COGNITIVE TRANSFORMATION IN PROFESSIONAL DEVELOPMENT: SOME CASE STUDIES

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Professional development programmes for in-service teachers constitute a complex task. We intend here to shed some light on the conditions that may entail a cognitive transformation in the involved teachers, building on our personal experience in these programmes and some case studies.

Keywords: Professional development, in-service teachers, metaphors, cognitive modes.

INTRODUCTION

In this paper I report on some didactic phenomena (in the sense of Margolinas, 1998) arising in our work in professional development for in-service primary teachers, at the University of Chile. These phenomena are related to the cognitive transformations that emerge in the being of the involved teachers, as well as researchers, under favourable circumstances, depending on "the time, the place and the people" (see Mason, 1998). Our work could be described as "theory-guided bricolage" in developmental research (Gravemeijer, 1998; Freudenthal, 1991), with the caveat that a detailed theory is not put forth first, because it rather grows out of the ongoing process. This approach to professional development or enhancement for in-service teachers is inspired by my former research on the fundamental role of metaphors and cognitive modes in the teaching-learning process (Soto-Andrade 2006, 2007). It involves "researching from the inside" (Mason, 1998), and it requires an embodied first-person approach (Varela, Thomson & Rosch, 1991), in an enactive perspective (Masciotra, Roth & Morel, 2007).

After recalling the fundamental components of a tentative theoretical framework, I set down below my main research hypotheses and proceed to report on some concrete examples of activities and germs of didactical situations (Brousseau, 1998), involving metaphors and switches in cognitive modes, that we have worked out with teachers. Translated quotes of several teachers' testimonies and reports are also included, as case studies. These give preliminary experimental evidence to support our hypotheses

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and suggest further research along these lines.

THEORETICAL FRAMEWORK

Nature and Role of Metaphors

It has been progressively recognized during the last decade (English, 1997; Lakoff & Núñez, 2000; Presmeg, 1997; Sfard 1994, and many others) that metaphors in mathematics education are not just rhetorical devices, but powerful cognitive tools that help us to build or grasp new concepts, as well as to solve problems in an efficient and friendly way (Soto-Andrade, 2006). We use conceptual metaphors (Lakoff & Núñez, 2000), that appear as inference preserving mappings going "upwards" from a source domain into a target domain, enabling us to understand the latter, usually more abstract and opaque, taking a foothold in the former, more down-to-earth and transparent in terms of our previous cognitive history. Metaphors are "met-befores", as Tall (2005) says.

Cognitive modes

A cognitive mode is defined nowadays as one's preferred way to think, perceive and recall, in short, to cognize. It shows up, for instance, when trying to solve problems. Flessas and Lussier (2005) gave a first operational description of what they call the 4 basic cognitive modes ("styles cognitifs" in French), combining 2 dichotomies: verbal – non verbal and sequential – non sequential (or simultaneous), closely related to the left – right brain hemisphere dichotomy and to the frontal – occipital dichotomy. We so obtain the *sequential-verbal, sequential-non verbal, non sequential-verbal* and *non sequential – non verbal* cognitive modes. They emphasize that effective teaching of a group of students, who may display a high degree of cognitive diversity, requires teachers supple enough to tune fluently to the different cognitive modes of the students.

An example: check that you have the same number of fingers in your hands by using the 4 basic cognitive modes (see Soto-Andrade, 2007, for more examples).

In what follows I adhere mainly to the framework laid by Lakoff & Núñez (2000), Presmeg (1997) and Sfard (1994, 1998) for metaphors, Flessas & Lussier (2005) for cognitive modes, Brousseau (1998) for didactical situations and to the research paradigms of Mason (1998), Varela et al., (1991) and Masciotra *et al*. (2007).

PROBLEMATICS

Professional development and enhancement for in-service teachers is a complex issue. In Chile, significant funding and human resources have been invested by the Ministry of Education, for more than two decades, to address this issue, but results have been rather scanty. Our students continue to perform poorly in international assessment tests like TIMMS or PISA, and also in national assessment tests like SIMCE [1]. Increasing evidence shows that after a typical 2 week intensive summer workshop, where they learn some more mathematics and design a couple of teaching modules, most teachers revert to their former inadequate teaching practices.

