EXPLORING THE USE OF THEORETICAL FRAMEWORKS FOR MODELLING-ORIENTED INSTRUCTIONAL DESIGN

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Designing modelling processes adapted to school restriction and able to produce a wide, rich and meaningful mathematical activity is far from been unproblematic. That situation seems to be even more problematic if the focus is on Early-Childhood Education. In this paper we explore the possibilities that existing theoretical frameworks can bring us. First, some theoretical consideration about modelling, the lack of sense of school mathematics and the use of theories for instructional design are outlined. Second, the design of a study process under the control of the Anthropological Theory of Didactics is described. Finally, the real implementation of this study process with 4-5 years old pupils is reported, showing how very young pupils can be involved in a wide, rich and meaningful mathematical activity.

INTRODUCTION

Modelling is occupying a central position in the current educational debate, from policymakers and curriculum developers to researchers and teachers. Focusing on research, important efforts in many directions can be observed: students' modelling competence, instructional design, modelling pedagogy, teacher training and support, students' and teachers' beliefs, among others.

On the other hand, research in mathematics education is developing more and more sophisticated theoretical frameworks which aim to understand the complex relations existing in every teaching and learning process. In a simplified way, every theoretical framework can be considered as a model of some *teaching and learning reality*. Structuring and simplifying processes are therefore necessary: every theory focuses on some objects and relations whilst other objects and relations are pushed into the background.

There is an ample consensus about conceptualizing modelling as a cyclic process where a dialectic between an extra-mathematical world and a mathematical one is established, as described by Blum, Niss and Galbraith (2007). Many different versions of the well-known modelling cycle have been developed, depending on researchers' interests and backgrounds. Conceived as different models of the modelling processes, each version tries to capture some features of these processes and/or the modelling-based teaching and learning processes.

However, it seems that there is a gap between research in modelling and applications, on the one hand, and research in mathematics education, on the other hand. That leads us to explore how existing theoretical frameworks not explicitly developed from a modelling perspective could be used to enhance research in the so-called modelling and applications domain. Particularly, we will focus on modelling-oriented instructional design through the Anthropological Theory of Didactics. Moreover, in this paper we will focus on early-childhood education levels, which are normally neglected in the existing research in modelling and applications.

MODELLING-ORIENTED INSTRUCTIONAL DESIGN

The process of designing modelling-oriented teaching sequences optimized to be used at school is far from being unproblematic. In our work with in-service teachers in a European training course in modelling and applications (LEMA project) one of their main concern was how to design interesting and authentic tasks, adapted to their school constraints and students' level and useful to develop the intended mathematical curriculum [1]. Although some teachers can show a great creativity to find real situations and problems and they feel able to adapt them for educational purposes, most of the teachers feel that it is a big, difficult and time-consuming work. Normally, anecdotic and isolated tasks are developed which address to some mathematical topics but these tasks fail in their intention of giving rise to a rich and wide modelling-based mathematical activity.

In the core of this situation a problem of didactic transposition (Chevallard, 1991) can be identified. Normally, real situations do not come themselves with interesting and crucial problems able to develop the desired wide and rich activity. We agree with Lehrer and Schauble (2007, p. 153) in that "models cannot simply be imported into classrooms. Instead, pedagogy must be designed so that students can come to understand natural systems by inventing and revising models of these systems".

Our approach in this paper is that of *applications and modelling for the learning of mathematics*, as Blum, Niss and Galbrait (2007) state and, particularly, the use of a modelling approach to help students to provide meaning and interpretation to mathematical entities and activities (also called *educational modelling* by Kaiser et al., 2007). That agrees with the current Spanish curriculum which reacts again the traditional lack of sense of school mathematics and asks for a meaningful mathematical activity where mathematical topics from different mathematical domains are connected and integrated.

