EFFICACY BELIEFS AND ABILITY TO SOLVE VOLUME MEASUREMENT TASKS IN DIFFERENT REPRESENTATIONS

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The aim of this study was to investigate the relationship between students' efficacy beliefs and their performance in volume measurement tasks which were given in different representations. A group of sixth grade students (N=173) completed a four-part self-report questionnaire and solved six volume measurement tasks in different representations format: text, diagram of 3-D cube array and net diagram. Perceived efficacy to solve volume measurement tasks was found to be a significant predictor of students' general performance. Furthermore, high-ability students had stronger and more accurate efficacy beliefs towards tasks with net diagram which were unfamiliar, whereas low-ability students had more accurate efficacy beliefs towards verbal tasks which were familiar.

Key words: efficacy beliefs, volume, 3-D cube arrays, net.

INTRODUCTION

The affective domain has in recent years attracted much attention from mathematics research community (Philippou & Christou, 2002). A number of researchers who have examined thoroughly the connections and the relationship among affect and mathematical learning found that affect plays a decisive role in the progress of cognitive development (Bandura, 1997; Ma & Kishor, 1997; Philippou & Christou, 2002). One of the components of affective domain are self-efficacy beliefs (Goldin, 2002), which were found to have significant correlations and direct effects on various math-related variables (Pajares, 1996). However, although much work has been done in this area, little attention has been given to the relationship between self-efficacy beliefs and the use multiple representations in mathematics (e.g. Patterson & Norwood, 2004).

In this paper we try to investigate the relationship between efficacy to solve volume measurement tasks and performance in volume measurement of cuboid tasks which are given in different modes of representations.

THEORETICAL BACKGROUND

Self-efficacy beliefs and mathematics performance

Self-beliefs, such as self-esteem, self-concept and self-efficacy, comprise components of the general beliefs system (Philippou & Christou, 2002). Students' perceived self-efficacy for a task, are defined as their judgments about their ability to complete a task successfully (Bandura, 1997).

A number of studies have found a positive relationship between students' self-efficacy beliefs and mathematics performance (Pajares, 1996). More specifically, Pajares and Miller (1994) reported that self-efficacy in solving math problems was more predictive of that performance than sex, math background, math anxiety, math self-concept and perceived usefulness of mathematics. Additionally to this, Pajares and Kranzler (1995) found that self-efficacy made as strong a contribution to the prediction of problem-solving as did general mental ability, an acknowledged powerful predictor and determinant of academic outcomes. In this line, Mayer (1998) stressed that students who improve their self-efficacy will improve their success in learning to solve problems.

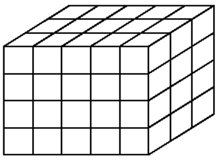
Researchers have also indicated that high-ability students have stronger self-efficacy and have more accurate self-perceptions (e.g. Pajares & Kranzler, 1995; Zimmerman, Bandura, & Martinez-Pons, 1992). Schunk and Hanson (1985) found that students who expected to be able to learn how to solve the problems tended to learn more than students who expected to have difficulty.

Self-efficacy beliefs have already been studied in relation to a lot of aspects of mathematics learning, such as arithmetical operations, problem solving and problem posing (e.g. Pajares & Miller, 1994; Pajares, 1996; Nicolaou & Philippou, 2007). However, these beliefs haven't been examined in relation to volume measurement tasks and this study tries to investigate this relationship.

Students' understanding of 3-D rectangular arrays of cubes

A number of researchers investigated students understanding of three dimensional rectangular arrays (3-D) of cubes, using interviews or tests (Ben – Chaim, Lappan & Houang, 1985; Battista & Clements, 1996). In particular, Ben – Chaim et al. (1985) indicated four types of errors that students in grades 5-8 made on the volume measurement tasks with three dimensional cube arrays. The first error was to count only the number of faces of cubes shown in a given diagram, while the second error was doubling that number. The third error was counting the number of cubes shown in the diagram and the forth error was doubling that number (see for example figure 1). In this study, when researchers asked students to determine how many cubes it would take to build such prisms, they found that only 46% of the students gave the correct answer, while most of them made the errors of type 1 or 2 (Ben-Chaim et al., 1985). These results are in line with those from a recent work by Battista and Clements (1996) where they found that 64% of the third graders and 21% of the fifth graders double-counted cubes. These types of errors made by students are clearly related to some aspects of spatial visualization (Ben-Chaim et al., 1985). In addition to this explanation, Battista and Clements (1996) stressed that many students are unable to correctly enumerate the cubes in such an array, because their own spatial structuring of the array is incorrect. In particular, they found that for some students the root of such errant spatial structuring seemed to be attributed to their inability to coordinate and integrate the views of an array to form a single coherent mental model of the array. However, Hirstein (1981) believes that these errors are caused by their confused notions of volume and surface area.

