THE PRACTICE OF (UNIVERSITY) MATHEMATICS TEACHING: MEDIATIONAL INQUIRY IN A COMMUNITY OF PRACTICE OR AN ACTIVITY SYSTEM

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Theoretical perspectives of 'community of practice' and 'activity theory' are used along with constructs of 'inquiry' and 'critical alignment' to theorise developing mathematics teaching at university level. The paper introduces and explains the theories and relates theory to issues in the ongoing development of a mathematics course for engineering students. It focuses on developmental research which seeks both to chart developmental progress and lead to more informed teaching relating to the goal-directed activity of those involved, the systems of which they are a part and the tensions/issues within which development occurs.

INTRODUCTION

In recent writing (e.g. Jaworski, 2007, 2008a) I have focused on *communities of inquiry* in developing mathematics teaching and learning. I have drawn particularly on Wenger's (1998) concept of *identity* based in modes of *belonging* to a *community of practice.* This has been in the context of *developmental research* – that is research that seeks to develop practice while charting that development (see also, Goodchild, 2008). Here, I want to look more closely at how theoretical and methodological perspectives not only complement each other but are intertwined in the complex process of improving practice in teaching and learning mathematics.

I distinguish two areas of theory here. The first is Wenger's theory of *belonging to a community of practice*. The second is theory of *inquiry*, based in Vygotskian ideas of *activity*, *mediation* and *tools*. The complex notion of *identity* and its relation to *community* is a central unifying force. I have used these theoretical ideas previously to address analysis of data in a longitudinal study of developing mathematics teaching and learning in schools through collaboration between teachers and didacticians in Norway. Many sources document this research (e.g., Jaworski, 2007; 2008a; Jaworski, Fuglestad, Bjuland, Breiteig, Goodchild and Grevholm, 2007; http://fag.hia.no/lcm/papers.htm). In this paper, I focus on the beginnings of research into developing mathematics teaching in a university mathematics department, focusing on my own practice as a (novice) mathematics teacher in this context.

The structure of this paper is as follows. First I give accounts, separately, of the two areas of theory, relating them explicitly to practices in mathematics teaching and learning. Then I turn to research into my own practice as a university mathematics teacher – a rather different form of practice from that of teaching mathematics in schools which has been my main focus in previous papers. I will expose some of the differences and related dilemmas and ways in which the two areas of theory cohere to support a theorising of practice and analysis of data. In doing this, I will address the nature of developmental research, its importance in contributing to development in mathematics teaching and learning, and issues in its operationalization

BELONGING TO A COMMUNITY OF PRACTICE

The term ‘community’ designates a group of people identifiable by who they are in terms of how they relate to each other, their common activities and ways of thinking, beliefs and values. Wenger (1998, p. 5) describes *community* as “a way of talking about the social configurations in which our enterprises are defined as worth pursuing and our participation is recognisable as competence”.

Within a university school of mathematics I recognize mathematicians, mathematics educators and our students at various levels as part of a community. In this community we engage with mathematics in various ways: learning mathematics, teaching mathematics and doing research into mathematics or into learning or teaching mathematics. Mathematics itself and what it means to do mathematics is central to this community. We can recognize both individuals and groups: that is to ascribe *identity* to both. Holland, Lachicotte, Skinner and Cain (1998, p. 5) write, “Identity is a concept that figuratively combines the intimate or personal world with the collective space of cultural forms and social relations”. Identity refers to *ways of being* (Holland, et al. 1998) and I talk here about ways of being in the university mathematical community. For example, people who teach mathematics have identity with relation to what it means to teach mathematics within a university environment, and within one particularly.

Within this community we all engage in some forms of *practice*: Wenger writes of practice: “The concept of practice connotes doing, but not just doing in and of itself. It is doing in a historical and social context that gives structure and meaning to what we do”. (1998, p.47). So *doing* within the school of mathematics means *engaging in the practice of university mathematics*. This *includes* doing mathematics, whether this is on the part of undergraduate learners or of research mathematicians; it *includes* students and academics researching aspects of the learning and teaching of mathematics, and associated contexts such as use of technology in teaching and learning and mathematics support for learners at all levels.