ATD AS A FRAMEWORK FOR INSTRUCTIONAL DESIGN

In the last years, a group of Spanish and French researchers have been exploring and developing the Anthropological Theory of Didactics (ATD from now on) as a reference framework for instructional design. The notion of *Study and Research Course* (Chevallard, 2006) as well as the basic assumptions of mathematics as a human activity linked this effort with modelling and gave rise to a new research agenda.

In brief, mathematics is conceived in the ATD as a human and social construction. Over centuries, mathematics praxeologies have been developed, refined, optimized, rejected, combined, etc. as new problems arose in many different domains: from daily life to natural and social sciences and, obviously, from intra-mathematical problems. In our modern societies, School has the responsibility of the diffusion of a part of this cultural heritage to young people so that they will be able to live and act as responsible and democratic citizens. A common and traditional way of doing that is *showing* to the students already finished mathematic praxeologies, as artefacts they can *visit* and they should preserve. Chevallard (2006) call this the *monumentalistic* approach which directly relates with the lack of sense of mathematics at school.

Opposite to that approach and looking for students' sense-making, Chevallard (2006) advocates for a renewed school epistemology where interesting and crucial problems and questions are in the core giving rise to a meaningful mathematical activity.

The ATD has developed the notion of *Study and Research Course* (SRC) as a model to analyse and design school teaching and learning practices. What are the main characteristics of a SRC? In short: (a) a SRC should start from a *generative* and *crucial* question Q_0 [2]; (b) the *community of study* has to take the study of Q_0 seriously (Q_0, and the situations where Q_0 is inserted, is not the excuse teacher uses to introduce some mathematics); (c) the study of Q_0 will give rise to answers (that is, praxeologies) but also to new questions Q_i, making the study process an open process and, to some extent, undetermined in advance; (d) as far as Q_0 or some Q_i can be extra-mathematical, not only "pure" mathematical answers and questions are expected through the study process but also *mixed mathematical praxeologies* (Artaud, 2007); (e) a SRC gives rise to a collaborative and shared study process, looking for good answers and for good questions (sometimes new answers are developed, sometimes already existing answers are found, depending on the *media* available in the community of study). Finally, it is expected that the community of study develops their own personal answer $A^{\heartsuit}$.

As far as Q_0 and some Q_i emerging from it are of extra-mathematical nature, the subsequent SRC can be considered as a wide modelling process. Indeed, as we reported elsewhere (García, Bosch, Gascón and Ruiz-Higueras, 2006), modelling can be reconceptualised as the progressive construction of a set of praxeologies of increasing complexity. The SRC is therefore a didactic device useful to develop and design wide, rich and meaningful modelling processes with educational purposes.

As far as mathematics will emerge through the process as needed answers for taken as seriously problems instead of an already existing construction, *living* elsewhere and brought to school ignoring the why, the lack of sense of school mathematics will be avoided. Therefore, the SRC is a didactic device useful to make modelling a reality at school fighting against the *monumentalistic disease*.

DESIGNING A STUDY AND RESEARCH COURSE FOR EARLY-CHILHOOD EDUCATION: COLLECTING SILKWORMS

Institutional, pedagogical, curricular and epistemological background

In Spain, the early-childhood education is a non-compulsory educational level for 3 to 6 years old children (3 grades) although almost every child in this age attends to the school. It is not conceived as a kindergarten but as an educational level ruled by a national curriculum. Three are the main domains in this level: *self-knowledge and personal autonomy*, *knowledge of the environment* and *languages: communication* *and representation*. Among the general aims of this level, three of them are of special relevance for our work: *(b) to observe and explore children's familiar, natural and social environment*, *(f) to develop communicative skills in different languages and forms of expression* and *(g) initiation into logical-mathematical skills* (MEC, 2007)

School activity has to be organized in a holistic and integrated way. Children's reality and near environment should be the starting point for every teaching and learning situation. Therefore, modelling could be present on every teaching and learning situation although it is not explicitly described in the national curriculum.