How many unit cubes does it take to make this rectangular solid? (Clements & Battista, 1996)



Four types of errors that students make on this problem:

Error type 1: Counting the cube faces shown in the diagram, e.g. 20+12+15=47

Error type 2: Counting the cube faces shown in the diagram and doubling that number, e.g. 47 x 2= 94

Error type 3: Counting the numbers of cubes showing in the diagram, e.g. 20+8+8=36

Error type 4: Counting the numbers of cubes showing in the diagram and doubling that number, e.g. 36 x 2=72

Figure 1: Four types of errors that students make on volume measurement problems.

THE PRESENT STUDY

The purpose of the study

The purpose of this study was to explore the relationship between students' efficacy beliefs to solve volume measurement tasks and their ability to solve volume measurement cuboids tasks; these were given in different modes of representations, namely text, diagram of 3-D cube array and net diagram. More specifically, the present study addresses the following questions: (a) Are students' efficacy beliefs to solve volume measurement tasks strong predictor of their performance in these tasks? (b) What is the relationship between students' efficacy beliefs to solve volume measurement tasks and their errors in dealing with 3-D cube arrays and net diagrams? (c) Are there differences in the efficacy beliefs and the accuracy of these beliefs among students of varied abilities?

Participants and Test

In the present study data were collected from 173 sixth grade students (84 females and 89 males) ranging from 11 to 11.5 years of age. These students were from 10 primary schools in Cyprus from rural and urban areas.

All participants completed a five-part test which was developed on the basis of previous studies (e.g. Ben-Chaim et al., 1985; Battista & Clements, 1996; Nicolaou & Philippou, 2007). For the purpose of this paper, we did not use students' answers from the first part of the test. The first four-parts of the test measured efficacy beliefs towards mathematical problems and volume measurement tasks and the fifth part measured students' ability to solve volume measurement tasks in different representations. Specifically, in the second part, students were asked to read each of the three volume measurement tasks: verbal task (SEiA), task with 3-D cube array (SEiB) and task with net diagram (SEiC) and state their sense of certainty to solve these tasks, without solving them. Responses were recorded on a 4 point Likert scale with 1 indicating not at all certain and 4 very much certain. In the third part, students were asked to state which one of the tasks from the second part was easy to solve (Es), was difficult to solve (Df), liked to solve (Lk) and did not find interesting to solve (Lint). The forth part comprised of five cartoon-type pictures and statements explaining the situation presented by each picture; the students were requested to select the picture that best expressed their efficacy beliefs (very high-SEI, high-SEII, medium-SEIII, low-SEIV and very low-SEV) to solve volume measurement tasks. The fifth part of the test had six volume measurement cuboids tasks which were given in different modes of representations: text, diagram of 3-D cube array and net diagram (see figure 2).

Verbal tasks

1. Mary tries to put 28 unit-sided cubes (1 cm edge) in a rectangular box with dimensions 2 cm x 5 cm x 3 cm. Is this possible? Explain your answer. (VPr1)

4. Four friends went to the cinema. They decided to buy some bags of nuts during the movie. The vendor said to them that there were two size bags of nuts, where:

• The prize of small bag was €1.

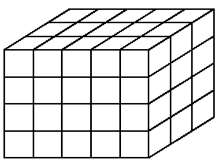
• The large bag's dimensions were two times the small bag's dimensions and its prize was €6.

The dimensions of small bag were 20 cm, 10 cm and 5 cm.

