WAYS OF WORKING WITH DIFFERENT THEORETICAL APPROACHES IN MATHEMATICS EDUCATION RESEARCH AN INTRODUCTION

Tommy Dreyfus

Tel Aviv University

The development and elaboration of theoretical constructs that allow research in mathematics education to progress has long been a focus of mathematics education researchers in Europe. This focus has found its expression in many CERME working groups: some are focused around a specific theoretical approach and others allow researchers from different theoretical traditions and backgrounds to meet and discuss. For example, the working group on *Argumentation and Proof* at the present (CERME 6) conference has reported on passionate discussions about different theories and their relationships (Mariotti, 2009). More specifically, relationships between theories have been made the explicit focus of attention of the theory working group that started at CERME 4 in 2005. This group has been reconvened at CERME 5 as well as at CERME 6; this year, we discussed fifteen papers, twelve of which make use of at least two theories and deal with how or why they can be connected in some way (see the part on Working Group 9 on *Different Theoretical Perspectives in Research in Mathematics Education* in these proceedings). The plenary activity from which this report emanated inserts itself in this line of work of CERME; one of its aims was to broaden the discussion about relationships between theories to include members of all CERME working groups.

The undertaking of mathematics education is very complex; this complexity is well expressed, for example, in Paola Valero's diagram (Valero, 2009). It is not without reason that the field has developed from having a curricular focus via a cognitive focus in various directions including philosophical, socio-cognitive, anthropological, ethnographic, and other perspectives, all the while producing home-grown theories to deal with all these aspects – and I am not even trying to distinguish between paradigms, theories, theoretical frameworks etc. For example, Realistic Mathematics Education has variously been characterized (including by people from the Freudenthal Institute) as a theory for mathematics education, as an instructional design theory or simply a philosophy for mathematics education.

When one reads a journal like *Educational Studies in Mathematics*, it seems at times that every paper presents a new combination of existing theories, a new theory, or at least a development of an existing theory. This raises the question how to look at and deal with the diversity of existing theories in mathematics education. Does this diversity express richness or does it express lack of focus (Steen, 1999) or even arbitrariness?

The question is made all the more urgent and difficult since theories come in different 'shapes' and 'sizes' and have different functions. Some concern the micro-genetic analysis of a learning processes in a classroom on a time scale of seconds, others the development of an individual student over months (or even years) and still others the momentary functioning of entire education systems. The ‘mesh sizes’ of theories thus range from the individual student via groups, classes, and schools to entire educational systems; and time scales under consideration range from seconds to years. Nevertheless, in the end they all deal with the same fundamental issue: How can students learn mathematics (better)?

However, even for (roughly) the same type of issue and scale, several theories with possibly different outlooks may exist; take for example the role of the social aspects in learning processes at the scale of a lesson: Is the social unimportant since deep mathematics is learned mainly when individual students are thinking by themselves, is the social the very vehicle of learning, or is it something in between, part of the context of learning (see Kidron, Lenfant, Bikner-Ahsbahs, Artigue, & Dreyfus, 2008)? Such a fundamental difference is likely to express itself in terms of different theoretical notions and hence different means and ways to analyze data.

Quite a lot of work has been done and published over the past ten years by people aware of the issues raised by the existence and use of many different theoretical frameworks, and trying to ‘do something about them’. Approaches have been very diverse. A few group studies have been published, in which researchers have worked on a common set of data, each researcher illuminating these same data from a different perspective such as a recent special issue on Affect in Mathematics Education (Zan, Brown, Evans, & Hannula, 2006). While this constitutes an interesting learning experience for the researchers as well as for the readers, it does not help us make progress toward connecting between the theories. We should be more ambitious. Nobody is probably aiming at a grand unified theory (see, e.g., Grand Unified Theory, 2009) as are theoretical physicists - this may be impossible altogether in the social sciences, and even if it is possible, mathematics education certainly has not reached this stage. We cannot even expect our community to converge to a set of common basic notions because the very idea of common basic notions negates the option of a variety of analytic approaches, and such a variety is needed in order to understand the complex multi-scale phenomena we are dealing with.

But we do need to make efforts to realize to what extent we are doing similar things in different languages and to what extent we use the same language to do different things. And once we realize that, we may want to establish connections, eliminate redundancies and distinguish what can and needs to be distinguished. Even more importantly, we want to find points of contact between theories that are dealing with different but related areas and find a language to talk about such theories together, to link between them in ways that are robust in the sense that they can be used by other researchers. These issues are very complex because theoretical frameworks are culturally situated – we have long known this from the difficulties many of us have to connect to and deeply appropriate the Theory of Didactic Situation (Brousseau, 1997) that has emanated from the French cultural background and grown in the environment of mathematics education in France. A recent issue of *ZDM - The International Journal on Mathematics Education* (Prediger, Arzarello, Bosch, & Lenfant, 2008) emanating from the CERME meeting at Larnaca offers a number of concrete case studies for how different research teams dealt with the fact that several theories were relevant for their study. There exist also examples from outside CERME, for example an attempt to coordinate argumentation theory and Realistic Mathematics Education to provide a microanalysis of a whole-class discussion (Whitenack & Knipping, 2002).

In the following two papers, two researchers experienced in consciously using, combining, comparing and contrasting several theoretical frameworks in the same study, will present different and possibly complementary approaches to such an undertaking. Angelika Bikner-Ahsbahs has taken the initiative of creating and coordinating a group of researchers who continue the work taking place at the CERME conferences also in-between conferences. She has coined the term networking theories to describe her view of how theories can be linked. John Monaghan presents a point of view formed outside of the CERME theory working group, on the basis of his research; this research has led him, for example, to refine the theory of abstraction in context, which has enabled him to take a step of integrating work on instrumentation with a dialectical, situated view of processes of abstraction; he has also recently connected the purely cognitive ideas of concept image and concept definition with a social view of learning mathematics. In his paper, he stresses the role of the person of the researcher when selecting (parts of) theories to network with; these two papers will be followed by some excerpts of the discussion that followed the presentations.

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