During this stage, pupils are supposed to develop quantification skills and the cardinal sense of numbers (measure of a discrete set) as well as languages and forms of expression to communicate about quantities. Pupils will develop numbers' cardinal sense through their activity in many situations where the measure of one or several discrete sets is necessary. Numbers (both the meaning and the signs) will emerge as models to deal with this quantification [3]. Validation and interpretation processes as well as communicative needs are crucial to make pupils' knowledge evolve from self-invented representation of quantities to numerals and numbers.

As Ruiz-Higueras (2005) describes, following basic works in Didactics of Mathematics developed by Brousseau and cols. in the University of Bordeaux, the question that should guide early-childhood reconstruction of numbers should be*: why do we need numbers and their representation?* Three would be, at least, the functions of numbers in this level: to measure a discrete set (from the set to the number), to produce a set (from the number to the set) and to order a set (to assign and locate the position of an element in an ordered set). Centred in the first and second function, school situations where numbers emerge as models to express the measure of a set, to verify the conservation of a set, to manage a set, to remember the quantity, to reproduce or produce a set of an already known quantity and to compare two or more sets has to be designed and implemented.

If the design process of teaching and learning situations takes care about the reality and authenticity of the situations considered, then modelling is an optimal pedagogical approach for teachers to develop teaching and learning situations concerning numbers and their representation in early-childhood education.

Design of the Study and Research Course

The Study and Research Course (SRC from now on) reported here has its origins in a real school situation lived by a teacher [4]. She was working with her 4 years old students about butterflies and she thought about introducing silkworms and the transformation process into *butterflies* (metamorphosis). It was spring and pupils are used to collect silkworms and to feed them with white mulberry leaves. So, it was easy to bring a box with silkworms into the classroom and observe its evolution. The teacher, in order to deal with de holistic and integrated principle, decided to make some mathematical work with this situation. She is used to work with a-didactic situations (in Brousseau's sense) and their students are used to face problems, to develop different solutions and representations, compare them, formulate messages in mathematical codes (including self-invented codes), validate the solutions against the *milieu*, discuss about the problem and the different solutions, etc. Students are developing their knowledge of cardinal numbers during this school year and they have been working in many situations where they have to produce a number that measure a discrete set, to build a collection equal to a given number, to compare different collections, to express orally or in a written form how many elements are needed to complete or to reproduce a given collection (both with the collection in front of them and with the collection hidden). However, they do not always use the number as the best way to answer *how many* questions and, depending on the student, they can count up to 20 (or more) but many of this numbers are meaningless.

At the beginning, only an anecdotic an isolated activity (*if we've got N silkworms, how many leaves do we need to feed them?*) seemed to appear. But, as soon as we start working with the teacher and taking the situation seriously, a rich variety of praxeologies emerged.

Compared with other situations used by the teacher, two are the main characteristic of this one. On the one hand, it is a real and authentic situation (silkworms are in the classroom and have to be fed). On the other hand, it is a dynamic system: silkworms will turn into cocoons and, finally, moths (butterfly for pupils) will emerge and die. That means that there are, at least, three different collections to be controlled over the time. In terms of dynamical systems, each state of the system can be described with the vector $(t, n(t), c(t), m(t))$ where t is time, n is the number of silkworms, c is the number of cocoons and m is the number of *moths*. A conservative law rules the system: for every t, $n(t)+c(t)+m(t)=N$, where N is the original number of silkworms.

Working in this kind of systems is quite challenging for 4-5 years old pupils. Techniques to deal with time have to be developed and ways of organizing data are needed in order to record changes. That means that during the study process at school, not only praxeologies around cardinal numbers will emerge but also praxeologies concerning time measurement and data handling. Along the whole study process, silkworms will not only be the excuse teacher uses to introduce some mathematical work, but the centre of the process. Interpretation and validation will be dense during the study process.

IMPLEMENTING THE SILKWORM SRC AT SCHOOL

We will report in this section about the real implementation of the silkworm SRC in two different classrooms (4 and 5 years old students). Data have been taken from a self-report written by the teacher as she was developing the SRC and she was managing the study process at school. The study process took place in spring 2008. There is no space here to explain the study process in detail (both students' work and teachers' decisions). So, we will try to focus on the main issues of this process.