Under closer scrutiny, we have observed that most of our in-service primary teachers are unfamiliar with metaphors and cognitive modes, or visualization, in their practice. They are "frozen" in the verbal - sequential cognitive mode, unaware of this and also of the fact that their teaching is shaped by unconscious and misleading metaphors, like the acquisition metaphor (Sfard, 1998) or the container-filling or gastronomic metaphor (Soto-Andrade, 2006). They have special trouble in creating "unlocking metaphors" for the not specially gifted.

The urgent question is: How to promote a real change in the teaching practices of in-service teachers, in the short or mid term?

RESEARCH HYPOTHESES

Our main research hypothesis is that metaphors and cognitive modes are key ingredients in a meaningful teaching-learning process. Moreover the deepest impact on this process is usually attained by metaphors that involve a switch from one cognitive mode to another.

We claim that competences regarding multi-modal cognition and use and creation of metaphors and representations are trainable and that measurable progress can be achieved in a one semester course. This, in spite of the fact that most of our teachers report that their initial training included no use of metaphors and privileged just one cognitive mode: the usually dominant verbal-sequential one.

We hypothesize that explicit work on metaphors and transits between cognitive modes will foster teacher's deep understanding of elementary mathematics. Furthermore, it will affect their professional practice in the classroom, in particular enabling average students to understand and handle mathematical objects and processes that would otherwise be within reach of only a happy few.

RESEARCH BACKGROUND AND METHODOLOGY

The background for our experimental research consisted in 5 classes (called "generations" in what follows) of in-service primary school teachers, of 30 teachers each, enrolled in a professional development programme, implemented by the University of Chile, on behalf of the National Ministry of Education, stretching from 2006 to 2008. This programme aims at "general" primary teachers, who are interested in enhancing their mathematical training, and certifies their mathematical proficiency after a 15 months period, where they must complete the requirements for 4 modules (numbers and data processing, geometry, ICT in education and problem-solving, 450 hours in all). They must also complete a 75 hour Seminar Project, which includes experimenting and theory-driven practice in the classroom.

Teachers applied for admission to this programme, with the support of their schools, and were selected according to their performance in a TIMMS like test, based mainly on mathematical contents pertaining to the curriculum of primary school. Selected teachers are usually highly motivated; they come to the University after hours, typically from 6 PM to 9:15 PM, at least twice a week, plus an intensive 2 week summer workshop. Gender distribution is 90% female, 10% male, on the average. Ages range from 25 to 60, even 68 in one case (see below).

This sort of programme opens up hitherto unknown possibilities for deeper work with teachers. In particular, as coordinator for the Numbers Module (160 hrs approx.) and advisor to the seminar projects of 6 teachers in each generation on the average, I had the opportunity to test several activities and a-didactical situations in work sessions with the teachers. This module aims mainly at reviewing the mathematics as well as the didactics of numbers, specially elementary integer arithmetic, fractions, ratios, decimal and binary description of numbers. Work sessions were interactive, with teachers usually working in small groups of four on the average.

The underlying idea for this module was to open up the opportunity for the teachers to have a first hand experience of problematic and challenging situations to be tackled, eventually "bare handed", where important mathematical objects or processes could emerge. So their experience would be an antidote to the usual cookbook recipe approach. Methodology consisted in observing the teachers, as they carried out various activities, with non intrusive guidance and support, recording their reactions, in video in some cases, and asking them to write reports on their work, besides communicating it orally to the whole class. After completion of the programme, I asked them to write a short report in the first person on their cognitive and affective experience, in the spirit of "researching from the inside" (Mason, 1998).

My viewpoint was that just recording contents taught plus results of post-tests administered to teachers provides a rather shallow understanding of their learning process. Instead, I tried to foster group work, monitoring the course of their work during sessions, by circulating and interacting with the groups, as a means to fathom their cognitive profiles and processes. This was complemented with the results of tests and challenges. The first-person report mentioned above also provided further insights into the process they had undergone. So my approach relies mainly in case studies rather than hard statistical evidence, emphasizing qualitative rather than quantitative assessment (see below however quotes on SIMCE [1] scores)

EXPERIMENTAL ACTIVITIES AND PRELIMINARY RESULTS

I comment here on some concrete albeit paradigmatic examples of the activities carried out, together with excerpts of the teacher's reactions to them.

Example 0: Do you have an innate approximate number sense?

