MODELLING AND FORMATIVE ASSESSMENT PEDAGOGIES MEDIATING CHANGE IN ACTIONS OF TEACHERS AND LEARNERS IN MATHEMATICS CLASSROOMS

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This paper explores how modelling and associated tasks and pedagogies can bring about a refocusing of the nature of assessment which it is argued, when viewed through the lens of Cultural Historical Activity Theory, appears to currently adversely mediate the object of activity in many school mathematics classrooms. An international professional development programme in mathematical modelling has been designed with formative assessment as a key theme. Drawing on data resulting from classroom activity developed from this programme I argue that modelling undertaken with a formative assessment approach can bring about a significant change in classroom activity for learners and teachers that might better prepare students to apply mathematics.

INTRODUCTION

A Cultural Historical Activity Theory (CHAT) analysis of classroom activity suggests that in almost all classrooms, at least in England, the collective activity of teacher and students is mediated to a large extent by "rules" of assessment and "performativity" which ultimately focus on learners' qualifications. Whilst this is not necessarily clearly discernible on any particular day in any particular classroom, recent research (Williams et al, 2008) points to an all pervasive culture of "performativity". Systemic measurements of performance and accountability are seen to drive the curriculum in our classrooms to the extent that this can be detected in classroom discourse with teachers making regular reference to the demands of assessment and examiners. In terms of CHAT, assessment and performance measures are part of, and lead, the "rules" that mediate the activity of the classroom activity system with its object of learning mathematics. These rules have culturally and historically evolved affecting, for example, the texts and pedagogic instruments used by teachers and learners. They also help mould and define the expectations of what lessons in mathematics should be, in the sense of Brousseau's didactical contract (1997).

The current culture is, therefore, such that a teacher's enactment of the curriculum does not necessarily match his or her espoused beliefs about the nature of the subject they teach, and consequently how it should be taught and learnt (see for example Tobin and McRobbie (1997)). Boaler (1997) documents case studies that illustrate how, in England, this has led to a narrowing of professional practice and risk taking, leading to a normative cultural script (Wierzbicka, 1999) where many lessons comprise of an initial period of transmission by the teacher of key mathematical ideas

or rules and procedures followed by a period where students practise these. This is not only detrimental to learning where shallow or surface learning dominates at the expense of deep learning and understanding (see for example, Entwistle (1981)), but can also be responsible for a narrowing of participation. As Brown et al (2008) report this can lead to situations, when students are asked about their likely future participation in mathematics beyond the compulsory curriculum (to age 16 in the UK) to responses such as

"I hate mathematics and I would rather die."

This paper explores, how, taking a mathematical modelling approach to classroom practice that incorporates formative assessment introduces a range of new mediating instruments allowing teachers and learners to refocus their classroom actions. The work reported here resulted from classroom experiences that emanated from the work of a professional development programme as part of an EU funded Comenius project, Learning and Education in and through Modelling and Applications (LEMA). Central to the approach advocated by this programme is the focusing of classroom activity on modelling with teachers and learners becoming fully involved with formative assessment practices. An overview of the framework that guided the development of the programme in this respect is outlined in the next section before some resulting classroom experiences are described and analysed in terms of CHAT.

ASSESSMENT FOR LEARNING

Following a thorough review of research relating to assessment, Black and William (1998a) claimed that focussing on formative assessment, i.e. assessment with the purpose of informing teacher and learner about learner progression, raises student attainment. Thus the assessment *for* learning movement, as it became, conceptualised assessment as crucially providing feedback at all stages of day-to-day classroom activity and promoted this in favour of summative assessment, or assessment *of* learning, where the focus is on measuring outcomes, often being used to give grades. In follow-up studies that involved teachers and their pupils working with researchers Black and Wiliam (1998b) clarified the key areas that need to be considered if classroom assessment practices are to be effective in improving learning. These are identified in the diagram of Figure 1 and outlined below.

This emphasises an overarching pedagogic philosophy in which teachers and students strive together to ensure that, as a community, they will use their monitoring, at every stage, of the mathematical modelling taking place in their classroom to inform them of how to improve students' learning. Fundamental to this is the clarifying of learning objectives so that all know what it is they are trying to achieve.

