MATHEMATICAL MODELLING IN TEACHER EDUCATION – EXPERIENCES FROM A MODELLING SEMINAR

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Mathematical Modelling has recently become a compulsory part of the mathematics curriculum in Germany. Hence future teachers must have a strong background about different aspects of modelling and also about appropriate methods how modelling can be taught. That means that the content and the methodology of university courses on modelling have to include all these aspects. In our paper, we will report on university seminars on modelling for students in their fourth year of study. Among other things, the students had to write a "learning diary" over the whole semester. The results give interesting insights in students' learning processes of modelling, their progress and their problems during the semester and their considerations about teaching modelling.

INTRODUCTION

Although mathematical modelling is now a compulsory part of the mathematics curriculum in Germany and one of the main competencies within the national Educational Standards, it is not at all guaranteed that pupils will be taught by teachers who have a sound knowledge of modelling. One reason for this is the fact that modelling has normally not been taught in teacher training courses at University, because modelling is not contained explicitly in the curriculum for future math teachers in Germany. However, there is no doubt (see, e.g., Krauss et al. 2008) that teachers have to be experts in modelling themselves in order to be able to teach students effectively and that their thinking has to be shaped towards creating rich classroom environments that enable students to be actively involved in modelling (Chapman 2007).

In the last few years, a lot of empirical studies have dealt with the question of how modelling can be taught in school (see, e.g., Maaß 2007 or Blum & Leiß 2007) or how students at University can be sensitized for modelling through complex modelling tasks (see Lingefjaerd & Holmquist 2007, Blomhoj & Kjeldsen 2007 or Schwarz & Kaiser 2007). The results of these studies opened new ways of thinking about modelling and the way it can be integrated in school mathematics in a profitable way. However, the question of how these aspects can be integrated in teacher education still remains open. Two of the main questions are:

1. How can future teachers be prepared in university courses for teaching modelling at school, which contents and which methods are appropriate?

2. How do students' processes of learning and understanding develop during such courses, what are their main difficulties and problems, and how can progress be observed?

In this paper, we will report on such a modelling seminar which has been taught at the University of Hamburg by the first author and with similar features at the University of Kassel by the second author, and with which we have tried to tackle these two questions. Our guiding principle for the conception of this seminar was: If we want our students to teach modelling in an appropriate way (with a correspondence between content and method, cognitive activation of pupils, reflection on learning and integration of summative assessment) we as lecturers have to conceive our own teaching in exactly the same way (correspondence between content and method, cognitive activation, reflection, summative assessment).

CONCEPTION, GOALS, CONTENT AND STRUCTURE OF THE SEMINAR

The main basis for our data collection was a modelling seminar for students in their fourth year of study at the University of Hamburg. In this course altogether 25 future teachers from all school levels were participating, including teachers for students with special needs. (The authors' experiences from other modelling seminars showed that this kind of mixture builds a good basis for discussions and is important for arguing that modelling is suitable for all kinds of school levels and types.) The course took place once a week for 90 minutes over one semester that means 14 lessons altogether. According to the meaning of a "university seminar", the students were expected to be actively involved in all activities and to cover a major part of the course by their own presentations. In the following we will describe more precisely the conception of this seminar and the way the students were observed over the semester. Mathematical Modelling as a subject in teacher education may, of course, be structured in many different ways because it is a vast field and contains a lot of important aspects (see Blum et al. 2007). In our considerations for planning and structuring a modelling seminar in a new way, the content and the methods should fit to each other. This is also a challenge for the lecturer. Concerning content, we regard the following competencies concerning modelling as particularly important:

(1) Theoretical competency (knowledge about modelling cycles, about goals/perspectives for modelling and about types of modelling tasks)

(2) Task related competency (ability to solve, analyse and create modelling tasks)

(3) Teaching competency (ability to plan and perform modelling lessons and knowledge of appropriate interventions during pupils' modelling processes)

(4) Diagnostic competency (ability to identify phases in pupils' modelling processes and to diagnose pupils' difficulties during such processes)

We did not include an "Assessment competency" (that is the ability to construct and mark tests appropriate for modelling). This competency is, of course, very important

for in-service teachers but can, in our view, not be expected from students who have not enough experience in assessment.

These four competencies were the basis for the structure of the seminar. The seminar was subdivided in the following five parts, also in order to have an appropriate balance between more theoretical and more practical phases:

Part 1 (Theory): Theoretical background about modelling (3 lessons)
Part 2 (Practice): Solving and creating modelling problems (3 lessons)
Part 3 (Theory and Practice): (1) Students analyse transcripts of pupils' work on modelling problems; (2) What are modelling competencies;* (3) Types of teacher interventions while modelling; (4) Methods how to teach modelling in school (4 lessons)
Part 4 (Presentations): Groups of students present their own modelling tasks and how pupils in school solved these tasks. (3 lessons)
Part 5: Last lesson – reflection of the whole work over the semester

*At the end of this part there was an intermediate evaluation of the seminar on the basis of a questionnaire.