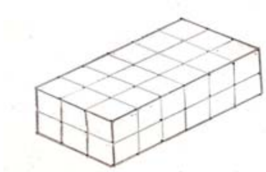
One child suggested to his friends that it was better to buy and share one large size bag, instead of buying four small bags. Do you agree? Explain your answer. (VPr4)

Tasks with diagram of three dimensional cube array

Find the volume (the number of cubes) of the following cuboids:



(SPr2a)

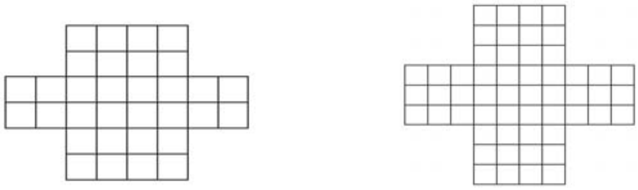


(SPr2b)

Which one of these cuboids has the greatest number of cubes? Explain your answer. (SPr2Ans)

Tasks with net diagram

The figures below show the nets of cuboids with one of its sides missing. Find the volume (number of cubes) of this net when folded:



(NPr3a) (NPr3b)

Which one of these nets when folded can carry the least number of cubes? Explain your answer. (NPr3Ans)

Figure 2: Volume measurement tasks.

The coefficient of reliability Gronbach's Alpha of the five-part of test was very high (a=0.794). Specifically, we found that the reliability of answers of students in the first four-part of questionnaire was $\alpha=0.782$ and the reliability of answers in volume measurement tasks was $\alpha=0.810$.

Data Analysis

Students correct responses in volume measurement tasks were marked with 1 and incorrect response with 0. However, the marks to responses of the questions: "Which one of these cuboids has the greatest number of cubes? Explain your answer." and "Which one of these nets when folded can carry the least number of cubes? Explain your answer." were: 1 for fully correct response, 0.5 for partly correct response (wrong explanation) and 0 for incorrect answer. We used the classification of errors made in previous studies (Ben Chaim et al., 1985; Battista & Clements, 1996) to code the students' errors while solving the volume tasks with 3-D cube array diagram and net diagram.

To answer the research questions of this study, four different analyses were conducted: a Regression Analysis, an Implicative Statistical Analysis with the use of the computer software CHIC (Bodin, Coutourier, & Gras, 2000), an Analysis of Variance one way and a Crosstabs Analysis. The implicative statistical analysis is a method of analysis that determines the similarity connections and the implicative relations of factors.

RESULTS

We used regression analysis with independent variable students' efficacy beliefs to solve volume measurement tasks (answers of students in forth part of test) and dependent variable their general volume measurement performance in the test. We found that students' efficacy beliefs to solve volume measurement tasks can be a statistically significant predictor of their performance in the test (10,1%). Furthermore, we examined the predictive role of students' efficacy to solve verbal volume measurement tasks to their performance in these tasks and regression analysis confirmed that (6%). Additionally, students' efficacy to solve volume measurement tasks with 3-D diagram can be a statistically significant predictor of their performance in one of these tasks (3%). We also found that students' efficacy to solve volume measurement tasks in net diagram predicted only 4% of their performance in these tasks.

To examine the relationships between students' efficacy beliefs to solve volume measurement tasks, their performance in these tasks which were given in different representations and their errors in dealing with 3-D cube arrays and net diagrams, we employed the statistical implicative analysis for the data of this study and gave us the similarity diagram (see figure 3), which allowed for the grouping of the tasks and the statements based on the homogeneity by which they were handled by students.