Wenger talks about identity in communities of practice as being about *belonging* to a community of practice. He suggests three modes of belonging: engagement, imagination and alignment. We *engage* in practice with others: our participation requires us to *do*, not just to observe the practices of which we are a part. Students have to engage with learning, teachers with teaching. All engage with mathematics. Engagement is the fundamental activity in doing. In order to engage we have to make sense of what we do; *imagination* allows us to interpret its various aspects and conceive of ways to achieve what we see as the goals of practice. We are not alone in our enterprise: the community of practice has developed over time and has norms and expectations of what will be done and how. We need to *align* with the norms of practice –

alignment provides the sociohistorical dimension within practice by which the practice is recognisable, sustainable and continuing.

Seeing university mathematics as a social practice is becoming a familiar basis for research in mathematics education related to learning and teaching mathematics in a university (e.g., Burton, 2004; Hemmi, 2006; Nardi, Jaworski & Hegedus, 2006) which has a long history and tradition, both in universities generally and in any one in particular. Recognisable aspects are university terms or semesters, lectures and tutorials, courses organised across several years of study in calculus, analysis, algebra and so on, and forms of assessment. Mathematics itself has an even longer history, with traditions in philosophical groundings, how topics are grouped and how learning and understanding mathematics are perceived. As mathematicians engage, whether in teaching or research, they bring imagination to interpret courses or research topics and they align with accepted practices, perpetuating a status quo and ensuring ongoing traditions. Students coming in fresh to the practices learn quickly acceptable forms of engagement and, imaginatively, how to make the system work for them according to their own, more familiar, communities of practice. They align with norms of practice developed over centuries and experience insights and obstacles familiar to cohorts of their forebears.

However, perpetuation of tradition is not always helpful in ensuring effective learning outcomes, especially if cohorts of learners no longer fit traditional moulds. Difficulties at the transition between school and university have been extensively reported (Hawkes & Savage, 2000). Existing research describes the mismatch between university lecturers' expectations of mathematics undergraduates and student competencies (London Mathematics Society, 1995). Brown, Wiliam, Barnard, Rodd & Macrae, (2002) reported how mathematics undergraduates' attitudes change and many become disillusioned with the style of teaching mathematics in university. In a study of teaching in university mathematics tutorials, Nardi, Jaworski and Hegedus (2005) suggested a variability of pedagogic awareness, in the teaching of university mathematicians, shifting from the naïve and dismissive to the confident and articulate. Hemmi (2006) studying mathematicians' and university students' attitudes to *proof* found distinct differences in the ways students and their teachers perceived mathematics learning and teaching at university level, and categorization of mathematicians interview responses showed significantly varying views on the nature of teaching. Burton's (2004) interview study of 70 mathematicians revealed both common traditions in mathematics teaching and research and particular viewpoints and idiosyncrasies. Such sources have highlighted both significant issues related to traditional practices and new concerns relating to changing traditions in which more research is urgently needed.

ACTIVITY, MEDIATION AND TOOLS: THE ROLE OF *INQUIRY*

Doing mathematics, for students at any level, requires engagement with abstract concepts which are not readily visible in the world around us. Although we can see particularities of mathematics in our familiar social worlds (examples of numbers or shapes, use of ideas of probability or statistical tools), expression of mathematical generality, necessarily, is abstract and requires abstract means of expression and justification.