One important goal of the seminar was that students do not only solve or construct modelling tasks but also learn methods how they can teach modelling. For us as lecturers it seemed important not to merely say which methods could be useful, but to integrate them directly into the work in the seminar. We decided to use teaching strategies from the field of "Cooperative Learning" (see e.g. Johnson & Johnson 1999, Kagan 1990), also because the first author had good experiences using this while teaching modelling at school. We think that Modelling as the content and Cooperative Learning as a teaching strategy fit together very well also at university seminars. Research has shown (see Johnson & Johnson 1995) that cooperative learning techniques promote pupils' learning and academic achievement, increase pupils' retention, enhance pupils' satisfaction with their learning experience, help pupils develop skills in oral communication, develop pupils' social skills, and promote pupils' self-esteem. Several studies on modelling made clear that modelling is better done as a group activity (Ikeda, Stephens & Matsuzaki 2007), also because this supports discussions about mathematics or extra-mathematical aspects, trains argumentations and gives the chance to profit from group synergy. That is why in the first lesson of the seminar the students had to build so-called "basis-groups" of five people who were supposed to work together over the whole semester; altogether there were six such groups. However, working in groups is only under certain conditions more productive than competitive and individualistic efforts. Those conditions are (Kagan 1990): Positive Interdependence, Face-to-Face-Interaction, Individual-&Group-Accountability, Interpersonal-&Small-Group Skills and Group Processing. So we had to take care that all group activities fulfil these conditions. We combined these activities with the content-parts of the seminar:

Part 1: Students had to know about different *directions* in the discussion on modelling and different *modelling cycles* (see e.g. Kaiser, Sriraman & Blomhoj 2006 and Borromeo Ferri 2006 as literature which was given to the students). They learned this content with the activity "Jigsaw": Each group member is assigned some particular material to learn and later on to teach to his group members (in this case each student had one direction of modelling as his particular topic, e.g. realistic modelling, and in the second round one version of the modelling cycle). Students with the same topic worked together in "expert-groups", so the basis-groups were divided. After working in these expert-groups, the original basis groups reformed and students taught each other. So at the end of this part the students had learned this content mostly on their own. It was, of course, important for the students that they also could ask all kinds of questions, especially in the last lesson of this part, and that we reflected both the theory and the activity Jigsaw.

Part 2 started with the question "What is a good *modelling task*?" For that we used the activity "Think-pair-share". This involves a three step cooperative structure. During the first step, individuals thought silently about a question posed by the instructor. Individuals paired up during the second step and exchanged their thoughts. In the third step, the pairs shared their responses with the entire group. After that the basis-groups solved the modelling task "Filling Up" ("Tanken", see Blum/Leiß 2007). For a better understanding we showed the students a possible solution process by means of the seven-step modelling cycle that we ourselves use in our work (Blum & Leiß 2007, Borromeo Ferri 2007), in order to help them to understand which part of their solution can be regarded as a real model or a mathematical model and so on. The six groups had then time for sharing ideas for their own modelling tasks which they had to construct and to test in school. For that "creating part" we used the method "RoundRobin Brainstorming"**:** One person of each group was appointed to be the recorder. A question or an idea was posed with many answers, and students were given time to think about the answers. After the "think time," members of the team shared responses. The recorder wrote down the various answers of the group members. The person next to the recorder started and one person after the other in the group gave an answer until time was called. At the end of this part, the groups had finished creating their modelling tasks and in addition they had learned how they could do a subject-matter analysis of the problem. Similar to the first part, we discussed questions and reflected the used methods for potential uses in school.

Part 3 contained a lot of interesting aspects of modelling. So we started each aspect with a short theoretical input and the students then had an activity on their own. Concerning aspect (1), the basis-groups worked on the transcripts of *pupils' solution processes* to the modelling task "Lighthouse", and we had a discussion afterwards especially about the distinction of the phases while modelling. Before we started with our input for aspect (2), we used the method "silent writing conversation". Every group got a big sheet of paper. In the middle of the paper they were to write "*modelling competencies*". The students had to do a brainstorming about what

