

# KATE'S CONCEPTIONS OF MATHEMATICS TEACHING: INFLUENCES IN THE FIRST THREE YEARS

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*In this paper I report on findings from a four year study of beginning teachers. The findings presented concern the conceptions of mathematics teaching for one of four case-study teachers and the influences on these conceptions. I present data from observations of lessons, interviews and written accounts that suggest Kate's conceptions of teaching became increasingly more consistent with a 'content-focused with an emphasis on conceptual development' view of teaching. Data are also presented which suggest that 'reflection' was the main influence on the development of Kate's conceptions both as an independent factor and in conjunction with the factors of 'experience' and 'working with others'.*

## INTRODUCTION

There is evidence that the conception of mathematics teaching held by individual teachers will contribute to the effectiveness of their teaching (Thompson, 1992; Askew, Brown, Rhodes, Johnson and Wiliam, 1997). The term conceptions is used here in the way suggested by Thompson (1992), as an inclusive term to include beliefs as well as other ideas such as mental images, concepts, meanings and preferences. Conceptions of mathematics teaching is clearly an area that needs to be addressed in any work which attempts to describe or influence the development of beginning teachers in relation to the teaching of mathematics. Assessing teachers' conceptions and the promotion of such conceptions that are believed to be positively influential in children's learning were seen as integral to my PhD study, an aspect of which I report on here.

Khus and Ball (1986) proposed four models of teachers' views about mathematics teaching, *a classroom-focused view*, *a content-focused with an emphasis on performance view*, *a content-focused with an emphasis on conceptual understanding view* and *a learner-focused view*. I used these models as a theoretical framework for the analysis of data collected in my study. Though I have analysed the data in relation to all four of Khus and Ball's models of conceptions of mathematics teaching, restrictions of space here only allow discussion in relation to the *content-focused with an emphasis on performance* and the *content-focused with an emphasis on conceptual understanding views*.

The aim of my study was to investigate the way in which beginning teachers' understanding of mathematics content knowledge needed for teaching might be developed through reflection using the *Knowledge Quartet framework*. This framework was used as a tool for identification and discussion of the teachers' mathematics content knowledge as evidenced in their teaching. The Knowledge

Quartet framework consists of four dimensions, *Foundation*, *Transformation*, *Connection* and *Contingency*. Details of this framework, and an account of how it was developed, may be found in the paper presented by Tim Rowland at the CERME meeting in Spain (Rowland, Huckstep and Thwaites, 2005).

Teacher's beliefs about mathematics and mathematics teaching were considered to be a component of mathematics content knowledge and are incorporated in the Foundation dimension of the Knowledge Quartet framework. Findings relating to the development of the Foundation aspect of one teacher's mathematical content knowledge were presented in a paper at the CERME meeting in Cyprus (Turner, 2007). The focus of the 2007 paper was on Amy, and drew on data from the first two years of the study. This paper focuses on the aspect of conceptions about mathematics teaching from within the Foundation dimension and presents findings relating to Kate over the full four years of the study.

## THE STUDY

The study began with 12 student teachers from the 2004-5 cohort of primary (5-11 years) postgraduate pre-service teacher education course at the University of Cambridge. The numbers reduced, as anticipated, to 9 in the second year, then 6 in the third year and finally 4 in the fourth and last year of the study. All participants were observed teaching during the final placement of their training year, twice during the first year, three times during the second year and once in the third year of their teaching. These lessons were all video-taped. In the training year the video-tapes were the basis for stimulated recall discussions using the Knowledge Quartet framework to focus on the mathematical content of the lesson. During the first year of teaching, feedback using the Knowledge Quartet framework was given following the two observed lessons. Participants were then sent a DVD with a recording of their lesson, and a request to observe the lesson and write their reflections on it. In the second year of their teaching only minimal feedback was given following the lesson as I wanted to see how the teachers would independently make use of the Knowledge Quartet in their reflections. They were sent DVDs of their three lessons and wrote reflections on each of these, drawing on their previous training in using the Knowledge Quartet framework. Participants also wrote regular reflections on their mathematics teaching which they sent to me. Group meetings were held to discuss the mathematics teaching and participation in the project of participants. These happened at the end of the training year and the first year of teaching, and at the end of each term in the second year of teaching. In their third year of teaching each teacher was interviewed individually in the Autumn and Spring terms and a group meeting was held in the Spring term.

