THE 'ECOLOGY' OF MATHEMATICAL MODELLING: CONSTRAINTS TO ITS TEACHING AT UNIVERSITY LEVEL

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Considering the general problem of integrating mathematical modelling into current educational systems, the paper focuses on the study of the institutional constraints that hinder the implementation of modelling activities. The study of these restrictions and the way new teaching proposals can overcome them appear as an unavoidable step for the large-scale dissemination of mathematical modelling activities at all school levels. Within the framework of the Anthropological Theory of the Didactic, it is proposed the use of a hierarchy of levels of didactical determination as a frame to set and analyse from the more specific constraints, related to the usual way of organising mathematical contents, till the more generic ones, linked to the 'dominant epistemology' concerning the role of mathematics in experimental sciences.

Key words: ATD, mathematical modelling, constraints, conditions, applicationism.

1. THE PROBLEM OF INTEGRATING MATHEMATICAL MODELLING INTO CURRENT EDUCATIONAL SYSTEMS

Nowadays, there seems to be no doubt about the possibility of introducing students to a mathematical activity orientated towards the study of applied and modelling problems. This agreement is shared by many researchers in the field of mathematical modelling and applications, and supported by the new curricular orientations that have recently been introduced in our educational systems, thus trying to focus mathematical teaching more on the study of 'real life situations' than on systems of well-organised mathematical contents. Several investigations from different theoretical perspectives have shown that mathematical modelling activities can exist at school under suitable conditions, at all levels and related to almost all curricular contents.

Beside all the progress of establishing modelling as a normalized activity in some controlled processes of teaching and learning mathematics, the problem of the large-scale dissemination of these processes has recently been addressed as both an urgent and intricate task. Some authors have started pointing out the existence of strong limitations hindering the inclusion and permanent survival of mathematical modelling practices in the classroom. For instance, Blum *et al.* (2002, p. 150) depicts the situation as follows:

> While applications and modelling also play a more important role in most countries' classrooms than in the past, there still exists a substantial gap between the ideals of

> educational debate and innovative curricula on the one hand, and everyday teaching practice on the other hand.

Kaiser (2006, p. 393) seems to go in the same direction when she states:

> Since the last decades the didactic discussion has reached the consensus that applications and modelling must be given more meaning in mathematics teaching. […] However, international comparative studies on mathematics teaching carried out during the last years, especially in the PISA Study, have demonstrated that worldwide young people have significant problems with applications and modelling tasks.

Related to this state of things, Burkhardt (2008) emphasizes the existence of two realities: on the one hand, the good progress and encouraging results in research about teaching modelling and applications; on the other hand, the difficulties of its large-scale diffusion in the classroom. He states quite brutally (op. cit., p. 2091):

> [W]e know how to teach modelling, have shown how to develop the support necessary to enable typical teachers to handle it, and it is happening in many classrooms around the world. The bad news? 'Many' is compared with one; the proportion of classrooms where modelling happens is close to zero.

To describe the difficulties encountered in the diffusion of modelling, many researchers use expressions such as 'counter-arguments' (Blum, 1991), 'obstacles' (Kaiser, 2006), 'dilemmas' (Blomhoj & Kjeldsen, 2006) or 'barriers' (Burkhardt, 2006), pointing out a new direction of research which moves from the problem of the design, implementation and analysis of modelling practices to the study of the conditions that affect the existence, permanence and evolution of these practices. In a research on teachers' beliefs about mathematical modelling, Kaiser (2006) defines different teachers' profiles to explain how some beliefs can become important 'obstacles' for the implementation of applied and modelling practices in teaching, because the nature of contextual and applied problems does not seem to be compatible with those beliefs. (p. 399). In the same direction, Blomhoj & Kjeldsen (2006, pp. 175-176) point out the existence of different 'dilemmas' that should be faced before widely incorporating the teaching of modelling. These dilemmas refer to: the understanding of mathematical modelling competency from a holistic point of view; considering mathematical modelling as an educational goal in its own right and the dilemma of teaching directed autonomy.