modelling competencies could be, however without saying a word. So they had to comment the products of the other group members also in a written way. After that we gave information on modelling competencies and had then a discussion in the plenum, mainly about how teachers can support modelling competencies and how they can assess these in school. Like before, we started aspect (3) with an activity, this time "Inside-Outside-Circle" before we gave a theoretical input about the meaning of "*intervention*" and "*self-regulated learning*". The activity "Inside-Outside Circle" follows the principle that all students are integrated in the learning process. So the students form an inner and an outer circle. Those in the inner circle look outside, those in the outer circle look inside. Then the whole group was asked: "What do you think a teacher has to know when teaching modelling so as to be able to intervene appropriately in case of students' difficulties?" The students stood opposite to each other and discussed this question in pairs. After five minutes, the outside circle moved on and students in new pairs exchanged their thoughts. The same was done with the second question, thus addressing aspect (4): "What do you think are good or bad methods for teaching modelling?" We closed this lesson with a discussion and a reflection about the five activities of cooperative learning we had so far during the seminar and how they fit to the contents of the seminar. Simultaneously this was meant to be a meta-reflection on different levels: 1. the students had to think about each method and about teaching them in school in connection with modelling; 2. we as lecturers had to reflect whether the chosen activities were useful to teach the contents of the seminar.

In *part 4*, all groups presented their *modelling tasks* and their *experiences* they had, in the meantime, gathered in school with these. Because of the participation of future teachers for all school levels, also the presented experiences were from primary to upper secondary school. The final *part 5* rounded off the seminar with a *summary* of all aspects.

Taking into account the rather elaborate conception of the seminar, we liked to know if the students felt sufficiently well-equipped to teach modelling at school. Furthermore we were interested in students' individual learning and understanding processes and how these develop during such a course as well as in their main problems and difficulties.

METHODS OF ANALYSING LEARNING PROCESSES

Reflection was a major issue for the students in the seminar, because thinking over one's own actions generally deepens the understanding a lot. To get insight into the thinking and learning processes, the students had to write a "learning diary" (see e.g. Gallin & Ruf 1990). One important goal of a learning diary is to write down one's individual learning story. It also helps stabilising the competencies related with the contents. For the goals of our explorative study, the "learning diary" was the adequate instrument to stimulate reflections on students' own learning processes over a long

time. Interviews could have been an alternative, but not with a whole seminar. The organization of a learning diary looks mostly as follows: write down the date, the topic of the lesson and the activity; write down why you had to do the activity; look back and think about where you are in the learning process. The students in the seminar had to do this in a similar way concerning their learning of the topic of modelling. In the last five minutes of each lesson, the students had time for writing their reflections into their learning diary. At the end of the semester, all diaries were collected in order to analyse them with respect to understandings and problems referring to a) the different parts on the content, b) the methods used, c) the way how the seminar was taught, and d) the students' own reflections on teaching and learning modelling in school. So we coded (Strauss/Corbin 1990) and categorized statements of the students according to these four aspects to get an overview and to find patterns. In addition, we analysed each diary with regard to hints concerning the learning process of the individuals.

RESULTS

Most of the students knew from their first semester course a little bit about modelling and what it means, but that was only a small part of the lecture. So 18 from 21 students wrote in their reflections after the first lesson that they had not known that modelling is such a big field.

"In this lesson I got a first insight in the theme "modelling". There it became apparent for me that this theme is very wide and does not only exist of the modelling cycle I know from my first semester course." (Katrin, 2nd of April 2008)

Not unexpectedly, dealing with part 1 was not easy for the students. To distinguish between different directions and then again between different modelling cycles was a high demand for them, what the reflections clearly show. But the method Jigsaw was a helpful strategy for the students to help each other and to become more clarity about the content. Anyhow the students felt that this strong theoretical part was helpful to get appropriate background knowledge.

"Sometimes it was not easy to understand one direction of modelling in the expert-groups, because of the shortness of the text. But this method [Jigsaw] is perfect! Everyone of the group has to explain something and so we discussed till I understood it better." (Swetlana, 9th of April 2008)

A progress in the learning process of the students could be reconstructed in Part 2. All students reflected that they understood the seven-step modelling cycle finally through the modelling task "Tanken" which we presented to them in a detailed manner after they solved this problem. Furthermore they felt that now the background from part 1 will help them to create an own modelling task, so for them theory and practice linked together here.

"It was good, that we went through the modelling cycle with an exemplary task. Thereby one became aware how complex a modelling task can be [...]. Now it will be easier for us to create our own modelling task." (Sarah, 16th of April 2008)

"Slowly I understand the modelling cycle better. Working with the "Tanken"-Task helped me to distinguish several steps of the modelling cycle." (Alexander, 16th of April 2008)

When analyzing the reflections on part 2 it became very clear that creating modelling tasks is as important for learning and understanding modelling as solving modelling problems. The students had to think over the school level in which they wanted to test the problem, how complex the task should be, how much time the pupils would probably need, and so on. Helpful for them was the method used in this context.