Case studies were built from observations of teaching, discussions following observed lessons, contributions to group meetings, written reflections and individual interviews. Data from transcripts of discussions following observed lessons and group interviews as well as from written reflections was all analysed using the

qualitative data analysis software NVivo. A grounded theory approach (Glaser and Strauss, 1967) was used which led to the emergence of a hierarchical organisation of codes into a number of themes. Analysis of data attributed to codes under the NVivo theme ‘beliefs’, and the Knowledge Quartet analysis of observed lessons were used to build a description of the participants conceptions of mathematics and mathematics teaching over the four years of the study. Analysis of data attributed to codes under the themes of ‘experience’, ‘reflection’ and ‘working with others’ allowed inferences to be made about the factors associated with changes to participants’ conceptions. Though data from all four case studies have been analysed in relation to changes in their conceptions of mathematics teaching, there is only room to report on Kate here.

Since in this discussion I hope to build a picture of the way in which the participants’ conceptions developed over time, it is necessary to refer to times at which different data were collected. To aid clarity, and achieve brevity in this, I will use the date of the year and a number only to identify the timescale. Table 1 is intended to help the reader place the data within this timescale.

**Table 1: Notation used to indicate the timescale of data collected in the study**

Notation used		Place in career
2004	Autumn term	Trainee teacher
2005/6(1)	Autumn / Spring term	First year in teaching post
2005/6(2)	Spring / Summer term	
2006/7(1)	Autumn term	Second year in teaching post
2006/7(2)	Spring term	
2006/7(3)	Summer Term	
2008(1)	Autumn Term	Third year in teaching post
2008(2)	Spring Term	
2008 (3)	Summer term	

## FINDINGS

Analysis of teaching and of data coded under the heading ‘beliefs’, provided an account of Kate’s conceptions of teaching over the first three years of her career. In Kate’s lesson observed in 2004, the Knowledge Quartet code ‘reliance on procedures’ featured strongly and suggested a view which *emphasised performance*. Kate was teaching a lesson about doubling single digit numbers and demonstrated recording the doubling process by writing an addition in a witch’s cauldron with the answer in a bubble above e.g. ‘3 + 3’ in the cauldron and ‘6’ in the bubble. To record doubling of two digit numbers an extra bubble was added for the ‘tens numbers’ e.g. ‘23 + 23’ in the cauldron, ‘4’ in the tens bubble and ‘6’ in the units bubble. When questioned about this in the post-lesson reflective interview Kate suggested an amendment,

If I was going to do the tens and units, I should have asked for the units first ‘cus that’s what they know they have to start with, the most significant number which is tens.

Kate focused on a procedure reflecting the standard algorithm which suggested an emphasis on performance view of teaching. However in this same lesson, there was an indication that Kate was concerned to develop conceptual understanding. The Knowledge Quartet code ‘making connections between concepts’ was attributed when Kate made connections between doubles and near doubles and even and odd numbers and used pictorial representation to demonstrate why doubles must be even and near doubles odd.

In the lesson observed in 2005/6(1), Kate introduced the concept of multiplication by making the connection with repeated addition. She used a number of different representations modelling repeated addition to develop understanding of the concept of multiplication. However, when they came to do some problems themselves, the children were given specific procedures for calculating and recording. This lesson seemed to reflect a mixture of *content-focused views* of teaching with both an *emphasis on performance* and an *emphasis on conceptual development*.

The second lesson observed in 2005/6 did not feature ‘reliance on procedures’ and Kate made use of demonstrations to develop the children’s understanding of capacity and conservation. However, at the group interview in 2005/6(2), Kate suggested that she thought the children preferred an approach which emphasised procedures.

They really like doing boring things, they like doing number sentence things, they don’t like the other [problem solving] it’s more difficult, but they really like number sentences.

The three lessons observed in 2006/7 all demonstrated a concern for developing conceptual understanding rather than focusing on performance. In the lesson observed in 2006/7(1), there were no instances of ‘reliance on procedures’ and Kate used a number of demonstrations to build the children’s understanding of measuring using appropriate non-standard and standard units. In the lesson observed in 2006/7(2), Kate made use of a number of different representations to develop the difference conception of subtraction and also asked the children to explain their own strategies for completing the calculations. In the warm up part of the lesson observed in 2006/7(3), Kate set problems involving making the largest and smallest numbers on a spiked abacus using specified numbers of beads. This was designed to develop their conceptual understanding of place value. The main part of this lesson involved shading of different fractions on various grids. The way in which Kate introduced this, and the activities set for the children seemed to be aimed at developing a conceptual understanding of vulgar fractions.

