ADAPTING THE KNOWLEDGE QUARTET IN THE CYPRIOT MATHEMATICS CLASSROOM

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This paper builds on the work carried out by colleagues on using an empirically-based conceptual framework, the Knowledge Quartet, as a tool for the analysis of mathematics lessons taught by preservice teachers in the UK. This framework categorises situations from classrooms where mathematical knowledge surfaces in teaching, and was used with the aim of understanding what relationship can be observed between Cypriot preservice teachers' mathematical knowledge and their teaching. In particular, in this paper I suggest that the framework needs to be supplemented in order to incorporate the interpretation of mathematics textbooks by teachers. I illustrate this by giving examples from lessons taught by participants in my study.

Key-words: Teacher Knowledge, Knowledge-Quartet, Textbook

INTRODUCTION

The object of the study discussed is based on the classic distinction by Shulman (1986) between two aspects of teachers' mathematical content knowledge, Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK). PCK includes the representations, examples and applications that teachers use in order to make the subject matter comprehensible to students. SMK consists of substantive and syntactic knowledge (Schwab, 1978). Substantive knowledge focuses on the organisation of key facts, theories, and concepts and syntactic knowledge on the processes by which theories and models are generated and established as valid.

From a variety of perspectives, research in the field of preservice teachers' knowledge focuses on their SMK and PCK. Some researchers have investigated preservice teachers' understanding of different topics in mathematics (Ball, 1990; Philippou and Christou, 1994; Rowland, Martyn, Barber and Heal, 2001) and others have focused on investigating the relationship between SMK and PCK and teaching (Rowland, Huckstep and Thwaites, 2004; Hill, Rowan and Ball, 2005) and have suggested that content knowledge might affect the process of teaching. These studies have shown that preservice teachers' substantive knowledge of mathematics was significantly better than their syntactic knowledge, and this was reflected in their teaching.

In Cyprus, concern among policy makers about students' achievement in mathematics has grown recently, and many attempts have been made to improve the instructional practices in public primary schools. Attempts of improving mathematics teaching in Cyprus have focused on learners and the curriculum, rather than focusing on teachers. Research on teacher knowledge has been neglected in the Cypriot literature. The few studies in this field (e.g. Philippou and Christou, 1994) focused on investigating aspects of Cypriot preservice teachers' substantive and syntactic knowledge of mathematics and have shown that the participants were poorly prepared to examine different mathematical concepts and procedures conceptually. However, if we want to understand better what goes into teaching mathematics effectively, the challenge is to identify the ways in which preservice teachers' knowledge of mathematics, or lack of it, is evident in their teaching. No one type of knowledge functions in isolation in teaching and thus, research in the field of teacher knowledge should focus on understanding the relationship between the different kinds of their knowledge. The identification of this relationship will help teacher educators to assess teacher preparation programmes, and to improve them where necessary. The study reported in this paper was carried out in the context of my ongoing doctoral study which is centred on understanding the relationship between Cypriot preservice teachers' SMK and PCK to teaching. In particular, the focus of this paper is on reporting results related to one of my research questions. I discuss whether the original conceptualisation of the Knowledge Quartet was relevant and adequate in the analysis of teaching in the Cypriot primary mathematics classroom.

THE STUDY

My approach to investigating the relationship between Cypriot preservice teachers' mathematical knowledge and teaching involved a mixed-methods approach. My study entailed four data collection methods. First, a questionnaire was designed to examine Cypriot preservice teachers' SMK of mathematics. 104, final year university students, following a teacher preparation programme, completed the questionnaire. It aimed to collect information about the participants' beliefs about mathematics and its teaching, and their substantive and syntactic knowledge of it. As a part of the questionnaire the participants were asked to respond to ten mathematics items that assessed their SMK. The aim of the interview questions was firstly to clarify the questionnaire data and second to gather some information about the interviewees' PCK of mathematics. The interview questions proposed two hypothetical scenarios that were relevant to teaching mathematics, representing real classroom situations which a teacher might encounter while teaching mathematics. The interview tasks provided information about what teachers know and believe about mathematics, and also about the knowledge and skills that they draw on in making teaching decisions.

While these interview tasks represented real situations in the mathematics classroom, their context remained hypothetical, and did not provide information on what teachers actually do in the classroom and how their knowledge of mathematics influences their teaching decisions in classroom where they interact with their students. This kind of information was provided by observing participants teaching mathematics in the classroom. Five of the interviewees were chosen to be observed while teaching mathematics. In Cyprus a large part of the teacher preparation programme (a four year university course) is spent in teaching in schools under the guidance of a school based mentor.

For the observations I used a framework that emerged from observing several lessons that were taught by preservice teachers in England (Rowland et al, 2004). This framework is called the *Knowledge Quartet* and is a tool that can be used in order to describe the ways in which SMK and PCK are revealed through teaching. As a part of my study I also evaluated the adaptability of the framework in the Cypriot classroom.