At a more general level, Burkhardt (2006, pp. 190-193) outlines and discusses the existence of 'barriers' that obstruct a large-scale implementation of modelling, such as the systemic inertia, the unwelcome complication of the 'real world' in many mathematics classrooms, the limited professional developments of teachers, the role and nature of research, and the development in education. To overcome these barriers and many others still unknown, he refers to some 'levers' (such as changes in curriculum descriptions supported by well-engineered materials to support assessment, teaching and professional development, etc.) that may show some promise progress in this field. Michelsen (2006) points out an even more general barrier when he questions the common separate vision of scientific disciplines, and states that traditional borders between disciplines suppose a clear constraint for the development of applied activities (op. cit., p. 269):

> The challenge is to replace the current monodisciplinary approach, where knowledge is presented as a series of static facts disassociated from time with an interdisciplinary approach, where mathematics, science, biology, chemistry and physics are woven continuous together.

This situation can be summarized in the formulation of the following didactic problem, which has to be located at the core of all research aiming to integrate mathematical modelling in teaching and learning practices:

What kind of *limitations* and *constraints* exist in our current educational systems that prevent mathematical modelling from being widely incorporate in daily classrooms' activities? What kind of *conditions* could help a large-scale integration of mathematical modelling at school?

Within the framework of the Anthropological Theory of the Didactic (ATD), most of the research related to mathematical modelling and teaching practices[1] (Artaud 2007, Bolea *et al.* 2004, Barquero *et al.* 2008, Barbé *et al*. 2005) takes into account the problem of the 'ecology' of didactic organisations, that is, the study of the conditions needed to implement teaching and learning activities and the constraints that hinder their normal evolution in a given educational institution. The origin of this ecological problematic, which was first applied to mathematical objects and practices before being enlarged to a wider institutional perspective, can be located in the study of the process of *didactic transposition* and its related phenomena (Chevallard 1985, see also Bosch & Gascón 2006).

In our research project on the study of a global modelling process at university level centred on the study of a population dynamics (Barquero *et al*., 2008), we have observed the existence of different kinds of transpositive constraints that hinder the normal evolution of modelling practices in the classroom. We will develop this point further in the next section, preceding it by a short presentation of the 'levels of didactic determination', a key notion introduced by Chevallard (2002) that we will use as a frame to analyse the different kinds of conditions and constraints that affect teaching and learning processes.

2. CONSTRAINTS ON THE TEACHING OF MODELLING ACTIVITIES

2.1. Levels of didactic determination

[1] Several works within the framework of the ATD as Chevallard (1992), Chevallard, Bosch & Gascón (1997) have analyzed and described mathematical modelling activities from this approach. From ATD, it is assumed that doing mathematics consists essentially in the activity of producing, transforming, interpreting and arranging mathematical models.

Mathematics teaching and learning processes can exist because a lot of conditions make them possible: the existence of a social educational project, the choice of a set of contents to be taught, a school organisation with grades, syllabi, teachers and students grouped in classrooms, teaching materials, teachers' training programmes, etc. These conditions are also factors that, while allowing some things to happen, are also impeding others to take place. In the research and design of new teaching proposals, taking into account these conditions and constraints seems necessary if we do not want to have a set of 'ideal' didactic organisations unable to 'survive' under normal conditions, being, as Chevallard (2002, p. 42) put it, only a 'world on paper'. To study the 'ecology' of mathematical practices that exist (or could exist) in a teaching institution and the possible ways of constructing them (the didactic organisations), this author introduced a hierarchy of 'levels of didactic determination' that consists in the following sequence (Ibid.):

Civilization ↔ Society ↔ School ↔ Pedagogy ↔ Discipline ↔ Domain ↔ Sector ↔ Theme ↔ Subject

This hierarchy goes from the most generic level –Civilization– to the most concrete one – the subject or questions that are to be studied by a group of persons. We refer to the lower levels that go from the *discipline* to the *subject* as the *mathematical levels* if the considered discipline is *mathematics*. They refer to the fact that, when a teaching project has been decided on, the contents or the aim of this project should be located in a discipline (or different ones) and, within this discipline, it should be related to the different domains, sectors and themes that structured it in the considered educational institution. For instance, in Spain, a first year course of mathematics for science students at university level is usually structured into three domains: calculus, linear algebra and differential equations. Frequently, the domains are in turn divided into 'sectors', which contain different 'themes', to which every subject or question to study belongs. At secondary school level, the domains are different and can change over time, with each curricular reform: the classical division into 'arithmetic, algebra, geometry' first changed to 'numbers and measure, functions, geometry, statistics', and has now turned into 'change and relations, space and form, statistics, measure, number'. We consider these low levels (as) the 'specific' ones. They are a useful tool to analyse the constraints coming from the didactic transposition process and the concrete way this process organises teaching contents at school: from the division into disciplines and blocks of contents, until (till) the low-level concatenation of subjects.