"It was good that we were to create our own modelling task in our basis-group. However we recognized that this will be a difficult undertaking. But the method RoundRobin was exactly adequate to get helpful suggestions from other basis-groups." (Anna, 23rd of April 2008)

Thus the three lessons of part 2 were once again a linking between theory and practice for the students, and a progress in their process of understanding could be reconstructed especially concerning the modelling cycle. Furthermore they had to deal with the question of authenticity and complexity while creating their own modelling task. The students were confronted with a lot of aspects of modelling in part 3 as described above. We have no space to go more into detail here, but we try to summarize the important points. Analyzing transcripts of pupils' modelling processes in aspect (1) was helpful for the students to distinguish several modelling phases.

"The transcripts of the pupils helped me in some part to distinguish several modelling steps." (Heidy, 7th of May 2008)

Modelling competencies and beliefs were interesting for the students. Most of them liked the question of how modelling competencies could be supported. They commented that one lesson was not enough for this content and that they would like to know more about this topic.

"The silent-writing-conversation was very fruitful at the beginning concerning the meaning of modelling competencies. Of high interest for me was the question of how modelling competencies can be supported. This is especially for a teacher an important question." (Jan, 21st of May 2008)

Starting aspect (3) with the method Inside-Outside-Circle was for all students a good start for the topic of teacher interventions. Most of the students started to reflect more about themselves as a teacher personality and also liked to have more time for this topic.

"After the discussion in the Inside-Outside-Circle I think that a teacher must be well prepared when he has a modelling problem for his lesson, because he has to analyse and to diagnose his pupils quickly to help them." (Carolin 21th of May 2008)

"Today I learned a lot about different kinds of teacher interventions, firstly theoretically and then practically through group work with a case study of a teacher. But I take much more out of this lesson today: The case study showed me how invasive a teacher can intervene, so that this intervention is restricted only to the content. But I will look to myself how I intervene to correct my interventions." (Andreas 21th of May 2008)

The reflection of the methods (aspect 4) was very constructive, because the students learned the methods on their own through the seminar. So they were able to decide about advantages and disadvantages. All of the students agreed also that these methods can be integrated while teaching modelling, but they have to be practiced.

"It is good that we are learning not only modelling as a subject in this seminar, but also the methods how we can teach this at school!" (Katja 28th of May 2008)

Testing the modelling task at school and then presenting the results in part 4 was particularly important for the learning processes of the students. Whereas the processes of understanding of the students concerning modelling partly stumbled in part 3 because of the diversity of the aspects, part 4 stood for their progressives. The reflections indicated that they learned and understood more about what modelling means on a theoretical level and also how to teach it.

"Today my group and I had our presentation. I think it was good! […] Overall the testing was helpful for me as a teacher, because I could see where pupils had problems while modelling. Also to get the self-awareness to walk between the small level of intervention and reservation was important for me. Furthermore it showed me that the task should be phrased precisely and to allow enough extra time." (Benjamin 4th of June 2008)

"Testing the modelling task in grade five was important and helpful for my understanding of modelling and the practical transformation in school. […] It was good to have a chance testing modelling problems at school." (Birgit 18th of June 2008)

Summary of the results

We summarize our results concerning the two questions at the beginning. First we asked how teachers can be prepared in university courses for teaching modelling at school, which contents and which methods are appropriate. On the basis of our experiences, we are sure that in general a balance between theory and practice must be given. Both should be connected by means of an appropriate teaching strategy, which must be reflected in the seminar. Of course, the contents of such a seminar may vary, but according to our experiences, the following contents are well suited for such a seminar (see the competencies referred to at the beginning): (1) Knowledge about modelling cycles, goals/perspectives and types of tasks; (2) Solving, creating and analysing modelling tasks; (3) Planning and practising modelling lessons; (4) Diagnosing actual modelling processes of pupils.

Second we asked how do students' processes of learning and understanding develop during such courses, what are the main problems and how can progress be observed. We decided that students had to write a learning diary to help us to answer that

question, also in combination with the evaluation of the seminar. These were the main problems of the students: to understand several directions of modelling and the distinctions between modelling cycles in the literature; to distinguish phases of the modelling cycle in general and also analysing transcripts of pupils' modelling processes; subject-matter analyses of modelling problems; and finally dealing with the question of authenticity while creating a problem. Progress of the students concerning these difficulties could be reconstructed mostly when, pragmatically speaking, they linked theory with practice. Reflecting these developments during the seminar helped the students, undoubtedly, on their way to become competent teachers of mathematics.

In conclusion, we would like to emphasize once again the necessity that university students who are to become mathematics teachers must have vast opportunities to deal with mathematical modelling both on a theoretical and on a practical level, including experiences with modelling at school. This will not only contribute to preparing them to be competent teachers for mathematical modelling but will also contribute to further develop their understanding of mathematical subject matter and of mathematics as a discipline (Lingefjaerd 2007).

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