RESULTS OF A COMPARATIVE STUDY OF FUTURE TEACHERS FROM AUSTRALIA, GERMANY AND HONG KONG WITH REGARD TO COMPETENCIES IN ARGUMENTATION AND PROOF

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The article describes the conceptions and first results of an enrichment study to the international comparative study on the efficacy of teacher education, Mathematics Teaching in the 21st Century (MT21). The study focuses on the professional knowledge of future teacher students in three countries – Australia, Germany and Hong Kong – with regard to the mathematical areas of modelling and argumentation and proof. After describing the theoretical framework and the applied methodological approach some selected results with regard to argumentation and proof are presented.

Keywords: Education, Mathematical content knowledge, Pedagogical content knowledge, Proof.

Background of the study

Although teacher education has already been criticised for a long time, only rarely systematic evaluation and studies concerning the efficiency of teacher education and how future teachers perform during and at the end of their education can be found (for an overview on the debate see Blömeke *et al*., 2008). Even in the field that is covered by most of the existing studies – the education of mathematics teachers – research deficits have to be stated: the research is often short term, of a non-cumulative nature, and conducted within the researcher's own training institution. Only recently more empirical studies on mathematics teacher education have been developed (cf. Chick *et al*., 2006, Adler *et al*., 2005).

In order to overcome this deficit the IEA (International Association for the Evaluation of Educational Achievement) currently carries out an international comparative study focusing on the efficiency of teacher education and the professional knowledge of future teachers called TEDS-M (Teacher Education and Development Study – Learning to Teach Mathematics). This study concentrates on future mathematics teachers and is conducted in 20 countries worldwide. We also refer to the COACTIV – study, another study on teacher education using similar conceptualisations of professional knowledge of mathematics teachers (see among others Krauss, Baumert & Blum 2008). Furthermore in order not only to develop a theoretical framework and adequate instruments for the TEDS-M study but also to offer a first research attempt to fill existing research gaps, a pilot study for TEDS-M was conducted called Mathematics Teaching in the 21st Century (MT211 [1]). This study also aimed to shed light on the important field of mathematics teacher education from a comparative perspective. For this among others the knowledge and beliefs of future lower secondary teachers were investigated (for results see e.g. Blömeke, Kaiser, Lehmann, 2008, Schmidt *et al*., 2008).

The study described in this paper is a complementary study to MT21 with the aim of gaining supplementary results basing on qualitative data as an addition to the quantitative data of MT21. This study is a collaborative study between researchers at universities in Germany, Hong Kong and Australia, using the theoretical framework and theoretical conceptualisation from MT21, but carrying out qualitatively oriented detailed in-depth studies on selected topics of the professional knowledge of future teachers, namely modelling and argumentation and proof, the latter being the theme of this paper. The study is only focussing on future teachers and their first phase of teacher education (for details see Schwarz *et al*., 2008).

THEORETICAL FRAMEWORK OF THE STUDY

The initial ideas of MT21 are considerations about the central aspects of teachers' professional competencies and by this the related theoretical framework is also the theoretical basis of the supplementary study. Concerning the professional knowledge of teachers the study follows the ideas basically defined by Shulman (1986). He fundamentally distinguishes two domains, namely general pedagogical knowledge and content knowledge. The latter is further divided into three parts:

- subject matter content knowledge
- pedagogical content knowledge
- curricular knowledge

For the study these areas of content knowledge are further sub-divided. In the area of subject matter content knowledge for example with regard to Bromme (1995) mathematics as a school subject and mathematics as a scientific discipline are differentiated.

Beside these cognitive components furthermore also an affective and value-orientated component is taken into consideration. This component especially accounts for the epistemological beliefs, more precisely the beliefs towards mathematics itself and the beliefs towards teaching and learning mathematics. Again in accordance with the theoretical conceptualisations of MT21 (see Blömeke, Kaiser, Lehmann, 2008) the differentiation of different beliefs towards mathematics of Grigutsch, Raatz and Törner (1998) is basis of the study. Here four kinds of beliefs are distinguished with relation to mathematics:

- formalism-aspect of mathematics
- scheme-aspect of mathematics
- process-aspect of mathematics

- application-aspect of mathematics

Based on these theoretical distinctions concerning professional knowledge of future teachers the overall aim of our study is to answer the following questions:

- What kind of knowledge with regard to the described domains of teachers' professional knowledge do future teachers acquire during their university study?
- Which connections between the described domains of knowledge and the beliefs can be reconstructed within these future teachers?