Finally, the data from the questionnaire, interview and observations were compared with data from the analysis of mathematics textbooks in Cyprus. Textbook analysis provided information on what policy makers consider desirable knowledge for teachers. However, what is considered desirable knowledge for teachers is often different from the knowledge that teachers use in and reveal through practice. A comparison of these two kinds of knowledge is considered to be helpful in modifying and improving teacher preparation programmes.

The combination of four methods and their integration during the interpretation phase provided strong inferences and produced a more complete understanding of the relationship between participants' content knowledge and their teaching. In the remainder of this paper I will focus on just one aspect of the study described here, and discuss issues related to the adaptability of the framework in the context of the Cypriot classroom.

THE KNOWLEDGE QUARTET

At the CERME meeting in Spain, Tim Rowland presented a paper (Rowland, Huckstep and Thwaites, 2005) about the Knowledge Quartet and suggested that this can be used as a tool for classifying ways that preservice teachers' knowledge comes into play in the classroom. At the following CERME meeting in Cyprus Fay Turner (Turner, 2007) also presented a paper about the Knowledge Quartet and explained how she is currently using the framework as a tool for professional development with a group of early career teachers.

The Knowledge Quartet consists of four dimensions, namely, Foundation, Transformation, Connection and Contingency. Foundation consists of trainees' knowledge, beliefs and understanding of mathematics. Transformation concerns knowledge-in-action as demonstrated in the act of teaching itself and it includes the kind of representation and examples used by teachers, as well as, teachers' explanations and questions asked to students. Connection includes the links made between different lessons, between different mathematical ideas and between the different parts of a lesson. It also includes the sequencing of activities for instruction, and an awareness of possible students' difficulties and obstacles with different mathematical topics and tasks. Finally, Contingency concerns teachers' readiness to respond to students' questions, to respond appropriately to students' wrong answers and to deviate for their lesson plan. In other words it concerns teachers' readiness to react to situations that are almost impossible to plan for.

Below, I argue that when adapting the framework in the Cypriot mathematics classroom, this needs to be supplemented by consideration of the use and interpretation of mathematics textbooks. I give three examples from lessons taught by participants in my study to illustrate this.

ADAPTING THE KNOWLEDGE QUARTET IN THE CONTEXT OF THE CYPRIOT CLASSROOM

When adapting the Knowledge Quartet it was not assumed that the knowledge used by Cypriot and English teachers is the same. Therefore, as part of my study I evaluated the adaptability and the validity of the Knowledge Quartet. In this section I describe the appropriateness of the Knowledge Quartet in the context of the Cypriot classroom, and explain that the framework needs to be expanded by adding a new code in the Transformation dimension.

For the most part, I found that the Knowledge Quartet could be used successfully to analyse mathematics lessons in the Cypriot mathematics classroom, in understanding how participants' SMK and PCK were related to their teaching. In particular, the issues raised for attention in lessons observed in the UK were also observed in the Cypriot mathematics classroom.

In my analysis of the lessons, I identified all the situations that I thought were significant with respect to participants' mathematical knowledge. The Knowledge Quartet proved to be comprehensive in describing most of the teaching episodes that were considered important for the purpose of my study. With reference to the 'Foundation', 'Connection' and the 'Contingency' dimensions, the codes proposed in the original study could be used to describe all the situations I thought were significant in understanding the relationship between participants' content knowledge and their teaching. For example, participants' ability to anticipate students' difficulties and obstacles, to hear and respond appropriately to students' thinking, to choose appropriate examples and representations, and to make connections between different mathematics concepts, were significant issues in understanding the ways in which their content knowledge came to play out in their teaching. In addition, issues related to participants' awareness of students' conceptions and misconceptions about a mathematical topic, their decisions about sequencing activities and exercises, or interrupting a classroom discussion to obtain clarification, or their decision to use a student's opinion to make a mathematical remark, were significant in identifying the relationship between participants' knowledge and teaching.

It was also clear from the data that Foundational knowledge underpinned the other three dimensions. In general, the application of teachers' knowledge in the classroom always rested on their Foundational knowledge, which was acquired in the academy in preparation for their role in the classroom.

On the whole the Knowledge Quartet was found to be a valid tool for analysing the lessons observed in the Cypriot classroom. However, an additional issue that proved to be significant in the analysis of my lessons was the use of mathematics textbooks, in particular how activities in the textbooks were adapted. Here, textbooks refer both to students' book and the teachers' guide. In the original study a code 'adherence to textbooks' was classified in the Foundation dimension of the framework. This code was used to describe episodes where teachers accepted textbook as authority for what and how to teach. However, the ways in which teachers adapted textbook activities are not addressed in any of the existing publications about the use of the Knowledge Quartet as a tool for observing mathematics lessons in the UK. This is not surprising, since the use of textbooks is not a common practice in the English primary school mathematics classroom. In contrast, the textbook is central and always present in the mathematics classroom in Cyprus.

