

THE JOINT ACTION THEORY IN DIDACTICS: WHY DO WE NEED IT IN THE CASE OF TEACHING AND LEARNING MATHEMATICS?

Florence Ligozat & Maria-Luisa Schubauer-Leoni

FPSE, Université de Genève (CH)

In this paper, we reflect on the Anthropological Theory of Didactics and the Theory of Didactical Situations in Mathematics as the roots of an emergent framework: the Joint Action Theory in Didactics. Disclosing some of the boundaries of the two major French theories in didactics allows us to sketch an integrative scheme of certain of their principles and concepts within the background of socio-cultural and pragmatist approaches to teaching and learning practices.

This paper aims at contributing to the discussion that has progressively given rise to a "theory networking space" in the previous Working Group sessions. We regard this work as an important step for several reasons. First, it accounts for the paradigmatic partition of the main theories currently used in mathematics education, ranging from the more *cognitive* ones that focus on the understanding processes of individual learners, to the more *cultural* ones, that are oriented by institutional and collective structures in which knowledge is subjected to social transactions. It sheds a new light on certain theories we are familiar with, since they are contrasted with some others on certain aspects like the role of social interaction, the role of learning environments, the role of the teacher...etc. Second, some very interesting mechanisms are disclosed about the ways researchers may attempt to connect these theories, while preserving their specificities. We especially value the tension between *integration* possibilities and *boundaries* to preserve, but also the triplet [principles, methodologies and paradigmatic research questions] that is worked out by Radford (2008).

As we support the development of comparative studies in didactics, these questions are of premium interest for delineating both the generic and the specific (i.e. content knowledge related) principles of the intricate processes of teaching and learning. More particularly, the work in progress in this CERME Working Group is an opportunity for us to reflect on the development of the Joint Action Theory for Didactics (JATD), for the purpose of grasping teaching and learning complexity under ordinary classroom conditions.

PART I : SKETCHING A NETWORKING SPACE FROM ATD AND TDSM

In the first part of this paper, we contrast the two major theories developed by the French didactics of mathematics, *i.e.* the Anthropological Theory of Didactics (ATD; Chevallard, 1985/1991; 1992) and the Theory of Didactical Situations for Mathematics (TDSM; Brousseau, 1997). Since these frameworks have developed over more than 30 years, this has to be drastically reduced to their major orientations, without having here the opportunity to decline the various branches that they inspired further

on. Indeed, what we are most interested in is their epistemological stance rather than outlining these theories *per se*. In line with one of the most important principle underlying both the ATD and the TDSM, we consider that theories, like knowledge, emerge as a collective elaboration to face a set of problems and questions that human groups experience in the development of societies. Thus, a good starting point for inquiring into theories may be to compare the realm of reality they account for, through *their paradigmatic research questions* (Radford, 2008) along with *their epistemological roots* in human sciences.

From an historical standpoint, the theorization of an "experimental epistemology for mathematics" that was worked out by G. Brousseau in the mid 70's is a mean to account for the generation of meaningful mathematical knowledge in classrooms. Then, in the early 80's, Y. Chevallard's anthropological analysis of the conditions of knowledge dissemination within institutions, shed a new light on knowledge taught as reworked from its genuine context of emergence in expert (or academic) communities. Therefore, the knowledge coherence and legitimacy as presented in school, has to be studied in terms of epistemic affordances and constraints. In both cases, the epistemological account of the knowledge content at stake as the third pole of the didactical system opened the era of the didactics of mathematics as a science taking off from the psycho-pedagogical stance on teaching and learning.

Since the early works, the ATD relied upon an assumed structuralist point of view of knowledge development within institutions that can be referred to the background of a Durkheimian sociology and eventually to certain socio-cultural approaches. In line with Douglas (1986), the basics of the ATD are that (1) ways of thinking of individuals are shaped by the collective practices to which they partake and (2) these collective practices are oriented by purposes whose coherence defines the primary goal of an institution as a social organisation bound to achieve a type of task. In the case of educational institutions, the transmission of a socially agreed culture is the core of the activity, relayed by an "intention to teach" and an "intention to learn" at the level of the teacher and the students respectively. Thus, the determination level of what the participants do is to be studied in the institutional patterns of the teaching and learning culture. Early works from Chevallard (1985/1991) have stated that the way mathematical knowledge is ordinarily presented within educational institutions does not match the epistemological way the mathematics are built (*i.e.* the mathematical praxeologies in the ATD). Differences in goals generate differences in tasks to be achieved and so the patterns of school mathematics are somewhat distant from academic mathematics. The *transposition process* as the starting point of the ATD accounts for the specific organisation of knowledge in the purpose of its transmission within educational institutions. In particular, the didactical transposition process is characterised by (1) a *decontextualisation* of mathematical practices from the problems they originally attended, into sequence of topics to fit the curricula constraints and the frames of teaching time; (2) a *recontextualisation* of these topics by the teachers, in order to make the students encounter the knowledge to be taught within