In this paper from a mathematical content related perspective we concentrate on the area of argumentation and proof. Furthermore because of the limited space we only focus on the first question and describe some selected results. For a more detailed description of results related to the area argumentation and proof see Schwarz et al. (2008). For first results related to the second question with regard to the mathematical area of modelling see Schwarz, Kaiser, Buchholtz (2008).

Concerning the area of argumentation and proof we refer to specific European traditions, in which various kinds of reasoning and proofs are distinguished, especially "pre-formal proofs" and "formal proofs". These notions were elaborated by Blum and Kirsch (1991): pre-formal proof means "a chain of correct, but not formally represented conclusions which refer to valid, non-formal premises" (Blum & Kirsch, 1991, p. 187).

Concerning the role of proof in mathematics teaching, Holland (1996) details the plea of Blum and Kirsch (1991) for pre-formal proofs besides formal proofs as follows: For him pre-formal proofs may be sufficient in mathematics lessons with cognitively weaker students, in other classes both kinds of proofs should be conducted. Pre-formal proofs have many advantages due to their illustrative style. In addition, pre-formal proofs contribute substantially to a deeper understanding of the discussed theorems and they place emphasis on the application-oriented, experimental and pictorial aspects of mathematics. However, their disadvantage is their incompleteness, their reference to visualisations, which require formal proofs in order to convey an appropriate image of mathematics as science to the students. The scientific advantage of formal proofs, namely their completeness, is often accompanied by a certain complexity, which may cause barriers for the students' understanding and might be time-consuming. However, there is no doubt, that treating proofs in mathematics lessons is meaningful with the aim of developing general abilities, such as heuristic abilities. The teaching of these two different kinds of proofs leads to high demands on teachers and future teachers. Teachers must possess mathematical content knowledge at a higher level of school mathematics and university level knowledge on mathematics on proof. This comprises the ability to identify different proof structures (pre-formal – formal), the ability to execute proofs on different levels and to know alternative specific mathematical proofs.

Additionally, teachers should be able to recognise or to establish connections between different topic areas. To sum up: Teachers should have adequate knowledge of the above-described didactical considerations on proving as well (for details see Holland, 1996, pp. 51-58). It can be expected that in addition to being able to construct proofs, teachers will need to draw on their mathematical knowledge about the different structures of proving such as special cases or experimental ‘proofs’, pre-formal proofs, and formal proofs and pedagogical content knowledge when planning teaching experiences and when judging the adequacy or correctness of their, and their students’ proofs in various mathematical content domains.

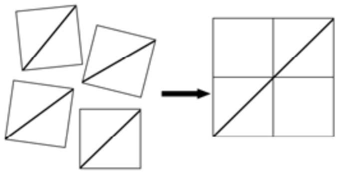
METHODICAL APPROACH

Based on the methodological approach of triangulation questionnaires with open questions and in-depth thematically oriented interviews were developed. This offers the opportunity to deepen the quantitative results of MT21 by means of this qualitative orientated data. The instruments are, as described above, restricted to the areas of modelling and argumentation and proof. The questionnaire consists of seven items that are domain-overlapping designed – as so-called ‘Bridging Items’. Each of the items captures several areas of knowledge and related beliefs on the base of the distinctions described above. In detail three items deal with modelling and real world examples, three with argumentation and proof and one is about how to handle heterogeneity when teaching mathematics. Furthermore, demographic information like number of semesters, second subject and attended seminars and teaching experiences – including extra-university teaching experiences - are collected. This questioning has been conducted with 79 future mathematics teachers on a voluntary base within the scope of pro-seminars and advanced seminars for future teachers at a German university. In Australia, 46 future teachers from two universities participated and in Hong Kong 84 future teachers from one institution.

Complimentary to this questionnaire an interview guide for a problem-centred guided interview was developed, which contains pre-structured and open questions (i.e., elaborating questions) on modelling and argumentation and proof. The questions are linked to the items in the questionnaire in the sense that the have the same theoretical base and cover the same sub-domains of teachers’ professional knowledge. The selection of the interviewees follows theoretical considerations and takes the achievements in the questionnaire into account. That means interviewees were selected according to an interesting answering pattern in the questionnaire or extraordinary high or low knowledge in one or more domains.