To make them feel the contrast between verbal-sequential and non-verbal non-sequential cognitive modes, we began with some experiments aiming at activating their innate approximate number sense or "numerosity" in the sense of Dehaene (1997), Lakoff and Núñez (2000), Pica *et al*. (2004), Halberda, Mazzoco and Feigenson (2008). For instance, they were asked to tell whether there were more yellow dots or blue dots in a random array of dots of both colours shown just for 200 ms (Testing your Approximate Number Sense, 2008). Our fifth generation of teachers scored here an impressive average of 95%, much higher than the statistical average success of only 75% (as it was the case in a class of average Master in Science students in our Faculty). This suggests that primary school teachers tend to have a significantly better approximate number sense than random adults.

Example 1: How to keep track of your lamas?

An 8 year old aymara shepherd is in charge of a herd of lamas (more than 40, it seems) at some barren place in the highlands in the north of Chile. But he is tired and would like to take a nap… How could he check that when he wakes up there are no lamas missing? He has no palm device, no paper and pencil, not even small stones, or sticks or a knife; just his bare hands. Moreover he does not know how to count calling numbers by their name. How could he manage to register the number of lamas in sight before going asleep and to recover it when waking up?

Every generation of teachers engaged in group work, in groups of 4 to 5, to discuss how to tackle the problem. As a supporting aid, we simulated the lamas with a bunch of coins on the plate of an overhead projector. Usually, after half an hour or so, in one or two groups, the idea emerged of using the phalanges of their fingers, thumb excluded. The idea spread quickly and finally all groups rediscovered the classical method of non-verbal counting by dozens still used in the Middle East and Far East, where you touch with your right thumb the 12 phalanges of your right hand, say, one by one, and fold one finger in your left hand to register each complete round of 12 (Ifrah, 2005, p. 74). Most did that from little finger to index, but some did it from proximal to distal phalanges (the classical way) and others, the other way around. So they learned how to count non-verbally up to 60, using their fingers and they applied this successfully to the simulated herd of lamas on the overhead projector. They also related this with the ubiquitous emergence of the dozen and 60 in human cultures.

This example may be looked upon as an implementation of realistic mathematics education (Gravemeijer, 2007; Freudenthal, 1991). The underlying hypothesis and motivation for this activity is that it is important to practice and get the feeling of non-verbal arithmetic before engaging into classical arithmetic. So our idea was to prompt the teachers to go back to the non-verbal sequential mode in the context of counting. Their reactions to this sort of activity were stronger than expected:

> My (programme) experience was totally significant in the most strict sense of the expression. It brought to me important changes in my way to approach lessons, in my professional practice and personal interests. But not everything was a "rose garden"… After the first lessons I was quite disappointed, because this course didn't make any sense to me. My expectations were to learn "more mathematics", fill in my gaps and not to debate endlessly about why, what for and how. I was even more disappointed with the Numbers Module, with metaphors! I didn't understand anything: I expected to solve hard arithmetical problems, to design endless exercise lists to calculate with fractions or decimals, to learn more and better algorithms, and it turned out that we were exposed to questions I had never asked myself: How do indigenes in the Amazonas do arithmetic, although they have no language for numbers? How can a shepherd boy know how many

> lamas he has if he doesn't know how to count? How could you teach counting to a little child, in a clever way? There, I had a cognitive break: I asked our teacher for an explanation of the aim of his lessons (I am now ashamed about that) and he kindly explained to me what he was after… (Evelyn, 32, 8 years of practice, 1st generation).

Example 2: Who has more marbles?

John and Mary have a bag of marbles each, all of the same size. How can they tell who has more marbles?

I invited the teachers, organized in small groups (3 to 4 each), to figure out other approaches than the usual sequential-verbal one (counting the marbles in each bag). Usually in less than half an hour they found at least one procedure for each cognitive mode (Soto-Andrade, 2007). The two pan balance for the non-verbal non-sequential mode emerged easily; also the idea of pairing off the marbles, without counting them, for the non-verbal sequential mode. Verbal - non sequential approaches took longer to appear (weighing simultaneously both bags in digital scales and reading off…).

Example 3. Registering quantities with dice.

The indigenes in an Amazonian village want to keep track of the quantities of seeds stocked for next year. How could they register quantities up to thousands if they have just a handful of dice at hand and they have not invented zero yet?