All the participants in my study considered the textbook as the main resource both for their planning and teaching. However, they all combined it with other resources, and included their own developed activities. The participants adapted the textbooks in very different ways. For example, there were cases where participants modified the textbook material in ways that made the lesson more meaningful and interesting for their students. However, in some instances participants were not sure how to adapt the textbook activities appropriately, modifying them in ways that altered their focus. This suggested that the ways in which preservice teachers used the textbooks was important in understanding how their knowledge came into play in their teaching.

The above led me to conclude that when adapting the Knowledge Quartet for observing lessons in Cyprus, and indeed in many other countries, there is a need to take careful account of these differences. Thus, issues related to the adaptation, modification, and interpretation of the textbook material are important in analysing a mathematics lesson in Cyprus. Having presented the appropriateness of the dimensions of the Knowledge Quartet in the context of the Cypriot classroom, I provide some examples from the lessons observed to demonstrate how the participants in my study used the textbook activities.

ADAPTING THE TEXTBOOKS: SOME EXAMPLES FROM THREE PARTICIPANTS

The lessons observed took place during the students' placements in school. These lessons were analysed using the four dimensions of the Knowledge Quartet. In this section, I give some examples related to how three participants (Rita, Elsa and Christiana) used the mathematics textbooks. Christiana chose to do additional courses in mathematics in her undergraduate teacher education course, and was classified in the group with a 'high' SMK score (this was assessed in the questionnaire, see page 2). Elsa was classified in the group with 'low' SMK score and Rita in the group with 'medium' SMK score. Neither of them chose to do additional courses in mathematics during their training. In general, the results showed the positive influence of strong

SMK in the effective use of textbooks. Christiana elaborated upon the textbook in ways that made her lesson more meaningful and interesting for students. She was able to drawn on her own understanding and use appropriately textbook activities and extends them to promote students' conceptual understanding. In contrast, Rita and Elsa seemed to have problems in understanding the textbook suggestions due to their lack of SMK. In many instances they could not understand the mathematics targeted by textbook activities, and so could not make much of them. Therefore, it becomes clear that in order to use textbook activities appropriately, teachers need to understand their content.

Not understanding the mathematics targeted by the textbook

Rita's lesson on multiplication by four offers an example of how she interpreted one of the activities in the textbook in ways that altered its focus. Figure 1 illustrates this activity.

Mr Michalis has recently opened a new restaurant. He has50 square tables in the restaurant. Each table can seat 4 customers. On Sunday night 36 customers went for dinner. By 23:00 half of them had left. One hour later all the other customers left and the restaurant closed.

1. How many tables does the restaurant have?
2. How many tables remained empty on Sunday night?
3. How many customers were in the restaurant just after 23:00?
4. Show on the clock the time that the restaurant closed.
5. On Monday ten friends went to the restaurant for lunch. Mr. Michalis needed to put tables together so that ten friends could sit next to each other. How many tables were needed?

Figure 1 Textbook Activity (2nd Grade, Students' Book, Part B, p.87)

In addition, in the teachers' guide it was clearly stated that:

> intentionally some information is not given […] students should think of all the possible answers to the questions asked, taking into consideration that each table can seat 1,2,3 or 4 customers (Grade B, Teachers' Guide, p. 103)

Rita seemed not to take into consideration what was suggested in the teachers' guide. She used a rather 'traditional' approach in solving the problem. She read the problem to her students, and did not leave them much time to think, before leading them towards the answers. More importantly, when dealing with question two of the problem she seemed to take for granted that exactly four customers were sitting at each table and said:

> 36 customers were in the restaurant. There were four people at a table. Thus, 36 divided by 4 will give us the number of tables that were full.

Rita's approach to solving the problem focused on procedures, required a single answer, and focused on relatively few skills. However, the focus of the problem was meant to provide students with the opportunity to explore a number of possible solutions. Rita showed a desire to develop conceptual understanding in several instances in her lessons, however, it seems that in this case her beliefs about good mathematics teaching could not be implemented because she did not understand the problem solving intention. I can infer from my post-observation discussion with Rita that she changed the focus on the activity due to her lack of understanding. In this discussion I asked Rita if she could think of an alternative way of solving the problem and she was adamant that she could not. Her answer suggested that she might not have read the teachers' guide. However, the aims that were proposed in her lesson plan were exactly the same as those proposed in the teachers' guide, so it seems that she did read the guide, but that her reading was superficial, and for some reason she missed some of the information provided. It could be argued that she followed the teachers' guide rather mechanically, moving through activities without understanding their focus. In this case her problems in understanding the teaching suggestions in the guide might stem from insufficient understanding of the problem.