The upper levels of determination refer to the more general constraints coming from the way Society, through School, organises the study of disciplines (pedagogical level). They concern the status and functions traditionally assigned to educational contents and the general way teaching and learning study activities are organised at school. In effect, there are a lot of common conditions for all disciplines that concretely affect what the teacher and students can do in the classroom. For instance, the amount of hours and sessions assigned to the teaching of a concrete discipline, the possibilities for disciplines to interact more or less easily, the way students are grouped (by age, by level, by gender, etc.), the organisation of the school space, etc. All those conditions and constraints belong to the *school level,* while the *pedagogical level* refers to those only affecting the teaching and learning of 'disciplines'. The way disciplines are grouped, valued, linked, diffused belongs to this level: the choice of an interdisciplinary way of studying questions or the way of presenting disciplines as independent. Very close to the previous levels, the *society and civilization levels* concern the way our society and civilization understand the rationale, functions, aims, etc. of school instruction.

The next two sections briefly introduce some of the institutional constraints encountered during an empirical investigation concerning to a local implementation of what it is called Study and Research Course (SRC) on population dynamics (see Barquero *et al.* 2008). As it is explained in this work, SRC are proposed as new didactical devices to teach mathematical modelling with a double purpose: to make students aware of the rationale of the mathematical contents they have to learn and to connect these contents through the study of open modelling questions. In more detail, our proposal for the teaching of modelling at university level (Barquero 2006 & Ibid.) consists in the implementation of a 'mathematical modelling workshop' that was run in parallel with the 'usual lectures' (dedicated to present the main contents of the course and exemplify them through carrying out some exercises on the blackboard). The workshop focused on the study of a population dynamics starting with the question of how to predict the evolution of the population given its size in some previous periods of time. To provide answers to this initial question and to the sequence of questions that followed it, the construction of different mathematical models was required. When studying the links between the questions and the answers provided by the models, new questions appeared that forced to broaden the previous models to more comprehensive, rich and complex ones, which made them continue with the process. At the end, this sequence of modelling activities covered most of the contents of a first-year course of mathematics for natural science students at university level.

Even though this local implementation was able to overcome some of these institutional constraints by setting up a set of suitable local conditions for the workshop[2], the large-scale implementation of such teaching proposals required the study of the real scope of these constraints in order to be able to introduce the appropriate changes at the appropriate level of didactic determination.

[2] For instance, the teacher of the workshop was a researcher in mathematics education and the teacher responsible of the course (the "lecturer") was a mathematician who does his research in mathematical modelling and was participating in the educational research project. On the other hand, the implementation was developed in an annual course where a group of only 25 students were attending it. Moreover, its program was enough flexible to change of order the introduction of most of the mathematical tools that were required by the development of the workshop.

2.2. 'Specific' constraints at the lower levels of determination

If we consider the 'low levels' of determination (see figure 1) that are specific of the teaching and learning of mathematics, three main interconnected constraints have been made clear by the workshop experiment(ed). The first constraint is located at the *thematic level* and has been studied in other didactic investigations under the name of 'thematic confinement' (Chevallard 2002, Barbé *et al*. 2005). It (comes) stems from the fact that the prevailing culture in educational institutions tends to confine the teacher's responsibilities at the strict level of the theme, without giving him/her the legitimacy to re-organise mathematical contents in a way different from the one imposed by tradition. In other words, teachers can (and have to) decide how to structure and sequence the themes, what subjects, problems and activities to include in each theme, how much time to spend on each one, etc. but they are rarely asked to decide on the choice of the themes or on the concrete division of mathematics into given domains and sectors. As has been shown by García *et al.* (2006), the problem of the disconnection of mathematical contents and the many efforts to solve it through modelling activities is related to this phenomenon of 'thematic confinement'.