After half an hour work on the average, in small groups, the teachers find out, and begin even to do arithmetic in dice-system! They report to understand now much better the decimal system and try this activity with their pupils, with encouraging results. Among others, Gina (49, 25 years of practice, 4th generation) reported:

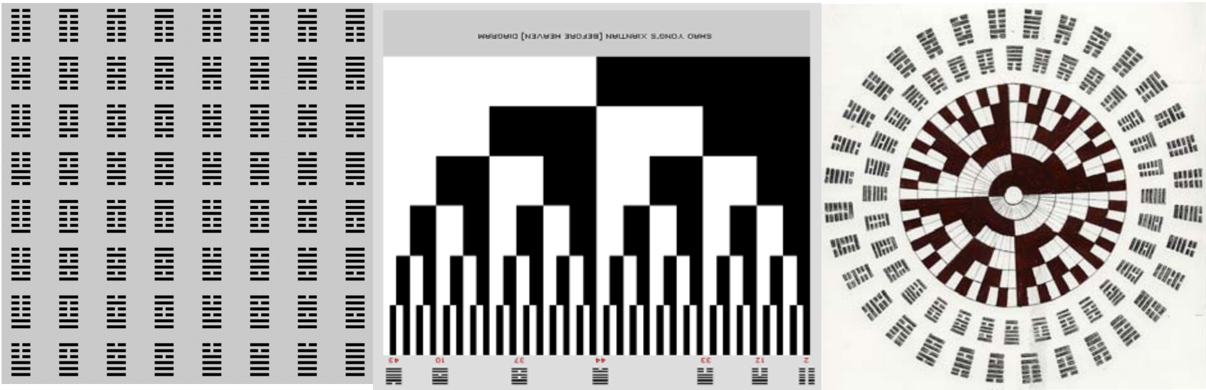
> This experience was very important to me, because you were able to "un-structure" my mind and take away my fear of numbers. Now I see that this fear came from a dull teaching, full of cookbook recipes, that never gave me the opportunity to enjoy discovering the way to solve problems all by myself. Numbers was my favourite subject in this programme, it allowed me to fly, to play, to err and not to feel silly…

Example 4: The number sequence, otherwise…

Is it possible to represent the numerical sequence 0, 1, 2, 3, …. up to 63, let us say, in a non verbal and non sequential way?

Teachers usually get to the point of discovering the given sequence, written in binary way, in Shao Yong's square (below left), and then of encapsulating it in a single image. (Soto-Andrade, 2007). In the first generation, 5 out of 30 teachers, after 30 minutes work in small groups, came up with diagrams equivalent to Shao Yong's Xiantian ("Before Heaven") or its inverted form (shown below, center, as in Marshall, 2006). Notice the underlying binary tree! In the 2nd generation, 6 out of 30 teachers, rediscovered Xiantian and, most remarkably, one of them, Ofelia (68, 50 years of practice), draw all by herself a circular version of Xiantian diagram (below right). In her own words:

> The Numbers Module shattered all my schemes. For the first time, my brain, archi-structured for algorithmic work, began to have a glimpse of a tiny light (showing the way) to working metaphorically, to solving a problem in different ways, to looking for different paths to reach the same target, not just be satisfied because I got there. I must confess that during the first weeks I was not able to fathom where we were heading to! When I first met a sequence of I Ching hexagrams, sincerely I was barely able to tell what I was looking at! So I never imagined that some weeks later I was going to be able to rediscover one the oldest binary trees, Shao-Yong's circular Xiantian. Later I spent hours trying to solve problems using different cognitive modes…



Here the teachers have the possibility of transiting from the usual verbal sequential mode (the given sequence) to the non-verbal sequential mode (iconic hexagram binary representation) and then to non-verbal non-sequential mode (Xiantian). When interviewed, they unanimously reported having understood, in this unexpected way, for the first time the binary description of numbers.

Example 4. Brownie's walk

Random walks provide a nice way to introduce probabilities. Instead of the well known drunkard's walk, we introduced to teachers with no previous training in probability a puppy called Brownie (a baby incarnation of Brownian motion), who escapes randomly from her home in the city when she smells the shampoo her master intends to give her. The stepwise description of her random walk can be tackled by rudimentary means, even by simulation, or with the help of efficient metaphors, like the Solomonic metaphor or the pedestrian metaphor (Soto-Andrade, 2006). In the first one, Brownie splits into 4 pieces, each going to each cardinal direction, and so on… In the second one, a pack of Brownies (a power of 4 preferably) runs away from home, dividing themselves equally into four packs at each corner, and so on… The latter has the virtue of allowing the teachers to work with natural frequencies, in the sense of Hoffrage, Gigerenzer, Krauss & Martignon (2002), avoiding fractions up to the last minute. We have here also an integrative problematic situation, involving geometry, arithmetic and algebra, besides randomness.