The evaluation of the questionnaires as well as of the interviews is carried out by means of the qualitative content analysis method by Mayring (2000). More detailed we apply a method of analysis that aims at extracting a specific structure from the material by referring to predefined criteria (deductive application of categories). From there, by means of formulation of definitions, identification of typical passages from the responses as so-called anchor examples and development of coding rules, a coding manual has been constructed to be used to analyse and to code the material. For this, coding means the assignment of the material according to the evaluation categories. More precisely the method of structuring scaling (ibid.) is applied by which the material is evaluated by using scales (predominantly ordinal scales). Subsequently, quantitative analyses according to frequency or contingency can be carried out.

In the following one exemplary item of the questionnaire is described, which shows, how the different facets of professional knowledge – pedagogical content knowledge, mathematical knowledge and beliefs - are linked. A similar item is included in the interview, so that it is possible to connect the evaluation of the data on a rich data base.

> Read the following statement:
>
> **If you double the side length of a square, the length of each diagonal will be doubled as well.**
>
> The following pre-formal proof is given:
>
> *You use squared tiles of the same size. If you use four tiles to make one square, you will get a square with a side length twice the length of the squared tiles.*
>
> *You can see immediately, that each diagonal has twice the length of the ones of the squared tiles because the two diagonals of two tiles are put directly together.*
>
> 
>
> a) Is this argumentation a sufficient proof for you? Please give a short explanation.
>
> b) Please formulate a formal proof for the statement above about diagonals and squares.
>
> c) What proof would you use in your mathematics lessons? Please explain your position.
>
> d) Can a pre-formal proof be sufficient as the only kind of proof in mathematics lessons? Please explain your position.

e) Please name the advantages and disadvantages of a formal and pre-formal proof.

f) Can the pre-formal and the formal proof for the statement about the length of diagonals in squares be generalised for any rectangle? Please give a short explanation.

g) What do you think about the meaning of proofs for mathematics lessons in the secondary school?

Figure 1: Task from the questionnaire concerning argumentation and proof

SELECTED RESULTS

Both, part b) and part f) of the task described above lay their focus on the future teachers' mathematical content knowledge. Part b) does especially not require any mathematics at a university level but only knowledge about fundamental geometrical theorems (e.g. Pythagoras theorem) and abilities concerning elementary algebraic transformations and abilities in formulation proofs. The items was coded on a five-point-scale while both codes, +1 and +2, means a right solution (answers coded with +2 in addition have a comprehensible structure) and -2 means serious mistakes like circular arguments or just a rephrasing of the pre-formal proof while a formal one is required. Examples of future teachers' responses and a more detailed description of the different coding of different answers are not presented here because of the limited space. Related descriptions can be found in Schwarz et al. (2008).

The results are the following:

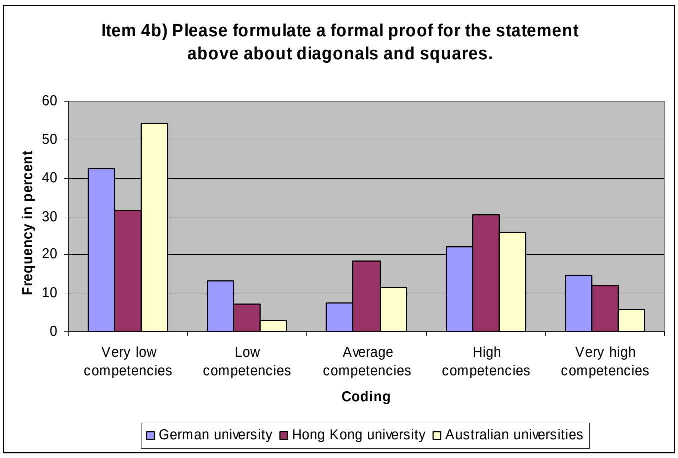


Figure 2: Results of item 4b)

One can see that for almost all institutions, the majority, in most instances, of future teachers in this case study were not able to execute formal proofs, requiring only lower secondary mathematical content, in an adequate and mathematically correct way.