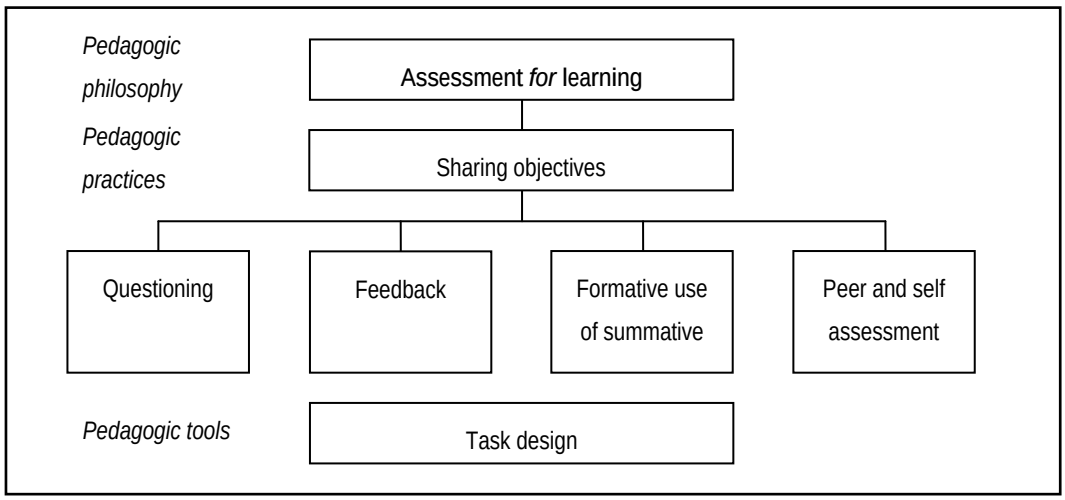


Figure 1. Schema illustrating key aspects of formative assessment

In terms of mathematical modelling this requires that students understand the overall nature and aim of modelling and the key sub-competencies they need to acquire. In supporting assessment *for* learning four key underpinning aspects of classroom activity were identified by Black and Wiliam:

(i) *Questioning*. Classroom discussion between teacher and students and between students is crucial in the learning of mathematics (see for example, Ryan and Williams, 2007) and fundamental to this are the questions that teachers pose. In summarising research in this area Tobin (1987) points to findings that suggest that the time between a teacher asking a question and intervening, perhaps to re-phrase the question, (often referred to as "wait-time"), is in many classrooms very short, and if lengthened leads to more effective learning. However, he points out that it is the quality of the question that is crucial in opening up opportunities for thinking and consequently learning.

(ii) *Feedback*. How teachers best give feedback to students to scaffold their learning (in the sense of Vygotsky) is always an issue of concern but this is possibly even more problematic when developing new pedagogic practices such as those associated with mathematical modelling. The research in this area that informed the development of good practice in formative assessment is clear in suggesting that the best feedback focuses on the task, is given immediately and is given orally rather than in writing. An important study by Butler (1988) reached the conclusion that as soon as teachers give a grade for a piece of work their comments about how to improve are ignored and that feedback that comprises of comments about how to improve instead of grades is more effective in raising student attainment.

(iii) *Formative use of summative assessment*. Much work has been done in developing ways in which such summative assessment of mathematical modelling can be carried out: see for example many of the bi-annual proceedings of the ICTMA. Whilst this has had little impact on summative assessment that leads to qualification at a national level, the frameworks and structures that have been developed may well provide suitable structures to inform formative assessment in classrooms.

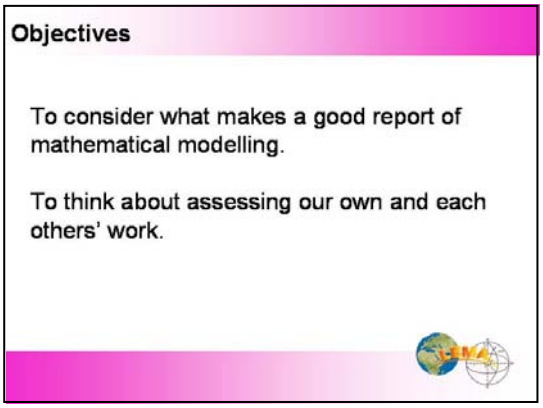
(iv) *Peer & self assessment*. Of course, learning is most effective when the learners themselves have a clear understanding of what it is they are trying to achieve, can measure their progress against clear objectives and know how to proceed to achieve their aims. Hence, the important focus on clarity of learning objectives. Peer assessment, where students assess each others' work, provides a valuable direct source of feedback for students, often using a language and given in a manner they readily understand, and also allows them to start to reflect on their own work and learning.