The suggestion that Kate’s emphasis was on conceptual understanding in 2006/7, was supported by analysis of the NVivo coding of data. Kate had five instances of the code ‘conceptual understanding’ attributed to her data from 2006/7. In her reflective account written in 2006/7(3), Kate wrote,

Following the quite broad objectives of the new strategy, we have been trying to teach about data handling in quite a conceptual way and get children to think about the advantages and disadvantages of different ways to represent data.

Though observations of teaching and analysis of the NVivo data suggested Kate had moved towards an *emphasis on conceptual understanding view* of teaching, there were also a number of instances of her data from 2006/7 and 2007/8 which suggested she still held an *emphasis on performance view*. Two codes considered to reflect an *emphasis on performance view*, which featured strongly in her data, were ‘teaching different strategies’ and ‘need for structured work’. Five instances from reflective accounts written in 2006/7 were coded as ‘teaching different strategies’.

We have been looking at different addition strategies ... We had specific teaching sessions on some of these areas, then had some activities in which children were encouraged to choose a method for themselves. 2006/7(1)

This instance, and others like it, indicated that Kate felt she needed to give children a ‘toolbox’ of strategies from which to choose in order to perform calculations. Later in the year she seemed to have moved her position towards one in which she felt it was more helpful to focus on just some specific methods.

The week was structured around teaching a few particular methods, which is a little different from the approach we have often taken before when we have given the children opportunities to choose their own methods. 2006/7(2)

Kate’s move towards an approach involving teaching specific methods with which children can be successful seemed to reflect an emphasis on performance.

Data coded as ‘need for structured work’, suggested that Kate seemed more concerned that children achieved success in solving problems than that they developed a conceptual understanding. During the interview in 2007/8(2), we discussed the teaching of ‘word problems’. Kate indicated that she focused on getting the children to look for specific words in order to decide what sort of calculation was involved.

So, rather than understanding the concept behind the problem, it was ... we wanted the children to know what they could do, and that’s why I repeated the same lesson again. This time we approached it a bit differently and said ‘if you can spot one of these words, then you can work out for yourselves what it means and you will be able to do it’.

During the interview in 2007/8(2) Kate suggested that she recognised her teaching focused on achievement or performance rather than on developing conceptual understanding through exploration.

I don’t think that we do much open-ended, and that is perhaps a bit of a weakness in the way that I teach at the moment, because quite often, quite often in lessons I tell them what I want them to achieve.

Though Kate sometimes focused on performance in 2007/8, there was evidence from the lesson observed in 2007/8(2) that she continued to emphasise conceptual understanding. In this lesson Kate demonstrated the commutativity rule for addition before introducing the strategy of putting the bigger number first. She showed this by pinning two sets of differently coloured clothes pegs on a coat hanger to illustrate an

addition e.g.  $2 + 3$ , and then turned the coat hanger around to show the addition  $3 + 2$ . Kate did not simply tell the children the rule but demonstrated why it was the case. Later in the lesson, Kate demonstrated adding ten by moving down one row on a hundred grid. She asked the children why adding 10 to 23 gave the answer 33. Kate tried without success to get a response which showed an understanding of place value in relation to the layout of the grid. In the post-lesson reflective interview, Kate stated that she was unhappy that pupils had responded in this procedural way, and said that she would work on an approach directed at understanding why this procedure works.

Kate's data suggest that over the first three years of teaching her conceptions of mathematics teaching had encompassed elements of a *content-focused view with an emphasis on performance* and a *content-focused view with an emphasis on conceptual understanding*. All of Kate's lessons observed over the three years indicated that Kate was trying to develop conceptual understanding in her pupils, and this was supported by analysis of the NVivo coding of her data. Kate's later comments suggest that she was consciously trying to focus more on developing conceptual understanding. However these comments also suggest that she continued to be concerned that children were taught specific strategies, suggesting a view which emphasises performance.