The study process started when the teacher was talking about butterflies in classroom and decided to link that topic with worms and metamorphosis process. She thought that bringing some silkworms into the classroom (figure 1) could be very motivating for their students. No mathematical work was planned in advance but she quickly noticed that a rich mathematical activity could be developed from this situation.

Fig. 1. Silkworms at school.

The first problem arose earlier: silkworms have to be fed, how silkworms' feeding should be organised? Some restrictions into the system had to be introduced: first, a leave for each silkworm each day; second, new leaves are needed each day and third, taking the leaves from the mulberry tree it is not possible (it is dangerous!) but the gardener will do it for us if we ask him. That gave rise to a quantification activity (praxeology around numbers as cardinals): the first and second restrictions were introduced in order to give rise to techniques dealing with cardinals and the comparison among different collections. These are really problematic situations for 4 and 5 years old students and numbers and numerals will emerge as the best models to deal with them (although some intermediate models, for instance, iconic representations, are also used). Although many pupils can recite the number names in sequence and they know numerals up to 9 or even more, many of them are not able to use them in context to measure a collection, to produce a new collection or to compare two or more given collections. For instance, the following dialog was recorded by the teacher:

Student: Teacher, we've got twenty-five silkworms minus two.

Teacher: Why? Can you explain it?

Student: Yes, there are twenty-five pupils in the class and, each day, two of us don't have a silkworm.

Teacher: Then, how many silkworms are there?

Student: Twenty-three.

Teacher: How do you know that?

Student: I don't know.

The silkworm activity offers a rich real situation to develop quantification skills. Moreover, as they have to write a message to the gardener with the leaves needed each day, communicative skills will be also developed.

Time is not a relevant variable for pupils yet, although the teacher asks pupils to write the date on the ordering-sheet. In the 5 years old classroom, students are quite engaged in silkworms care. The class was divided into groups which have to take care of the silkworms each day. A list of things to be done and a diary was made (figure 2): *1st, counting the silkworms; 2nd, cleaning the house; 3rd, bringing as many leaves as silkworms; 4th, filling in the diary; 5th, writing down the numbers; 6th, writing a letter to the teacher* (asking for new leaves). The teacher introduces also a table where pupils record the date, the name of the caring group, the number of silkworms and the number of leaves.

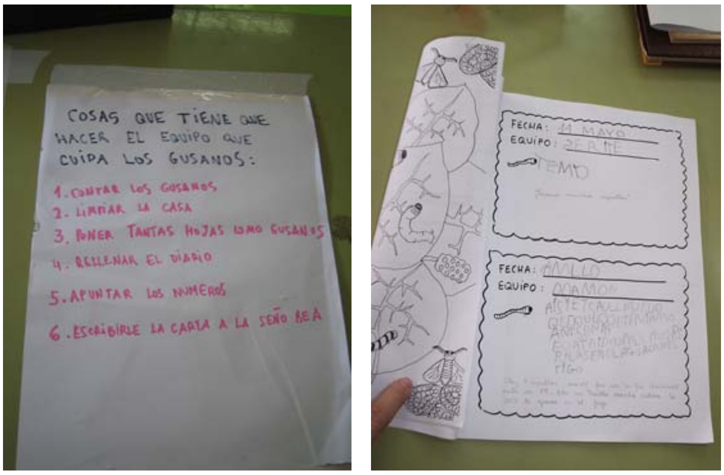

Fig 2. Things to be done and diary

Days were going by until the day cocoons appeared. That caused the first evolution of the mathematical activity (and, obviously, pupils' happiness). On the one hand, pupils decided to put the silkworms apart in other box because cocoons could be damaged when they had to clean the box and fed the silkworms. That caused the division of original collection in more than one collection and additive strategies to control the whole collection were needed. On the other hand, time arose as an important issue: they needed to control time in order to measure how many time will pass until the moth emerges from its cocoon. The static system has changed into a dynamical system. Pupils' quickly asked for time control:

Student 1: When will butterflies emerge?