Very similar results can be seen with regard to item f). Here also no university mathematics is needed but just an understanding of a proof suitable for lower secondary mathematics teaching. Again answers were coded on a five-point-scale with +1 and +2 meaning right solutions and -1 and -2 meaning wrong solutions. Then the results are the following:

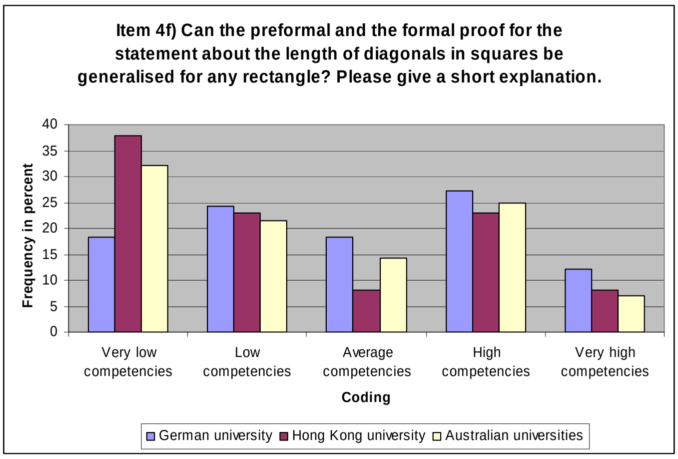


Figure 3: Results of item 4f)

Again, in most cases, the majority is not able to recognise and satisfactorily generalise a given mathematical proof.

In contrast, in all samples there was evidence of at least average competencies of pedagogical content reflection about formal and pre-formal proving in mathematics teaching with the exception of the Australian sample with respect to the sufficiency of pre-formal proof as the only type of proof in mathematics lessons. The related results are presented in a more qualitative way in the following paragraphs.

Preferences for pre-formal proving are evident, both with respect to mathematical content knowledge and pedagogical content knowledge. In contrast to the Hong Kong and Australian samples, there was a strong tendency in the German data for pre-formal proving to be incorporated into the pedagogical content-based discussion particularly with respect to problems of using proof with students of different abilities. In both the Hong Kong and Australian data, future teachers indicated a broad open-mindedness to various didactical conceptions but the pre-formal proof was perceived as an atypical part of mathematics teaching, possibly reflecting the use of alternate terms and conceptions for argumentation and proving that is not formal proof in the teacher education courses in these contexts. In both samples, mathematical content considerations tended to be the basis for didactical reflections.

With regards to affinity towards proving in lower secondary mathematics lessons Australian, Hong Kong and German students indicated a high to very high affinity to proving. It was assumed a higher affinity to proving would be expressed in more distinct pedagogical content reflection; however, the nature of these reflections differed with the samples. Future teachers in the German sample assumed dealing with proofs helped develop students' argumentation abilities especially with respect to their own hypotheses rather than their completeness of mathematical theorems. The difficulties students might have with proving in the classroom also came to the fore. In contrast, the Hong Kong and Australian future teachers rarely mentioned difficulties students might have with proving. The responses of future teachers from both Hong Kong and Australia reflected a formal image of mathematics being reinforced through use of formal proofs in teaching and the practice of proving leading to the comprehension of mathematical theorems.

SUMMARY AND OUTLOOK

The paper describes first results of an additional study to the international comparative study on the efficiency of teacher education MT21. With regard to a theoretical framework distinguishing between different areas of teachers' professional competence results concerning future teachers' knowledge in different areas are presented restricted to the mathematical field of argumentation and proof.

As the presented additional study only focuses on future teachers, which means university students, no statements concerning the professional knowledge of practicing teachers can be made.

With regard to the further work to be done one of the next steps of the evaluation will be a more detailed distinction between different subgroups of the sample and the particular characteristics of their professional competence. For this evaluation the sample will be divided twice. On the one hand different school types the future teachers are studying for can be differentiated. On the other hand future teachers in different phases of their university studies, which means beginners or students at the end of their studies, can be distinguished. Besides that the results of the analyses of the interviews are to be linked to the results of the questionnaires. First results of these analyses can be found in Corleis et al. (2008). Finally the results of the additional study are to be related to the results of the main study MT21.

NOTES

1. The previous name of this study was PTEDS.

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