the classroom practices. This process has long been regarded to be consubstantial to the functioning of didactical systems as ruled by institutional practices. In recent works, it has been refined by featuring the didactical praxeologies as a set of practices, combining with each other, in order to describe the possibility of studying the process of mathematics in the classroom. It is structured in terms of moments that are *theoretically inherited* from the praxeological structure of the mathematical knowledge, *i.e.* the two levels of practices that correspond respectively to the techniques for solving a type of problem and the formulation / justification of these techniques. In furthering this, the ATD also attempts to account for the role of words, graphics and gestures as "ostensive objects" that shape the mathematical activity. Ostensives encapsulate the socio-cultural definition and values of the mathematical knowledge and they provide tools for a praxeology to develop. In our view, the ATD's paradigmatic research questions attend to a *top-down systemic approach* of the mathematical studying process. A description of the mathematical tasks and the possible didactical praxeologies are attempted as forms of institutional practices.

The epistemological roots and the research questions of the TDSM are more complex to depict. Brousseau's well-known starting point is that a given mathematical knowledge can be functionalised by a *fundamental situation* gathering the epistemological conditions for the emergence of the considered piece of knowledge in the human culture. This major underlying principle is somewhat compatible with the definition of the mathematical praxeologies in the present works of the ATD. Whereas ATD considers this principle as a mean to *describe the possible structures of human practices in studying mathematics*, at the level of institutions, the TSDM refers to the same principle for *modelling the epistemological conditions* in which the students may develop some meaningful mathematical knowledge, within the classroom.

A major concern in G. Brousseau's work is to identify such fundamental situations in the primary school mathematics and to derive some didactical situations from them. In such situations, students encounter some constraints requiring an adaptation of their prior knowledge towards the learning of a new one. The students have to work out the solution of a problem in which specific knowledge cannot be avoided. Brousseau explicitly refers to the Piagetian theory of learning. The core of the learning process relies upon the students' adaptation to a *milieu* as a set of epistemological constraints. The milieu is designed to orient the students' actions by providing some positive or negative feedbacks to the strategies used. To achieve meaningful learning, the students have to take the responsibilities of their game (devolution) without relying on the teacher's feedbacks. This is what Brousseau defines as an *a-didactical situation*, in which the student is supposed to focus his/her interest on a "game" against the milieu and "forget" the teacher's expectations at least for a while. From the student's point of view, the outcome of the game is a new "connaissance" that is being progressively socialised within the classroom debate. Typically, the student first *acts* to find a local solution to the problem, then *formulates* his/her strategies through a communication game and finally, the strategies may be *validated* within a

controversial debate in the classroom. Moving from the peculiar answer to the problem to a generalised pattern of knowledge is supported by *some changes in the milieu* with which the student interacts. Then, the institutionalisation process managed by the teacher makes sure that the "connaissance" constructed by the students within the didactical situation, is adequate to the definition of knowledge in curricula. Thus, the outer horizon of Brousseau's didactical situations remains coherent with a cultural approach of knowledge. However, the kernel of this theory relies upon *a constructivist epistemology* where the student-milieu relationship primes the learning process, by the mean of the a-didactical situation. Social interactions come into play for anchoring the "connaissances" built by students as individuals, within the pre-existing socio-cultural knowledge. As noticed by Radford (2008), they are "a mere facilitator of individual's development of mental structures"(p320). In our view, the paradigmatic research questions that the TDSM addresses is *the design of epistemic models of knowledge, i.e.* situations that enable an adaptive shift of the student towards the construction of new knowledge, without relying onto the teacher's indications at some points of the didactical contract.