After engaging in activities of this sort, teachers reported:

> Cognitive metaphors simply surprised and fascinated me. I had learned with the traditional, mechanical system, and in that way I was teaching my students. Now, I learned about cognitive modes, how to reach every one of my students, and how, with the help of a metaphor, I succeeded in making mathematics closer, friendlier and more reachable. I got so convinced that I chose Numbers for my Seminar Project and I modified radically my professional practices. I wanted to prove that metaphors and these new approaches would give good results, not just for the emotional atmosphere in the classroom but also for "hard" tests. And indeed, my K-4 2007 class got the *first place in the country*, in the SIMCE assessment test [1], increasing by 25 points the previous score, up to 328 points, with no previous training for the test! (Evelyn, 32, 8 years of practice).

> I took advantage of this way of working to carry several activities to my classroom, using various metaphors, which made the students enjoy more my lessons, learning more easily. I transferred all this to my pupils. And this year 2007, our K-4 classes, taught by my colleague Lily (also a student in this programme) and myself increased dramatically their SIMCE score [1], from 281 to 304 points (former SIMCE scores for this grade, since 2002, were 287 and 282). This happened with no special training for the test, contrary to the case of many other schools; the students had just the regular lessons with us (Gina, 49, 25 years of practice).

> I had certain expectations: this program would deliver knowledge to me, besides methodologies to apply to my pupils. But you broke my schemes. What I expected did not happen. What you achieved was to take me out from my "pigeonholing" and to make me think further. If we as teachers are rigid and un-imaginative, hardly will we be able to have our pupils free their imagination and become enchanted with mathematics. This is badly needed, that's why they reject maths so much. I have questioned my way of interacting with my pupils and the way of structuring my lessons (Karem, 32, 6 years of practice, 4th generation).

CONCLUSIONS AND DISCUSSION

Observation of the teacher's performance shows that even those who never had this sort of experience before were able to activate less usual cognitive modes, to transit from one to another and to take advantage of new metaphors to understand better and to efficiently solve problematic situations. In particular, after some prompting, a high percentage of them were able to switch from their dominant verbal-sequential cognitive mode to a non-verbal or non-sequential one. These findings support our optimistic hypothesis that *cognitive flexibility*, i.e. the ability to approach the same object through various cognitive modes and transiting from one cognitive mode to other, is trainable, even for in-service teachers and that it is facilitated by group work.

However, their first person reports suggest that we had sub-estimated the magnitude of the cognitive shock they experience during the first weeks of our programme. It is interesting to note that testimonies of older and younger teachers are surprisingly alike in this respect. The same holds for their reactions thereafter and changes in their professional practice, as reported above. As a typical example, we recall a 50 year old teacher, Yihecika, from our 3d generation, saying at his final Seminar presentation: "I am very moved, because I am an old teacher doing new things!". At least in the case of these primary teachers, this disproves the hypothesis that changes in cognition and professional practice are out of reach for older teachers.

A rather unexpected outcome of the work carried out with our in-service teachers is the dramatic improvement of their student performance, in several cases, in traditional standardized multiple-choice tests like SIMCE [1]. We may notice that the *relative* improvement was approximately the same for Evelyn and Gina (25 and 23 points resp.) albeit *absolute* scores differed noticeably (328 and 304 resp.), as it is on the average the case between fully private schools and state supported private schools in Chile. Although our programme is intended for teachers in service at state-owned or state supported private schools, Evelyn has been teaching at a fully private high income school for 2 years because she was fired from her previous teaching job at a state supported private school right after completing her professional development programme (as it is the case of roughly 10% of our teachers!). On the other hand, Gina teaches in a low income state supported private school whose explicit aim in mathematics was to reach sometime the threshold of 300 points.

In conclusion, we have gathered some new positive experimental evidence related to this "theory-oriented bricolage", that appears to entail significant cognitive transformations in the being of the teachers (Mason, 1998) and as a consequence, changes in their classroom practice and performance of their students, even measured in traditional ways.

1. SIMCE is a national assessment test, applied to K-4 every year and to K-8 every two years. It is much closer in spirit to TIMMS than to PISA. SIMCE national average score in mathematics for K-4 stagnates at 246 in 2006 and 248 in 2007. Standard deviation is about 50 points. In mathematics only 26% of the students attained the advanced level, whose threshold is 286 points.

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