In addition to these important pedagogic practices one further key area that needs to be considered is the design of the tasks that are used. Here, where assessment is being refocused and considered as being an integral part of daily classroom activity, the tasks students are asked to engage with are therefore absolutely critical. If, for example, the teacher wants students to focus on their ability to interpret from mathematical model to reality, the tasks used need to be designed to allow a range of possible and appropriate interpretations to be made by the students being taught. On other occasions other particular modelling sub-competencies or meta-cognitive awareness of the modelling process as a whole may need to be the focus of attention of classroom activity, requiring tasks to be designed accordingly.

A MODELLING CLASSROOM

Here I describe some detail of a lesson that was designed to involve students in mathematical modelling incorporating formative assessment approaches. Due to restrictions of space I focus on just two aspects of the lesson particularly related to formative assessment practices: namely teacher questioning and peer assessment. The lesson was one of a sequence taught by both the teacher of the class and researcher following the teacher's partial attendance at the LEMA professional development programme in England, which the researcher had led following his work as part of the development team. The lessons were developed using materials and approaches advocated by the programme, and in the particular lesson outlined here the intention was to involve students in peer assessment as a prelude to future self assessment. The students were aged 13-14 and in an upper mathematics set in a comprehensive school catering for students of all abilities (aged 11-18), in a town in the north west of England. The teacher started the lesson by introducing its objectives (Figure 2a). The emphasis of the first objective was on the development of good communication skills about mathematical modelling rather than on the mathematics itself; additionally the remaining objective of the lesson was for students to focus on their assessment of their own modelling activity and that of their peers. Following this the teacher reminded the class of the sub-competencies of mathematical modelling to which they had previously been introduced, and which had been clarified using the schema of Figure 2b. This is based on that used as the theoretical basis of the PISA study (OECD, 2003); here it has been adapted to highlight processes that are used in developing a solution to a modelling task as the

"modeller" moves from one key stage to the next. The teacher highlighted these suggesting that the students might wish to think about them when making a poster of their "solution".



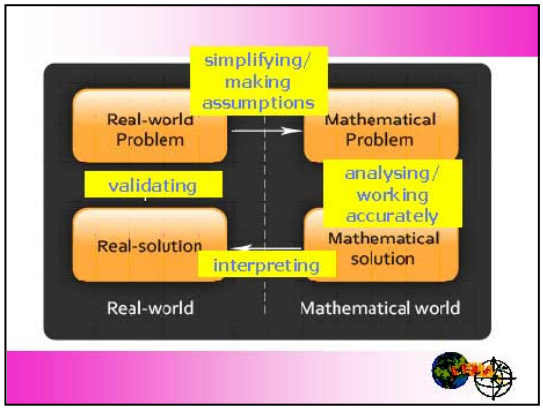


Figure 2a. Lesson objectives. **Figure 2b. Schema outlining modelling cycle**

Finally in this introduction to the lesson the teacher set the task:

> In a school playground there are two trees: one is small and one is large. There is also a straight wall.
>
> A group of pupils organise a race: each pupil starts at the small tree; then has to touch the large tree; followed by the wall; before finally running back to the small tree.
>
> Where is the best place for a pupil to touch the wall?

The pupils started to tackle the problem, working in groups of four or five: as the lesson was shorter than usual, the pupils had only about half an hour to complete their work and poster. The teacher circulated the room as the groups worked. Here I illustrate the teacher's interactions with one group. He approached their cluster of tables and discussed where they had got to.

Teacher: OK, what's your group doing?

Pupil 1: Going for the middle point of the wall (gesturing to a diagram of the situation)

Teacher: And you think that's the solution?

Pupil 1: Yeah

Teacher: How could you convince somebody that's the solution?

Pupil 1: I don't know.

Pupil 2: Does it have to be in a triangle [referring to the path taken by someone in the race]

Teacher: [reflecting the question to other members of the group] Does it have to be in a triangle?

Pupil 3: Yeah, because there are three points….

Pupil 2: Yes, that's the only way you can do it.

Teacher: Well, I suppose somebody could run

Pupil 2: If the wall was there, then they could just go like that [pointing to a sketch diagram]

Teacher: [indicating to the rest of the group Pupil 3's sketch with a section of wall lying along a straight line joining the two trees] oh right, so if the wall was there…. so the first thing you are doing is making some assumptions. So you have to say what your assumptions are: you've assumed everything is in a straight line [indicating this on Pupils 3's diagram] and you've assumed that it's like that [indicating the triangle path on Pupil 1's diagram]. What is it you actually want to….

Pupil 1: Find out where the wall is.

Teacher: Right, so at first you have to decide what the situation looks like…..