Student 2: Well, tomorrow.

Student 1: No, they will take more days.

Student 2: Yes? How many?

Student 1: Now, here (the student points at a day in the calendar).

Student 2: Well, we can count the days (in the calendar) and when the butterfly emerges we will know how many days are.

Fig. 3. Table to control system's evolution (1)

Teacher knows that time control can be excessively demanding for 4-5 years old students. She needs to introduce some *tools* in classroom in order to let students control quantity and time together. A table (figure 3) is introduced by the teacher (date, group name, number of silkworms, number of new cocoons, number of leaves and total amount of cocoons). It will emerge as a tabular model of system's variation and records its evolution.

From a mathematical point of view, the original praxeology about quantity is evolving and widening including time measurement and strategies to handle with data (obviously, adapted to 4-5 years old students). From now on, students activity can be characterized as a dialectic between the system divided into different sub-collections (different boxes with silkworms and cocoons) and the tabular model (where system evolution is been recorded).

The day the first moth emerged provoked the necessity of calculating the time passed since the cocoon appeared. Again, this is a problematic task for 4-5 years old pupils. At this level it is usual to introduce some techniques to measure time working over calendars. Pupils are used to work with them and they can get some control on time passed or needed just counting on the calendar.

Student: It's been three days.

Student: No, I said ten days.

Student: Days have gone and they can't be counted.

Teacher: Yes, we can. Let's see, how can we know the days from my birthday?

Student: Well, we look for it in the calendar. We say one, two, three,… (some pupils went to the calendar in the wall and counted, pointing with the finger, since teacher's birthday).

Teacher: Now it is the same. Let's see, Antonio as responsible of the day, tell us how many days. First, you have to look for the day the cocoon appeared.

Student: It has to be one of the first cocoons because it is in the brown box.

Teacher: Ok. Antonio, look for the day in the calendar and count…

The result was twelve days. The next days the same activity gave rise to different results. That was interpreted as there were not a fixed number of days for the moth to emerge but a range. Pupils' interest decayed us they knew the days but they were interested in moths' care.

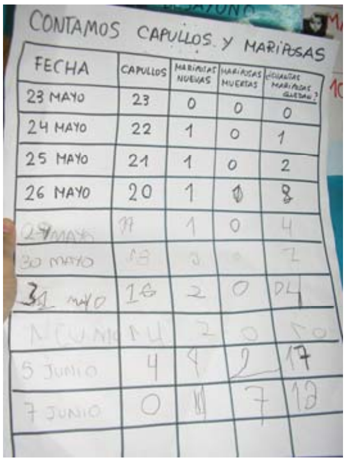

Fig. 4. Table to control system's evolution (2)

As they knew that moths will die very soon, the teacher decided to repeat again the time-quantification activity with the collections: cocoons, new moths, death moths and moths alive (figure 4).

When all the moths died, the system was over and the activity around them finished. However, the class had lots of information about the system and its evolution. The models constructed during the study process recorded this evolution. An interpretation activity was introduced by the teacher in order to make these tabular models useful to recover information about a system that had passed by.

The teacher proposed the pupils to make a poster representing the collection in different stages: silkworms eating leaves, cocoons and silkworms and cocoons and moths (figure 5).

Students had to interpret data on the table and produce the corresponding collections. From a modelling point of view, the fact that the system will never be back again in the classroom makes this interpretation activity completely crucial to summarize what happened and to talk about the system to somebody else. From an educational point of view, this is one of the most important moment of the study process: models can, to some extent, relieve the system and produce information about it even if the system will never be back again. The learning of time-quantity relations is one of the main aims at pre-school. During this final activity, students need to control time and quantity at the same time and the interpretation of the model is the key for that control.