Both these theories attempt a model of teaching and learning mathematics as a three poles system where the "being teaching" (teacher) and the "being taught" (student) are two epistemic instances constrained by the knowledge structure. In the ATD framework, the diffusion of mathematical knowledge is studied merely at the collective level of the social structures whereas the TDSM attempts to link the conventional patterns of knowledge and the connaissances constructed by individuals in a rather functionalist way (the milieu originates in the student's actions /formulations/validations). These structural and / or functional stances on the teaching and learning process were crucial in the development of the French didactics of mathematics. We regard it as a major epistemological break from the merely psychological approaches to students' difficulties in mathematics and the pedagogical positivism more generally. It afforded the premises of a science of the teaching and learning phenomena in mathematics, and it also inspired other subject matter didactics in the French speaking community. However, moving back to the major features of each theory allows to highlighting some irreducible boundaries between them.

The epistemological boundary: The TDSM draws strongly on the student – milieu interactions, as an epistemic model of the adequate conditions for reconstruction of knowledge to occur within didactical conditions. The teacher's role in the devolution and the institutionalisation phases is an add-on. In between, the teacher organises the constraints of the milieu to sustain the optimal interactions. The dualistic relationships between the student and the milieu exclude the vision of the classroom social environment as a "thought collective" (Douglas, 1986) to which each student is subjected *ipso facto* through the use of language and more generally signs that are socially agreed. *The predominance of the milieu, as a pre-structured environment made of material, symbolic and social objects to which students have to adapt themselves, shadows the reflective activity that they may also activate to make meanings from*

collective practices. The adaptive function of the milieu addresses the individual minds as independent structures that become intertwined through the formulation and validation games. The reference to the collective practices is not continuous in the participants' experience as it is supposed to be in the underlying principles of the ATD framework. However, one can also argue that the ATD focuses on the institutional practices mainly but the way individuals may get the ownership of these practices and eventually make them evolve, is not accounted for. Very few elements describe what the participants effectively do within the didactical system, in order to teach and learn. As stated by Arzarello, Bosch, Gascon & Sabena, "the non-ostensive objects exists because of the manipulation of the non-ostensive ones within specific praxeological organizations" (2008, p181). *The interpretative process of the collective meanings by individuals are shadowed by the schemes of institutional practices that (over)structures local purposes and psychological processes*. Although the concept of "mesogenèse" was promisingly introduced (Chevallard, 1992) to account for the dynamics of the relations between individuals and objects in their environment, it did not deepen, for instance, how the semiotic systems handled by students (i.e. ostensives) may generate meanings, i.e. non-ostensives (Schubauer-Leoni & Leutenegger, 2005).

The methodological boundary: Early works from Chevallard stated that, ordinarily, the knowledge presented to students in classrooms does not appear according to the epistemological conditions in which it was born, due the decontextualisation and sequentialisation processes in curricula. From this point of view, the works carried out by Brousseau's team may be regarded as an attempt to *counter* the transposition process by redesigning school mathematics into meaningful situations that are not ordinarily supported by didactical institutions. Indeed, a didactical situation is supposed to restore some of the epistemological conditions for knowledge to be built, by designing specific learning environments. A series of fascinating designs were produced in which cultural knowledge is genuinely functionalised (numbering with integers, measuring capacities, introducing rational and decimal numbers, Euclidean divisions, linear functions...etc.). But the way ordinary school institutions may incorporate these situations is not investigated, leaving some opportunities to misleading interpretations of certain examples of didactical situations in some teaching materials. Furthermore, the design process tends to minimize the teacher's work which is then strongly supported by the research team. One can say that it shunts the "*repersonnalisation*" process of the institutional patterns of knowledge, which is ordinarily carried out by the teachers. The relationships between the milieu to be organised and the interaction arena which is ruled by the reciprocal expectations of the didactical contract is the main concern. But the relationships between the ordinary resources that the teachers use and the effective teaching environments they implement cannot be investigated from Brousseau's paradigmatic research questions because they strongly rely upon research designs.

From these boundaries, we argue that (1) the TDSM cannot be regarded as a direct continuation of the ATD framework in terms of classroom practices and interactions among individuals ; (2) the structuro-functionalist stances that are consubstantial to both these theories does not allow an account of the interpretative motions of the subjects within the didactical system as an social institution. These two points could be said to be out of synch with the purposes of those researchers who actually work with one or another theory. Nevertheless, we argue that if didactics is to be a science of the teaching and learning phenomena about a given content knowledge, then some new research questions have to be addressed.

PART 2 : THE GROWTH OF J.A.T.D. AS AN INTEGRATIVE THEORY

In this part, our purpose is not to feature details and examples of use of the Joint Action Theory in Didactics, since this is presented in Sensevy (this group of papers). We rather would like to present the conditions of emergence of its paradigmatic research questions and how some principles and concepts may be borrowed from the ATD and TDSM, by the mean of a conversion process in the light of some pragmatist theories to match a socio-historical perspective of knowledge development in teaching and learning (Forget & Schubauer-Leoni 2008; Ligozat, 2008).