Another example, of not understanding the suggestions in the textbook occurred in Elsa's lesson on the parts of a circle. In this lesson Elsa tried to define the different parts of a circle. Table 1 shows the definitions that she proposed alongside the definitions that were suggested by the teachers' guide.

The definitions that Elsa gave to her students were mathematically incorrect. Even though she used the activities proposed in the textbooks she did not use the suggested definitions. It seemed that her understanding of the different parts of a circle is limited. Below I provide an extract from our post-observation discussion to support my argument:

> Elsa: Generally, I think that everything went well. However, my impression is that students were confused about the chords.
>
> MP: What do you think confused them?
>
> Elsa: Uh, I think that the definition of a chord is confusing itself. To be honest, I am confused myself. On the one hand, according to the definition provided in the textbook, a chord does not pass through the centre. On the other hand, the teachers' guide mentions that the diameter is the biggest chord. I think this is very confusing.

The extract above indicates that Elsa's understanding of the parts of a circle was limited. She seemed not to be aware of the correct definitions of different parts of a circle, and, due to her limited understanding, was unable to follow the suggestions included in the textbook. It was likely that Elsa chose not to use the definitions as suggested in the textbook because she believed that these were too difficult for her students. In trying to make these easier for her students, she made it more difficult.

	Elsa's definitions	**Definitions suggested by teacher's guide**
Diameter	Is a straight line that starts from the beginning* of the circle and reaches the end of the circle passing through its centre	Each chord that passes though the centre of the circle. A straight line passing through the centre of a circle and connecting two points on the circumference
Radius	It is a line that starts from the centre and reaches the end of the circle	A straight line segment connecting the centre of the circle with a point on the circumference
Chord	Is a line that starts from the beginning of the circle and reaches the end but does not pass through the centre	A straight line segment connecting two point on the circumference
Circumference	The 'round -round' * of a circle	Not included

* This is the exact translation for Elsa's definition from Greek, which in effect means the boundary of a circle

Table 1: Defining the parts of a circle

In general, in mathematics definitions should be inclusive. However, Elsa's definition of the chord was exclusive. Her statement 'does not pass through the centre' excludes the diameter which indeed is a chord. In contrast the definition of the chord in the teachers' guide was inclusive. In addition, it was clearly stated that the diameter is the biggest chord. Therefore, it can be argued that her problem in understanding the definition proposed in the textbook stemmed from her limited understanding of the topic. This was indicated by her tendency to refer to the 'beginning' and the 'end' of a circle, meaning points on the circumference.

Elaboration upon the textbook: making activities more meaningful and interesting for students.

An example of developing the textbook material is offered by Christiana's activity illustrated in Figure 2. The version of the activity as proposed in the students' book is also presented. Both activities have been translated from Greek. It is clear that in her modified version of the textbook activity Christiana put emphasis on developing students' conceptual understanding. I consider Christiana's version to be an improvement because she elaborated on the textbook activity in a way that made it more meaningful to her students, by helping them to explore division and multiplication as reverses operations.

A FACTORY PRODUCING JAM

The students in Philippos' class visited a factory producing jam. The jam was bottled and then packed into large boxes. Each box could hold 50 bottles. On that day the production was 9250 jars of jam. How many boxes were needed for packing the jars? The table below shows the production of jam for each day of the week. Fill in the information in the table provided

Days	Jars for each day	Jars in each box	Number of boxes
Monday	24 500	50	
Tuesday	18 900	50	
Wednesday	11 750	50	
Thursday	21 600	50	
Friday	12 600	50	

Textbook activity (4th grade, Students' book, Part C, page 33)

Christiana modified the activity and asked her students to fill in the information in the table presented below

Days	Jars for each day	First filling		Second filling	
		Number of jars in each box	Number of boxes	Number of jars in each box	Number of boxes
Monday	24 500	50		100	
Tuesday	18 900	50		100	
Wednesday	11 750	50		100	
Thursday	21 600	50		100	
Friday	12 600	50		100	

Then the students were asked to write down their observations relating to the numbers of boxes needed for the first filling and the second filling

Figure 2: Elaborating textbook activities

CONCLUSION

In general the Knowledge Quartet was comprehensive in the classification of teaching situations in which participants' mathematical knowledge surfaces in teaching. Issues related to the interpretation of textbooks were not addressed by the framework, however were important in analysing mathematics lessons in a Cypriot classroom. This suggests that when adapting the Knowledge Quartet for observing lessons in Cyprus, and indeed in many other countries, there is a need to take careful account of possible differences between the context in which the framework was originally developed, and the context in which this is applied.

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