The data discussed above presented a picture of Kate's conceptions of mathematics teaching over the first three years of her career. An analysis of data under the NVivo coding headings, 'experience', 'working with others' and 'reflection', gave some insight into the influences on these conceptions. Three instances of data under the heading 'experience' suggest that this was an influence on Kate's conceptions of mathematics teaching as *content-focused with an emphasis on conceptual understanding*. In her reflective account 2006/7(1) Kate wrote,

From last time we covered place value I realised that the majority of my year ones were not very clear on this concept. I wanted to make sure they understood the importance of tens and units on how we write our numbers.

During the interview in 2007/8(1), I asked what Kate thought had influenced the way in which her teaching had changed.

I think having done it before and knowing it works and sometimes I think when I have been teaching things, I have thought 'do I really understand this', or I have thought, 'I think I might be giving a misconception here or something', and then the next time I am really careful not to.

I would argue however that 'experience' alone did not influence Kate's conceptions of mathematics teaching. Rather, an examination of the three instances, demonstrate that it was Kate's reflection on her experience that influenced her conceptions of mathematics teaching. Phrases such as 'I realised', 'I have thought' and 'extrapolating in my head', all suggest active reflection.

There were several instances of data attributed to codes under the heading 'working with others' that suggested this too influenced Kate's conceptions of mathematics

teaching. Some such instances seemed to suggest that her colleagues had a view which emphasised on performance, while Kate's view was more one which emphasised conceptual understanding. In her reflective account 2006/7(2), Kate wrote,

Various materials suggest you should use them [empty number lines] in a 'come and show me how you are going to use this in your own way' kind of approach. However my colleague believes that we should only be teaching counting on along the empty numberline because that is what the children will be taught in year three.

Kate seemed to be in a dilemma because she was concerned with conceptual understanding while her colleague seemed to focus on content of the school curriculum. Two instances from the interview in 2007/8(1) suggest that Kate's 'enculturation into a community of practice' (Lave, 1988) involved exposure to views which emphasised performance. In the first of these, Kate's use of the term 'we', suggested that an emphasis on performance had resulted from shared planning.

We are trying to work on getting them to have skills of the physical, and the sort of organisational skills of recording their maths and they sort of need a structure to do it in.

In the second instance Kate was replying to my question about whether she ever talked to other people about reflections on her teaching.

Yes, occasionally. I think I would say, 'they found that really difficult, the numbers were too high and they didn't get a chance to work on the process because they were using those numbers', or 'that was really quick and they could have done another'.

This suggested that Kate saw her conversations about mathematics teaching with colleagues as being focused on the performance of the children rather than their conceptual understanding.

There were a number of instances of data under the heading 'working with others' that suggested Kate had an *emphasis on conceptual understanding* view of teaching. However, these did not necessarily suggest that Kate's colleagues had been influential in developing this view. In her reflective account 2006/7(1), Kate discussed a difference of opinion about a planned investigation.

The person planning for our team had planned for the children to investigate the question 'do all rectangles have four sides'. When this was first suggested it struck me as a rather trivial question, but as I continued to think about it I thought it was not a very good question at all because it suggested there was something intrinsically 'rectangular' about the examples they would be spotting which would allow them to recognise them as rectangles without taking into account their four-sidedness.

I haven't discussed this with my colleagues as I didn't want to be awkward, but I made a note to myself to keep my eyes open at planning meetings so I can politely say something straight away if I am uncomfortable with the mathematical ideas behind our planning in any other cases!

Kate focused on the conceptual appropriateness of the task despite the influence of her colleague, rather than because of it. During the interview in 2007/8(1), I asked Kate whether she ever talked to her colleagues about issues such as the use of representations in her teaching.

Not as often as we should because nobody wants to do the planning again. Um, I guess I would just use the other representation rather than discussing it with anybody.

This instance suggested that Kate did not automatically take on the ideas of her colleagues, but considered their conceptual appropriateness and changed them in her own teaching if she thought it necessary. Kate's 'enculturation into her community of practice' seemed to have been mediated by critical reflection. Kate engaged in the process Wenger (1998) referred to as *critical alignment* in such a way that she developed a view of teaching that continued to be strongly *content-focused with an emphasis on conceptual development*, despite this not seeming to be the general view of her community of practice.