Many empirical studies have reported that the specific role played by the milieu in TDSM's is a feature that is hardly observed as controlled by the teacher in ordinary classes. Most of time, the set of objects partaking to the situation is not self-sufficient to enable students develop an epistemic relation to the problem or task to be achieved. Or, to reformulate this in the terms of the ATD, consistent bodies of mathematical praxelologies are hardly managed by the teacher. However, in these ordinary conditions, that we consider to be the most common teaching and learning reality for mathematics, we cannot envision that no learning happens at all. It progressively leads us to consider that *didactical situations that would be a priori endowed with some a-didactic affordances may not be an adequate model* to theorize the ordinary teaching and learning practices. In other words, the "obdurate reality" of classrooms as an empirical field has to be investigated. What kinds of meanings are constructed in students' "ordinary" learning experience? How does the teacher support them? What kind of common ground is being built for the whole class and how does it fit with the cultural definition of knowledge? What do we know about the way teachers select, structure, refine and adjust instructional settings? ...etc. Such questions arose from empirical observations of classrooms at primary school mainly and with an increasing demand for professionalizing teacher education. The institutional location of researches in didactics in teacher training institutes (IUFM in France, since the early 90's) and/or in some department of educational sciences (e.g. Geneva) has broadened the scientific scope of the subject matter didactics toward a comprehensive account of the didactical phenomena as an educational matter. The realm of studies of the didactics of mathematics as a science meet the opportunity to grow from a merely epistemological programme to a quest for an account of human practices that are specified by the conveyance of a socio-historically built culture. In this

context, the *paradigmatic research questions of the JATD are new ones compared those featured by the ATD and the TDSM*. The teacher and the students cannot be regarded any longer as epistemic instances merely subjected to the structure of knowledge. The interpretative part of their activity within the educational institutions as a social framework has to be accounted for too. To be clear, we are not arguing that the JATD could replace the fields of investigation that are at the focus of the ATD and/or the TSDM. We would like to point out that it is a complementary framework aiming at giving a status to the subjects' actions and interpretations relatively to the institutional contexts for teaching and learning a given subject matter.

In producing such a framework, we call in some principles that are rooted in both human activity as primarily social and historically built *and* in a pragmatist view of the situations in which the activity develop. Against this background, *the transposition process* sketched by Chevallard and *the didactical contract* theorized by Brousseau, can be viewed as the starting point of *a hybridizing plot*.

First, we postulate that the interpretation of classroom events cannot be performed by focusing solely on either the teacher's actions or the students' ones. We propose to look at *the teacher and students "joint" action* to account for both the historical and the situated interdependence of the classroom actions. Such a joint action may involve separate and distinctive acts that are bound together to make the collective action progressing in some cooperative patterns. The genesis of joint action is based partially on orderly, fixed and repetitious definitions of previous acts through the collective memory that is relayed by the use of signs (graphical, gestual, or vocal). Of course, such joint action is also open to uncertainty and so the transformation of the use of signs to sort new tasks and problems. These statements are general to many actions in human activity (Clark, 1996). A way of specifying them is to consider both the specific purposes of educational institutions and the forms of knowledge to be taught.

i) From TDSM, the didactical contract is probably the most likely principle to address the problem of the individuals' interpretation of contextual practices. We consider that the intention to teach a given topic supported by the teacher generates an expectation to learn "something" from the students. Regularities in the functioning of the classroom as a didactical institution progressively makes the students aware that a teacher usually has "something" in mind beyond the concrete tasks or questions they have to sort. On his/her side, the teacher organises didactical time slots for making the students develop a reflection, an inquiry, the achievement of a task...etc. As soon the student is aware of what is being taught, he/she supposed to know, and the teachers moves on toward another topic. Therefore, teachers and students always remain in an asymmetrical relationship due to the difference in the respective status of their knowledge. We consider *the cultural stance of the didactical contract* as a system of reciprocal expectations merely, according to which the teacher and the students adjust their actions. The asymmetrical status of the teacher and the students relative to their respective relationship to knowledge is consubstantial to the chronogenesis and topo-

genesis processes that were initially sketched by Chevallard (1985/1991) to describe the structure of recontextualisation of knowledge in the classroom.