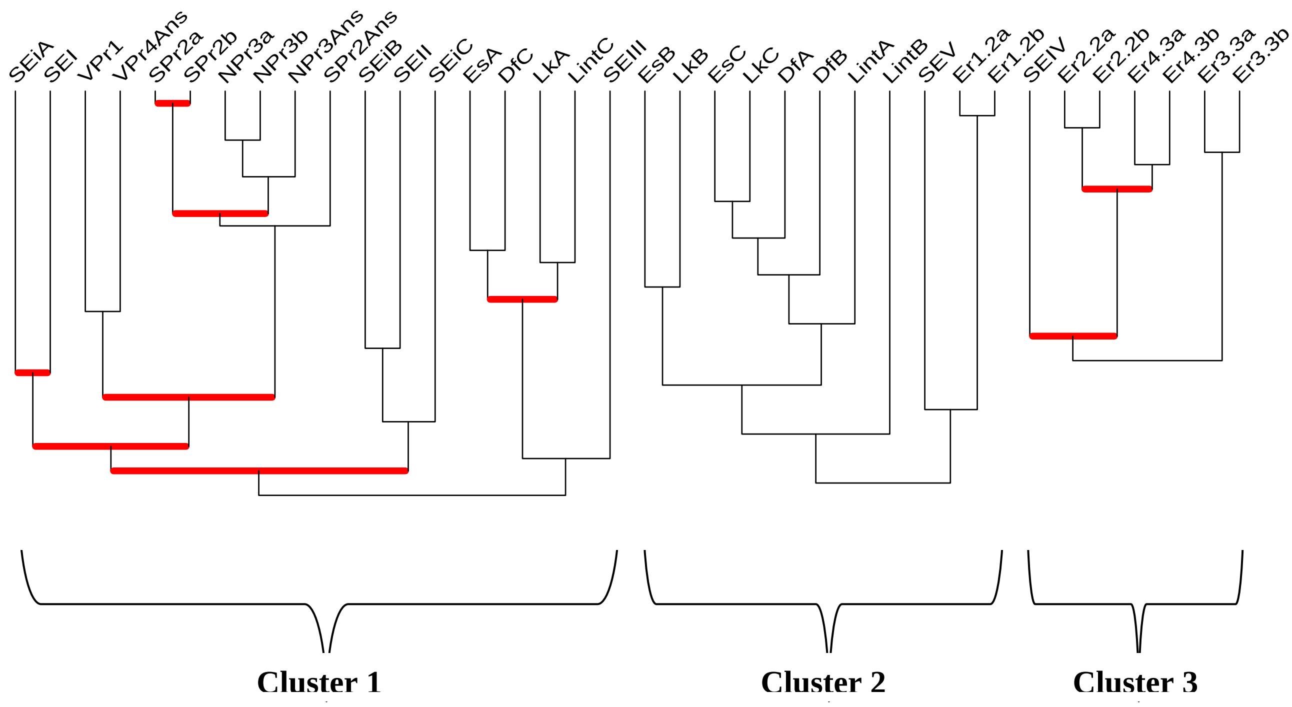


Figure 3: Similarity diagram of students' responses to the four-part of test.

Note: The similarities in bold color are important at level of significance 99%.

In figure 3, three distinct clusters of variables were formed. The first cluster consists of correct responses of students to volume measurement tasks and high efficacy beliefs, while the second and the third cluster consist students' errors and low efficacy beliefs. More specifically, the first cluster involved five similarity groups. The first group included the two statements of high efficacy beliefs to solve all volume measurement tasks and verbal tasks. The second group involved the verbal volume measurement tasks, while volume measurement tasks with 3-D cube array diagram and net diagram formed the third similarity group. These groups provided further support that different cognitive processes were required in order to solve verbal volume measurement tasks and volume measurement tasks with diagram.

However, their similarity connection indicated that equivalent content knowledge was needed to develop volume measurement ability in different representations. The forth group included the three statements of high efficacy beliefs to solve all volume measurement tasks, tasks with 3-D cube array diagram and tasks with net diagram. Finally, the fifth group of the first cluster involved mainly four statements which referred to students' evaluation for verbal tasks as easy and interesting and for tasks with net diagram as difficult and less interesting. All above groups of similarity of the first cluster show that students with high efficacy beliefs to solve volume measurement tasks in different representations solved these tasks in a similar way. Furthermore, these students assessed the verbal tasks as easy and interesting, while the task with net diagram as difficult and less interesting. It is hypothesised that students solved mainly verbal volume measurement tasks in their textbooks and so they had more experiences to solve these tasks than tasks with net diagram. Therefore, they felt more certain to solve familiar tasks than unfamiliar ones.

The second cluster involved two similarity groups. The first group mainly included four statements which referred to students' evaluation for tasks with net diagram as easy and interesting and for verbal tasks as difficult and less interesting. The second group involved the statement of low efficacy beliefs to solve volume measurement tasks and the wrong strategy: count the number of faces of cubes shown in diagram, which used from students to solve tasks with 3-D cube array diagram. The third cluster involved the statement of lowest efficacy beliefs to solve volume measurement tasks and errors to tasks with diagram. From the second and third cluster indicated that different cognitive processes were required to calculate the number of faces of cubes shown in 3-D cube array diagram and in net diagram. However, in the case of errors: count the number of faces of cubes shown in diagram and double that number, similar cognitive processes were required to apply it in 3-D cube array diagram and in net diagram.