The teacher continued circulating the room encouraging groups as they worked on the problem and towards the end of the period completing their posters which explained what they had done to arrive at their solution. Following this the teacher focused the whole group on the second objective of the lesson: "To think about assessing our own and each others' work". This was "operationalised" by adopting the pedagogic practice of asking each group to consider the poster of another using pink sticky notes to identify up to 3 positive features of the poster being considered and 3 or fewer features where there could be improvements using yellow sticky notes (see Figure 3 below). As these early attempts demonstrate much of the feedback focused on issues relating to communication ("Not enough diagrams") and aesthetics ("Cool trees! [referring to drawings] and "Colourful"). In many ways this was a disappointing outcome, but this was the first time the class had been asked to take part in such formative assessment processes, and in a lesson a week later students gave slightly more attention to issues of mathematical content but there still remained room for there to be more of a focus on the processes involved.

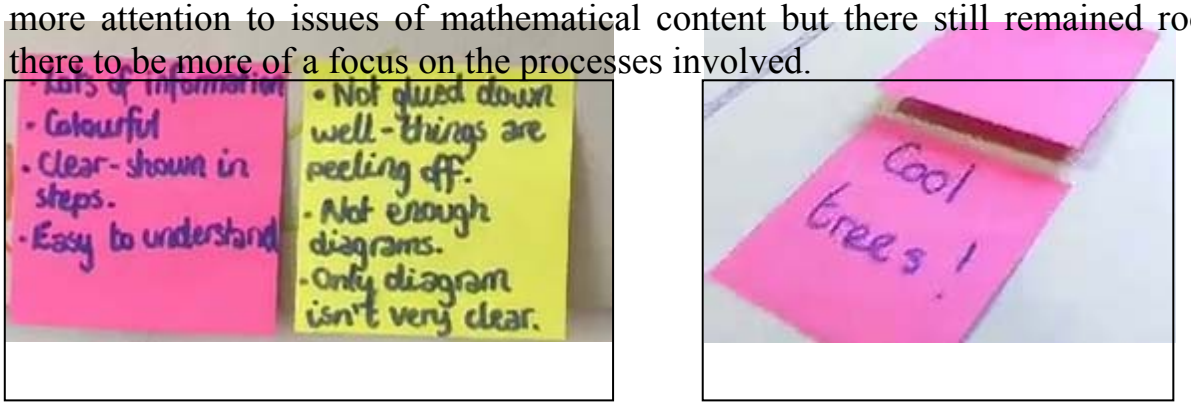


Figure 3. Peer feedback on modelling task.

DISCUSSION

In the brief extracts with which I illustrate a modelling lesson here we observe activity that is very different from the normative script of lessons that I describe earlier and which a recent nationwide inspection report corroborates as being the norm (Ofsted, 2008). Consider, for example, the interaction of the teacher with the

group of students, where the teacher prompts discussion and problem solving rather than "transmitting" rules and procedures. I now consider how Cultural Historical Activity Theory might enlighten our thinking about the nature of such lessons and highlight potential areas of conflict for teachers who attempt to follow such approaches.

CHAT builds on the fundamental thinking of Vygotsky, who suggested that the action of a subject is mediated by 'instruments' which may include artefacts and tools, or in the case of communicative action, as is often the case in classrooms, by cultural tools, concepts and language genres (see for example, Engestrom, 1995). This is indicated by the top triangle in the schema of Figure 4.

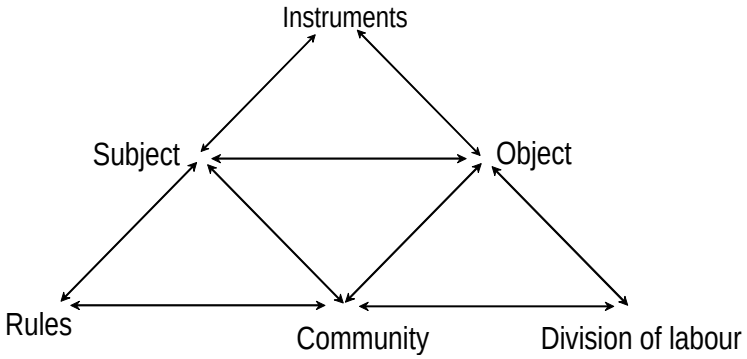


Figure 4. Schema of activity system

Leont'ev extends thinking to take account of the communal nature of activity: the schema of Figure 4 thus indicates the additional nodes of mediation in a culturally-mediated and historically-evolved Activity System. These indicate the importance of the ways in which the division of labour and associated norms/expectations/rules mediate the subject's activity in relation to the community.