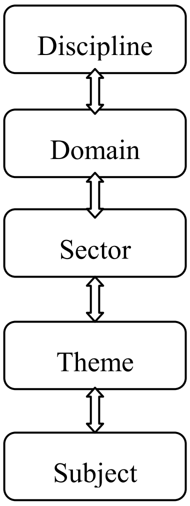

Figure 1

The second constraint is related to the concrete organisation of mathematical contents into *domains and sectors*. As we just said, the modelling activities of the workshop led the students to consider most of the contents of the mathematics course (calculus in one and several variables, basic linear algebra). However, during the workshop these contents appeared in a very different organisation from the one in the syllabus. If the lectures followed the classical 'logic of mathematical concepts', the workshop was more guided by the 'logic of the extra-mathematical questions or types of models' that progressively appeared. To be more specific, the whole modelling process was divided into three main 'stages' that correspond to the main lines of investigation followed during the workshop: the discrete evolution of populations with separate generations (discrete one-dimensional models: recurrent sequences); the discrete evolution of populations with mixed generations (discrete multi-dimensional models: transition matrices) and the continuous evolution of populations (differential equations). This forced the teachers to continuously work in a sort of 'double curriculum' project and it seems obvious that, in the long run, much more effort was needed to preserve the new organisation.

Finally, if we move to the *discipline level*, the running of the workshop showed the necessity of strongly modifying the traditional didactic contract that currently exists at universities. To carry out a modelling activity, it is necessary to break with the rigidity of the structure "theory lessons – problem lessons – exams" and to give the students some mathematical responsibilities that are usually assigned exclusively to the teacher: addressing new questions, creating hypotheses, searching and discussing different ways of looking for an answer, comparing experimental data and reality, choosing the relevant mathematical tools, criticizing the scope of the models constructed, writing and defending reports with partial or final answers, etc. Thus, the teacher had to assume a new role of acting like the director of the study process instead of lecturing the students, which highlighted that the teaching culture at university level does not offer a variety of teaching strategies for this purpose.

2.3. 'General' constraints at the upper levels of determination

When we move to the most generic levels (see figure 2), the *pedagogical constraints* appeared when it was necessary to find a suitable timetable for the workshop, with long sessions of two or three hours instead of the usual classes of 50 minutes, as well as some computers available in the class. Organising the students' work in teams, including the assessment of the teams' work and its inclusion in the individual evaluation of the course also appeared as difficult obstacles to overcome.

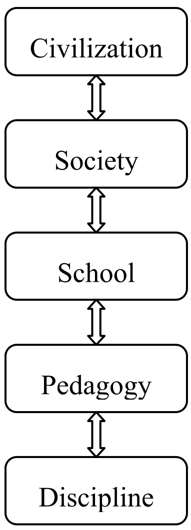

Figure 2

Considering the *society* and *school levels*, by now, we have only studied those related to the 'dominant epistemology', that is, the way our society, the university as an institution and, more concretely, the community of university teachers (and students) have to understand what mathematics is and what its relation is to natural science. Our first hypothesis is that the widespread understanding of mathematics and its relation to natural sciences is what we can call "applicationism". It may be depicted in the following way: a strict separation between mathematics and other disciplines (in particular natural sciences such as biology and geology) is established; when mathematical tools are built, they are 'applied' to solve problematic questions from other disciplines, but this application does not cause any relevant change neither for mathematics nor for the rest of disciplines where the questions to study appeared. For example, in the majority of the Spanish university courses we have examined, the study of population dynamics is a subject located in the sector of differential equations under the label of 'application', as if some dynamics laws could exist without any mathematical tool to describe it and, in the same way, as if differential equations could independently exist without any extra-mathematical problem to solve. One of the main characteristics of this dominant epistemology at university level is that it extraordinarily restricts the notion of mathematical modelling. Under its influence, modelling activity is understood and identified as a mere 'application' of previously constructed mathematical knowledge or, in the extreme, as a simple 'exemplification' of mathematical tools in some extra-mathematical contexts artificially built in advance to fit these tools. To be more concrete, the main characteristics of 'applicationism' can be described using the following indicators:

I_1*: *Mathematics is independent of other disciplines ('epistemological purification'): mathematical tools are seen as independent of extra-mathematical systems and they are applied in the same way independently of the nature of the considered system.[3]

I_2: *Basic mathematical tools are common to all scientists*: all students can follow the same introductory course in mathematics, without considering any kind of specificity depending on their speciality.

