the mathematical content, the role of mathematics in the Vioxx case and the societal role of mathematics as a coherent and convincing whole and 2) gives the students a profound insight into mathematical activities, theory and knowledge.

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1 http://www.emu.dk/gym/tvaers/sciencegym/english.html (loc. 22 May 2008)
2 www.navimat.dk (in Danish, loc. 22 May 2008)
[3] The Lundbeck Foundation in 2006 provided a five year sponsorship for the DASG networks activities[3] and each gymnasium in DASG has applied for its membership on a voluntary basis.
4 155 general and 42 technical upper secondary schools (gymnasiums)
5 http://www.fda.gov/cder/drug/infopage/vioxx/vioxxQA.htm (loc. June 2008)
6 http://www.weitzlux.com/vioxx/trial_402790.html (loc. June 2008)