I suggest that in the modelling classroom which attempts to involve formative assessment practices there is a shift in the attention of both teacher and students to view assessment in terms of informing learning and this in turn considerably alters the dynamics of the learning community. Highly visible in bringing about this refocusing are the pedagogic tools that the teacher employs. Crucial in this regard is the use of a rich modelling task, but equally important are (i) the sharing of learning objectives that in this case (at an early stage of the students' development as mathematical modellers) focus on the object of the activity (the learning of mathematics), (ii) the teacher's decision to involve groups of students in working on this, (iii) their need to develop a poster communicating their solution together with their way of working and (iv) the peer assessment activity which clearly refers back to the shared learning objectives.

Greater insight might be gained into the nature of the classroom activity by exploring further Leontev's (1978) theoretical development of Vygotsky's thinking in which he explores the nature of a subject's action in relation to the communal activity and the

manner of the operation that achieves this. He suggests three parallel hierarchies shown schematically in Figure 5.

ACTIVITY	--	ACTION	--	OPERATION
COMMUNITY	--	SUBJECT	--	INSTRUMENTS
MOTIVATION	--	GOAL	--	METHODS

Figure 5. Schema illustrating the nature of the action of a subject in relation to communal activity.

Thus in terms of classroom mathematical activity we need to understand how things are "normally" for the subject and how the modelling classroom differs from this. In both classrooms the activity has as its object the learning of mathematics: normally this is motivated for the community, as I suggest earlier, by the pressure to perform well in summative assessment and with institutional measures of performance having a major influence in defining goals related to achieving high grades in national assessments. This has led over time to a use of a restricted range of instruments: in particular, reflecting the highly structured nature of the summative assessment (Wake, 2008) the texts used involve students in practice exercises that in the main involve students in the recall and use of instrumental understanding (Skemp, 1978). Equally pedagogic practices are in general restricted with "the teacher doing most of the talking, emphasising rules and procedures rather than concepts or links with other parts of mathematics" (Ofsted, 2008 p. 20), and with teacher talk constituting "a substantial proportion of pupils' time for learning mathematics" (ibid. p. 20). Thus, the actions of teacher and student might to a large extent be considered as active and passive respectively.

In the modelling classroom, however, the introduction of new instruments (tasks and pedagogic practices) brings about a change of motivation and goals. On these occasions the motivation for teacher and learners, as encapsulated in the learning objectives of the lesson illustrated here, has been, perhaps only temporarily as I shall discuss below, re-focused on the students' learning. This alters the nature of the actions of both teacher and learners: *both* are now active with learners struggling to solve a task and make reflective judgements about their ability to do so using new rules of assessment that focus on process as opposed to outcomes. At this early stage of this class of students working on modelling the *operation*, the method by which the action is instrumentally accomplished, requires careful attention by both teacher and students. The introduction of new instruments for use by both teachers and learners destabilises their usual ways of operating, introducing new challenges for all. Thus the development provides a 'break-down' in the usual routine of the classroom activity which now becomes the focus of attention and hence conscious action. Previously we (Williams and Wake, 2007) and others (eg Hoyles et al, 2001) have recognised this in workplace activity. Here in classrooms, I propose, this as a useful

way of deliberately provoking a means of mediating changes in the *actions* of teachers and learners.

Finally, a word of warning! The developments in classroom activity arising from the LEMA programme, such as described here, are in many ways encouraging, demonstrating the potential to enrich the learning experience of students of mathematics. The claim by Black and Wiliam that a focus on formative assessment practices will ultimately lead to increased attainment in summative assessment is helpful to teachers working in a system where measurement of performance is so pervasive. However, bringing about the necessary changes in teacher and student actions involves teachers, either individually or as a collective, in considerable risk taking: when all around are following the "safe" option there is a great deal of pressure to conform to the "norm". Additionally, as Hodgen (2007) points out the simple messages often associated with "assessment for learning" are not necessarily sufficient in allowing teachers to make the shift. Perhaps programmes of professional development such as that developed by LEMA will help in this regard. However, it seems unlikely that teachers will be able to sustain developments in such a way unless summative assessment is realigned to support this. Elsewhere, (Wake et al, 2004) our research has shown that attention needs to be paid to each mediating node of an activity system if curriculum development of this sort is to be effective: in paying such attention there needs to be alignment of purpose and an awareness of how each part of the system interacts with each other.

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