***I_3*:** *The organisation of mathematics contents follows the logic of the models* instead of being built up from considering modelling problems that arise in the different disciplines. All happens as if there were a unique way of organising mathematical contents and different ways of applying them.

***I_4*:** *Applications always come after the basic mathematical training*: the result is then a proliferation of isolated questions that have their origin in the different systems. The first thing is to learn how to manipulate the mathematical concepts and later learn about their use. The models are built from concepts, properties and theorems of each theme independently of any extra-mathematical system.

I_5: *Extra-mathematical systems could be taught without any reference to mathematical models*, that is, there exists the belief that natural science can be taught without any mathematics.[4]

To empirically contrast to what degree 'applicationism' prevails in university institutions (see Barquero *et al.* in press), we used these indicators to analyse teaching materials (syllabi, textbooks' prefaces and curricular documents) and to design an interview and a questionnaire addressed to geology and biology teachers and students of a science faculty in Catalonia. The study was developed during the years 2007 and 2008. The analysis of about 30 syllabi of mathematics for natural science courses of 10 different Spanish universities mainly confirmed I_2, I_3 and I_4. Some of the prefaces of the most recommended books for these courses helped to corroborate I_1 and I_5. A good example is the case of Salas & Hille (1995) (our translation):

> In this edition, you will find some easier applications to physics and, as extra chapters, some more difficult applications […]. Despite the incorporation of more applications, this book is still a mathematics book, not a science book or an engineering book. It is about calculus and its main basic ideas are limits, derivatives and integrals. The rest is secondary; the rest could be left out.

The interview with a sample of 8 geology and biology teachers and researchers and the answers of 30 other teachers to the questionnaire showed the following results: Related to I_1 and to I_3, up to 97% agreed that "Mathematics is introduced independently of geological or biological systems that could be modelled using

[3] This indicator is more general than the other ones as it refers to a characteristic of mathematics as a discipline and not to the way it is taught.

[4] This is an extreme indicator of the independence between mathematics and natural sciences (especially in the case of biology and geology) that is surprisingly widely shared to the point that, in most cases, people state that scientific systems could be studied without any mathematical tool.

mathematics" and that "the teaching of mathematics is more structured according to mathematical notions than to natural science problems". Related to I_4, up to 80% disagree that "mathematics is introduced only after its necessity has been shown and as a tool for the study of science problems". Finally, the most worrying fact (related to I_5) is that almost 40% agree that in natural sciences degrees, mathematics could only be used to analyse the quantitative aspects of science phenomena.

3. CONCLUSIONS

Using this "ecological" metaphor, we can say that for modelling to be able to normally 'live' in a teaching institution, it is necessary to study the conditions that facilitate and the constraints that hinder the type of mathematical activities that can be carried out in this institution. In this sense, the Anthropological Theory of the Didactic appears (as) a prioritary line of investigation to study these institutional constraints that affect the teaching and learning of mathematical modelling in current educational systems. From the ATD, the study of this "ecology" needs to take into account the different levels of didactic determination, not only to reach the variety of constraints acting on the classroom activities, but also to know better at what level – that is, in what intermediate institutions (from the 'mathematical lesson' to the 'Western civilization' in our case) it is necessary to act in order to improve the conditions that make the large-scale development of this activity possible.

In order to carry out this study, it appears necessary to provide a general model of mathematical activity that integrates mathematical modelling into the other dimensions of mathematical practices. Researchers in mathematics education have to emancipate from the dominant epistemologies that are implicitly imposed by educational institutions to which we belong. With this purpose, it is important to set out an alternative epistemological model, that is, an operative definition of what mathematics is and what the main characteristics are of the different mathematical activities that exist in our social institutions. As well as, the integration of a description of mathematical modelling within a general epistemological model of mathematics that takes into account the institutional environment of this activity.

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