The sample of this study was clustered into three groups according to their volume measurement performance in the tasks of the fifth part of the test. The performance of the three clusters of students was examined in respect to their efficacy beliefs to solve volume measurement tasks. The comparison of the means by one way ANOVA indicated statistically significant differences between these groups ($F_{(2,169)}=6.240$, $p=0.002$) at efficacy beliefs towards volume measurement tasks. Using Bonferroni procedure, we found only statistical significant differences at efficacy beliefs between students with the lowest performance ($\overline{X} = 3.10$) and highest performance ($\overline{X}=4.18$) in volume measurement tasks. Therefore, high-ability students have stronger efficacy beliefs towards volume measurement tasks than low-ability students.

However, at the same time, according to the results of the crosstabs analysis, students who solved the tasks of test correctly or wrongly indicated both very high efficacy beliefs and very low efficacy beliefs. We found that students who solved the tasks of the test correctly had more accurate self-efficacy than students who solved the tasks of the test wrongly. More specifically, high-ability students were more accurate in their efficacy beliefs towards tasks with net diagram in relation to their performance in these tasks (73% of students who solved the tasks with net diagram correctly indicated very high and high efficacy beliefs and only 7.5% of them indicated very low and low efficacy beliefs). The tasks with net diagram considered as an unfamiliar form of the volume measurement tasks for the students, because they did not solve any similar tasks in their mathematics textbooks. Also, crosstabs analysis showed that low ability students were more accurate in their efficacy beliefs towards verbal tasks in relation to their performance in these tasks (37% of students who solved verbal tasks wrongly indicated very high and high efficacy beliefs and 35% of them indicated very low and low efficacy beliefs). The verbal tasks are more familiar to the students, since their mathematics textbooks have a number of these tasks.

Additionally, the sample of this study was clustered into five groups according to their efficacy beliefs towards volume measurement tasks. The efficacy beliefs to solve volume measurement tasks of the five clusters of students were examined in respect to their general volume measurement performance. The comparison of the means by one way ANOVA indicated statistically significant differences between these groups ($F_{(5,166)}=3.697$, $p=0.003$) on volume measurement performance. Using Bonferroni procedure, students with very high efficacy beliefs ($\overline{X}=2.43$**)** and students with very low efficacy beliefs ($\overline{X}=0.55$**)** differed significantly in their general volume measurement performance.

DISCUSSION

The purpose of the present study was to investigate the relationship between students' efficacy beliefs to solve volume measurement tasks in different representations and their performance in these tasks. We found that students' efficacy beliefs to solve volume measurement tasks was a statistically significant predictor of the general volume measurement performance of students. The predictive role of efficacy beliefs was indicated from various studies in different concepts of mathematics (Pajares & Miller, 1994; Pajares & Kranzler, 1995; Nicolaou & Philippou, 2007).

In the similarity diagram three distinct clusters of variables were formed. The first cluster included students who solved correctly the tasks of the test and indicated very high and high efficacy beliefs towards volume measurement tasks, whereas the second and the third group involved students who used wrong strategies to solve volume measurement tasks with 3-D cube array diagram and net diagram and indicated very low and low efficacy beliefs towards volume measurement tasks. Specifically, these different similarity groups which were formed show that the confidence with which students approached volume measurement problems connected and had direct effects on their volume measurement performance.

We found, also, that high-ability students had stronger and more accurate efficacy beliefs towards volume measurement tasks in comparison to low-ability students.

These findings confirm the earlier results by Pajares and Kranzler (1995) and Zimmerman et al. (1992). Furthermore, high ability students had more accurate efficacy beliefs towards volume measurement tasks with net diagram which were unfamiliar, whereas low-ability students had more accurate efficacy beliefs towards verbal volume measurement tasks which are more familiar to them.

Moreover, students who had high efficacy understand the volume measurement tasks better that the students who have low efficacy beliefs. This finding confirms the results of the study of Schunk and Hanson (1985). Also, students with high efficacy beliefs tend to assess the verbal tasks as easy and interesting, whereas the tasks with net diagram as difficult and less interesting. Therefore, these students' perceptions probably play an important role to their volume measurement performance and/or the development of their efficacy beliefs. This finding needs to be further explored.

In conclusion, the above findings about the predictive role of efficacy beliefs towards volume measurement tasks in different representations are very important in mathematics teaching and learning. Efficacy beliefs is an important component of motivation and behaviour (Pajares, 1996) and thus teachers need to develop ways to enhance efficacy beliefs of students of varied abilities. More specifically, high ability students need to solve "new" and creative tasks in which they will give the necessary attention and low ability students need to solve more easy and familiar tasks in which they can succeed.

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