INTRODUCTION

APPLICATIONS AND MODELLING

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The red thread in the programme for working group 11, *Applications and Modelling*, was to identify and discuss different theoretical perspectives found in research on the teaching and learning of mathematical modelling. Particular emphasis was placed on the relation between research and development of practices of teaching. The presentation: *A survey of theoretical perspectives in research on teaching and learning of mathematical modelling* were given by Morten Blomhøj and Gabriele Kaiser to set the scene for the working group. And the work was ended with a closing panel discussion with Javier García, Gabriele Kaiser, and Hugh Burkhardt as panellists and Susana Carreira as moderator. In addition Hugh Burkhardt gave a historical perspective on the field by his presentation: *The challenge of integrating modelling in mathematics teaching practices – a historical view* by.

The presentation and discussion of the papers was structured according to five themes: (1) Teachers' professional development for teaching and assessing mathematical modelling, (2) The role of ICT in teaching and learning mathematical modelling, (3) Researching the teaching and learning of mathematical modelling within the Anthropological Theory of Didactics (ATD), (4) Researching the teaching and learning of mathematical modelling within the framework of Realistic Mathematics Education (RME), (5) Researching the teaching and learning of mathematical modelling under the Models and Modelling Perspective (MMP). Each theme was introduced shortly and rounded off with a general discussion. The proceedings is organised in accordance with the thematic structure of the programme and include the19 papers presented and discussed at the conference.

Theme 1 was introduced by Katja Maass and Geoff Wake and included six papers.

In the first paper, Rita Borromeo Ferri and Werner Blum present and discuss their experiences with modelling seminars as a way of integrating the teaching and learning of modelling in mathematics teacher education. As a basis for their design of the modelling seminars the authors have identified four main competencies related to the teaching of mathematical modelling: (1) Theoretical competency, (2) Task related competency, (3) Teaching competency, (4) Diagnostic competency. It is argued that mathematics teacher education should support the development of such competencies and include experiences with modelling activities in school practices.

Katja Maaß and Johannes Gurlitt write about the problem of how to evaluate teachers professional development in the teaching of mathematical modelling. Based on the authors experience from the international LEMA project the paper discusses the challenges related to the design and application of an evaluation questionnaire for teachers participating in a professional development project.

The paper by Barbara Schmidt is also related to the LEMA project. She analyses - also by means of questionnaires - the motives and obstacles experienced by the teachers for including realistic modelling activities in their teaching. According to the regulations of mathematics teaching it should include realistic modelling activities. However, different institutional and educational factors seem to form obstacles for this ambition. The findings suggest that it is possible to identify types of teachers that experience motives and obstacles for realistic modelling differently.

The paper by Jeroen Spandaw and Bert Zwaneveld reports on the development of a text book for secondary mathematics teacher education. One of its objectives is to further the coming teacher professional development for teaching mathematical modelling. The paper discusses issues such as the teachers' dispositions for modelling, educational goals for teaching modelling, design aspects, testing in modelling, the role of domain knowledge, and computer modelling. The paper also reflects on the relationship between mathematics, teaching of mathematics and modelling, and on the role of modelling in the Dutch mathematics curriculum.

The next paper is concerned with formative assessment in relation to mathematical modelling activities. Using a Cultural Historical Activity Theory perspective, Geoff Wake argues that modelling activities and related pedagogies and in particular the quest for formative assessment in relation to learners modelling processes have the potential to bring about a significant change in classroom activity for learners and teachers; and that such changes might support the learning of mathematics for more students and better prepare them to apply mathematics. This paper is also related to the LEMA project.

In the last paper of theme 1 Jonas Bergman Ärlebäck reports on a study on teachers' beliefs and affects about mathematical modelling. Five different domains of beliefs are identified as important for if and how teachers will include mathematical modelling in their teaching: (1) the nature of mathematics, (2) real world (reality), (3) problem solving, (4) school mathematics, (5) applying, and applications of, mathematics. Two teachers' beliefs are analysed according to these five domains.

Theme 2 was introduced by Morten Blomhøj and included four papers each presenting concrete cases of ICT supported modelling activities.