Fig. 5. Reconstructing the system from the model

CONCLUSIONS

Designing modelling processes adapted to school restrictions, able to produce a wide, rich and meaningful mathematical activity is far from been unproblematic. In this paper we argue for the necessity of sophisticated theoretical frameworks for modelling-oriented instructional design. Moreover, for very young students, there is a lack of research concerning modelling-based teaching and learning.

We have described a process of study designed under the Anthropological Theory of Didactics and carried out by 4-5 years old pupils. First of all, the example shows how the theoretical framework allows us to control the design process and its real implementation. Secondly, the study process reported here shows how very young pupils can be involved in a wide, rich and meaningful modelling activity where different praxeologies of increasing complexity emerge as the system is evolving over time. Pupils use, learn and widen their mathematical knowledge as they want to take care of the silkworm collection and to know more about silkworms' transformation: quantification skills, additive and subtractive strategies, time-quantity relations and data handling procedures are brought into play. Finally, once the system has disappeared, models previously developed emerge as tools to reconstruct the system in every stage and to recover time and quantity information. Very young pupils are engaged in a modelling activity, producing and using models, a long time before they are able to really understand what modelling is and the role modelling plays in daily-life, society and science.

NOTES

[1] In Spain, the national and autonomic curriculums are not modelling-oriented. Although many teachers and textbooks are interested in applications and modelling, their main concern is to develop the mathematical topics listed in the curriculum.

[2] When a question can be considered as “generative” and “crucial”? It depends mainly on the institution where the study process will take place, the educational system and, finally, the society. School level and curricular constrains, the way the educational system is organized and the main aims of school within society need to be considered.

[3] Number’s ordinal sense will not be considered in this paper.

[4] We will like to thank to the teacher, Mrs. Blanca Aguilar (from “El Olivo” school in Torredonjimeno, Jaén, Spain), her effort and enthusiasm which made the experimentation possible.

REFERENCES

Artaud, M. (2007). Some conditions for modelling to exist in mathematics classrooms. In W. Blum, P. Galbraith, H-W. Henn and M. Niss (Eds.), *Modelling and Applications in Mathematics Education* (pp. 371-378). New York: Springer.

Blum, W., Niss, M. and Galbraith, P. (2007). Modelling and Applications in Mathematics Education. Introduction. In W. Blum, P. Galbraith, H-W. Henn and M. Niss (Eds.), *Modelling and Applications in Mathematics Education* (pp. 3-32). New York: Springer.

Chevallard, Y. (1991). *La transposición didáctica: del saber sabio al saber enseñado.* Buenos Aires: Aique.

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In M. Bosch (Ed.), *Proceedings of the IV Conference of the European Society for Research in Mathematics Education* (pp. 21-30). Barcelona: FundEmi IQS - Universitat Ramon Llull.

García, F.J., Bosch, M., Gascón, J. and Ruiz-Higueras, L. (2006). Mathematical modelling as a tool for the connection of school mathematics. *ZDM – The International Journal on Mathematics Education*, 38(3), 226-246.

Kaiser, G., Sriraman, B., Blomhøj, M. and García, F.J. (2007). Modelling and applications – Differentiating perspectives and delineating commonalties. In D. Pitta Pantazi y G. Philippou (Eds.), *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 2035-2041). Larnaca: University of Cyprus.

Lehrer, R. and Schauble, L. (2007). A developmental approach for supporting the epistemology of modelling. In W. Blum, P. Galbraith, H-W. Henn and M. Niss (Eds.), *Modelling and Applications in Mathematics Education* (pp. 153-160). New York: Springer.

MEC (2007). Real Decreto 1630/2006, de 29 de diciembre, por el que se establecen las enseñanzas mínimas del segundo ciclo de Educación infantil. *Boletín Oficial del Estado*, 4 (January 4th), 474-482.

Ruiz-Higueras, L. (2005). La construcción de los primeros conocimientos numéricos. In C. Chamorro (Ed.), *Didáctica de las Matemáticas para Educación Infantil* (pp. 181-219). Madrid: Pearson Educación.