ii) However, we do not maintain the constructivist stance of the didactical contract, *i.e.* the contract as regulating an antagonist set of objects that would constrain the students' actions. A converting plot is then required to describe the relationships of the participants to the objects partaking to the situation. Following Mead's definition of the social act (Mead, 1934), we consider that individuals indicate the objects to themselves in line with the function these objects have in collective practices. The meaning-making process is supported by actions –gestures and discourses- in communicative situations. Objects have a meaning for one-self only because they have also have a meaning for otherselves in the situation but also in the culture pre-existing to the situation. Such processes, as indications of objects within the background of language games (Wittgenstein) are actually under investigation for describing the articulation of collective practices and meanings made by individuals. The distinction of "which object counts for which participant", or "from whom this kind of relation comes out" and "who grasps it" is important in determining 1) the set of objects that participants indicate to themselves, 2) the meaning that they may ascribe to their own actions with these objects, 3) the control they gain from it and that may be re-allocated in further experiences. This threefold meaning-making process over time is described as *a mesogenesis*.

iii) Then, it follows that the topogenesis and the chronogenesis are strongly related to the teacher's actions because of his/her leadership in the didactical relation. The teacher is the one supposed to orient the student's actions in order to help him/her learn, but also to notice the student's elaborations in order to designate them a new knowledge. Therefore, some *chronogenetic and topogenetic techniques contribute to the building of a common reference* (objects, relations) in the mesogenetic process. Chronogenetic techniques are anything that the teacher may do in order to *orient the students' actions* toward the piece of knowledge to be learnt. The topogenetic techniques are anything that the teacher does to *regulate his/her involvement* in the joint action and to assign a role to the students all together or as individuals. The devolution and institutionalisation categories for the teacher's action primarily exist in Brousseau's didactical situations, but they may be revised as generic to any teaching process.

iv) The specification of the joint action also operates through the epistemic tasks that are to be achieved. The pre-existing culture necessarily comes in when studying how knowledge to be taught is presented in the teaching materials and curriculum texts. But the purposes of the ordinary practices in classrooms may be rooted in some multi-determination levels other than merely mathematical ones. Thus, acknowledging for the individuals' interpretations of the situations they encounter lead us to reconsider the transposition of knowledge within the didactical institutions from a *bottom-up* point of view that is coupled with the *top-down* analyses typically performed by the ATD framework. We conduct an analysis of the epistemic tasks that are em-

bodied in the teaching materials that the teacher uses (Ligozat & Mercier, 2007). For instance, from the worksheet proposed by the teacher to the students, we may inquire 1) what could be learnt in performing it and then 2) what could be taught according to the curriculum of a given grade. At this step, the fundamental mathematical situations or the mathematical praxeologies provide some useful ways of modelling the epistemic knowledge. The possible gaps and contradictions that are issued by the decontextualisation process may be disclosed against the background of the mathematical practices. Then a bottom up process aims at reconstructing the meanings that objects, situations and practices may have for the participants to the classroom joint action. In this second process, the epistemic model of mathematical knowledge is used as reference to understand 1) what is actually taught and learnt in the joint actions; 2) what the distance left toward the cultural knowledge is and 3) what the epistemic necessities that bend the joint action in some specific ways are. This type of analysis may be carried out at various scales of analyses (a classroom episode, a whole lesson, a teaching unit spread over several lessons...etc.) that can be nested together. The coupling of both the transposition and the social transactions analysis with the classroom supports the investigation method in the JATD framework. A full study of the course of joint actions in the classroom against the transposition of measurement at primary school was achieved in Ligozat (2008).

CONCLUSION

The JATD attempts to encompass a huge programme for didactics as a scientific domain studying the human transactions organised about the transmission of a socio-historically built culture. The need for a theory that aims at theorising teaching and learning practices as they occur in ordinary classroom seems unavoidable. However, in its present state, the JATD has to face different kinds of problems: 1) defining its *identity* as a generic theory for the study of the didactical facts but which develops and produces results by accounting for the specificity of knowledge domains; 2) the further clarification of *its epistemological stances* with respect to the principles and concepts that are borrowed from other theories and 3) the definition of some methodological *units* from its very extended realm of reality, that may be worked out independently without taking the risk of generating some misleading interpretations. The very intention of this paper can be regarded as an attempt to contribute to the first and second points with respect to relationships the JATD has with other theories concerning specific domain didactics. However the clarification of the epistemological stances of the action theories that we invoke still remains a major stake for the works in progress.

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