Maria Lucia Lo Cicero and Filippo Spagnolo in their paper report from an experimental project with three upper secondary classes that have been working with motion sensors and computers to produce graphs for different motion phenomena. From pre- and post-tests and analyses of the classroom interactions it is argued that students developed modelling competencies and that the modelling activities can enhance the students' mathematical and physical understanding of important concepts such as Cartesian graph, function, derivative, velocity and acceleration.

In the second paper Christina Roeckerath presents and analyses a simulation software package that can support the students' modelling and analyses of different types of biological interactions between species such as predator-prey, competition or parasitism. It is argued that such modelling activities can provide the students with an insight into the interdisciplinary relationship between mathematical modelling and theoretical population biology, and support their learning of biology.

Mária Lalinská and Janka Majherová discuss in their paper different aspects of visualization in relation to projectile motions modelled by secondary students using a spreadsheet and a graph drawing software. The motion of fireworks is used as a situational context to set the scene for the modelling activities and it is argued that ICT-supported modelling activities allow the students' to experience and understand better the mathematical and physical elements involved in the phenomena.

In the last paper of theme 2, Hans-Stefan Siller and Gilbert Greefrath present and analyse in detail modelling cycles in which technology is integrated by means of handheld or computer based software. The potentials in different types of software (CAS, DGS and SP) for supporting the students learning of modelling and mathematics are discussed and illustrated with the example of modelling "dangerous road intersections".

Theme 3 was introduced by Berta Barquero and Javier García and included five papers.

The first paper by Berta Barquero, Marianna Bosch and Josep Gascón introduces the metaphor of ecology and the notion of levels of didactic determination from ATD, and show theoretical constructs can be used to better understand the institutional constraints that hinder the large scale implementation of mathematical modelling activities. The theoretical ideas are exemplified through an analysis of "applicationism" - a notion used by authors to capture the set of beliefs that guides applications of mathematics in traditional mathematics teaching.

In the paper by Richard Cabassut it is argued that mathematical modelling activities can be analysed as a double didactical transposition within the framework of ATD. Real world problems and related techniques undergo a transposition when used in mathematics teaching similar to the transposition that mathematical concepts, techniques and theories undergo. This transposition process is analysed with respect to the modelling cycle, and examples of mathematisation tasks from the LEMA project are used to illustrate the elements in the transposition process.

García and Ruiz-Higueras in their paper illustrate how the ATD can be used as a theoretical framework for designing mathematical modelling activities for teaching. A design - also from the LEMA project - for 4-5 years children is presented and analysed to illustrate the theory based design process. The experiences from the

implementation of this design show how even very young pupils can be involved in rich and meaningful mathematical modelling activities.

The paper by Vázquez reports about the ATD based design of modelling activities for engineering students. The processes of transposition of the praxeologies involved in a particular modelling task – the modelling of a motor – are analysed, and it is argued that in order to understand the technologies linked to the students techniques, it is necessary to take in account the different disciplines involved.

Serrano, Bosch and Gascón in their paper analyse from a ATD perspective the mathematisation process in the modelling cycling process. A modelling task for university students on forecasting the sale of a given product from an empirical time serie is used as an example. The experinces show that a modelling activity initiated with a real-situation can lead to mathematising that affects both the system and the model and that challenge the students' modelling competency and their learning of important mathematics.

Theme 4 was introduced by Mette Andresen and included two papers and a poster by Simon Zell and Astrid Beckmann.

In the first paper Mette Andresen presents a long-term research and developmental project concerning mathematical modelling in a multidisciplinary context in upper secondary teaching. A course of lessons based on the Vioxx case is used to illustrate the different levels of reflection in the students' modelling work in this context.

The paper by Roxana Grigoras deals with the modelling of real world phenomena where no numerical data are given. In the case studied, lower secondary students' are trying to make sense out of a picture of the surface of the planet Mars. In this very open modelling activity the students use a number of fundamental mathematical ideas. The activity is analysed using RME as a theoretical framework.

Theme 5 was introduced by Nicos Mousoulides and included only one paper. Here N. Mousoulides, M. Chrysostomou, M. Pittalis and C. Christou present and discuss a case where a class of 11-years students worked with the fresh water shortage problem in Cyprus. It is a real life problem, the students' used relevant technology (Google Earth and spreadsheet) and they were in fact able to compare, judge and reflect on the different models developed. The activity was design and analysed within the framework of MMP.