The factors of 'experience' and 'working with others,' seemed to have had some influence on Kate's conceptions of mathematics teaching. However, both these factors also involved the mediation of reflection. Reflection also emerged as a separate heading in the NVivo coding process and Kate had a greater number of instances of her data attributed to codes under the heading of 'reflection' than to 'experience' and 'working with others' taken together. Codes under the heading 'reflection' which related to conceptions about mathematics teaching included, 'changed thinking', 'justification of teaching', 'questioning own teaching', 'suggested improvements' and 'judgements about effectiveness'.

Some of the instances of Kate's data coded under the heading 'reflection' suggested a view of teaching that emphasised performance. In her reflective account written in 2006/7(1), she focused on how well the children had performed on the tasks.

They seemed much more prone to making mistakes [in subtraction than addition] such as being one out because of counting the one they started on. They found taking away using number lines really tricky and were quite unreliable at taking away using objects.

Though such comments focused on the children's performance of tasks there were also suggestions in them that Kate was thinking about why they had difficulties. Similarly, some comments made during the interviews in 2007/8, focused on children's performance on tasks but also mentioned understanding. For example,

The year ones did a sheet of number sentences ... that was a bad choice of sheet because it was an 'empty box' sheet and we hadn't been doing any empty boxes ... they still got it wrong because they didn't understand what it was asking them ... but I understood why they did it. So, it was OKish because they were quite purposefully engaged ...

Though this instance suggested Kate focused on engagement rather than learning, it also indicated that she had given some thought to children's conceptual difficulties.



There were few instances of data that suggested Kate focused only on children's performance without in some way considering their conceptual understanding.

In her reflective accounts Kate made several comments which explicitly demonstrated her concern with the conceptual understanding that had, or had not been achieved through her teaching. For example,

In the first lesson we did several activities which involved putting numbers into order and then went on to positioning numbers on a numberline for their independent activity, but I think this activity had more to do with place value than ordering numbers as they had to work out how many tens marks to count along and then think about the units. 2006/7(2)

Kate also made a number of comments during the interview in 2007/8(1) which suggested she held a view of teaching which emphasised conceptual development.

The children thought that triangles would have a line of symmetry but the one we tried didn't. In retrospect I wish that we had discussed that a bit more because it would have been interesting to get all the triangles out of the box and compare them.

Data from the heading 'reflection', suggested that Kate's had a strong view of mathematics teaching as *content-focused with an emphasis on conceptual understanding*. Though, this does not necessarily suggest a causal link between reflection and her view, it can be argued that reflection did influence Kate's conceptions. Kate wrote these reflective accounts because of her involvement in the study. The kind of thinking she engaged in was therefore prompted by the requirement to reflect on her teaching using the Knowledge Quartet. During the interview in 2007/8(1), Kate confirmed that this framework had influenced her thinking,

The first few things I would be thinking of are the organisational things, and then I try to think 'did they learn anything' and 'was the learning alright even if the organisation wasn't' kind of thing. So, I think it is useful to have some kind of structure to help you know what you need to know and what they need to know and how to learn it.

Later in the interview, Kate reiterated that the structure provided by the Knowledge Quartet helped her reflect on whether or not her teaching had been effective in promoting understanding.

I think what I have said and how I have explained things, I am more aware than I would be if I didn't have such a clear idea of what I was looking for.

### **Summary and implications**

Analysis of longitudinal data from one case study of a beginning teacher has given some insight into the conceptions of mathematics teaching held by that teacher, as well as insight into the influences on those conceptions. Though finding about Kate's conceptions and the influences on them are inferential, the use of the Knowledge Quartet framework for the analysis of lessons, and the systematic analysis of all data from interviews and reflective accounts, gives a strong basis for these inferences. It is

reasonable to suggest that Kate has developed a view of mathematics teaching that is increasingly *content-focused with an emphasis on conceptual understanding* and that the development of this view has been influenced by reflections on her teaching supported by the Knowledge Quartet framework. ‘Experience’ and ‘working with others’, have also been influential in developing Kate’s conceptions of mathematics teaching. However, reflection was an important mediator in these two factors. There is evidence, not discussed here, that Kate had also moved towards a *learner-focused view* of mathematics teaching. The direction of development of Kate’s conceptions is one which we might wish to replicate in other beginning teachers. If so, it would seem that finding ways of encouraging the sort of reflection on mathematics teaching that Kate has undertaken over the first years of her career, is an idea worth pursuing.

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