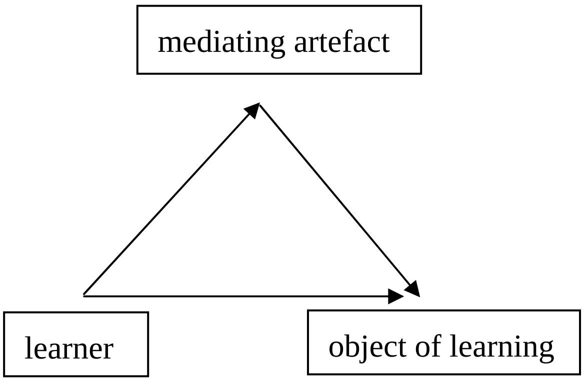

Figure 1: A simple mediational triangle

Schmittau (2003), drawing on Davidov, speaks of mathematics as involving *scientific concepts* which require “pedagogical mediation for their appropriation” (p. 226). Scientific concepts are concepts which cannot be learned spontaneously in engagement with everyday life (Vygotsky, 1986). Some form of mediation (going between) is needed for students to meet mathematical concepts and engage with them in meaningful ways. Particularly, Vygotsky talks about tools and signs which mediate the process of learning – *mediating artefacts* (see Figure 1). Such artefacts include both physical and intellectual tools; for example books and writing on paper, and language in which ideas and concepts are expressed. Technological tools can be helpful mediators for learning mathematics and teachers can orchestrate the use of technology to promote learning. *Pedagogical* mediation refers to the role of a teacher in creating opportunity for students to learn. The simple mediational triangle (Figure 1) deriving from Vygotsky and Leont’ev (e.g. Leont’ev, 1979) has been extended by Engeström (e.g., 1998)to include mediation *in social worlds* captured by the terms “rules”, “community” and “division of labour” to which he refers jointly as “the hidden curriculum” (1998, p. 76). (See Figure 2). It is “hidden” because the factors involved are often not considered or questioned overtly as mediating factors in the education enterprise.

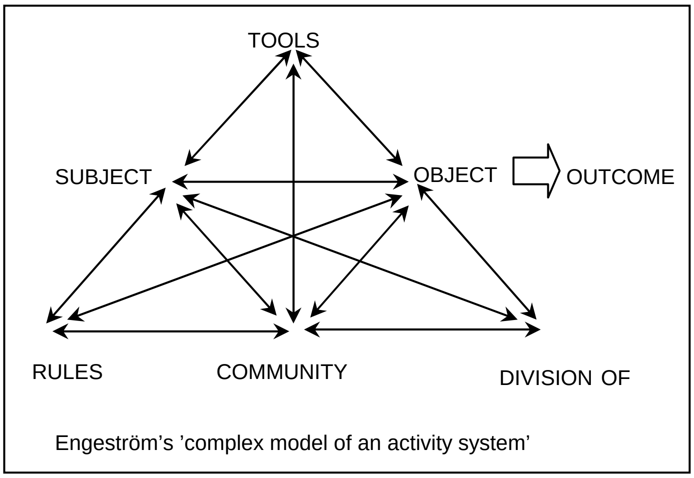

Figure 2: An expanded mediational triangle

In university mathematics education, the *rules* include courses to be taken, measures of success in a course or programme, expectations of participation; *community* encompasses those who engage in processes of mathematics learning and teaching with the purpose of advancing mathematical knowledge and understanding, primarily students and teachers; *division of labour* encompasses the differing roles and responsibilities of those within the community, for example teachers to teach and students to learn. Thus, for a learner (the *subject* of the learning process) with an *object* of learning mathematics, the activity of engaging in mathematics in a mathematical community is mediated by all of these factors as well as the artefacts commonly used to support learning.

Engeström refers to the system defined by the relationships illustrated in Figure 2, as an *activity system*, following a theory of *activity* deriving from Vygotsky and Leont'ev. Briefly, all activity is motivated, and comprises actions which are explicitly goal directed. Thus, in any such system, participants act according to goals and their actions are mediated by the various elements of the system (Leont'ev, 1979; Jaworski & Goodchild, 2006). An issue that arises in the learning and teaching of mathematics in a university is that of potentially conflicting communities where the goals of activity are concerned. So within a broad activity system of university mathematics (including students, teachers, researchers, learning, teaching and so on) we see subsystems which relate to the activity of certain groups. For example, teachers working within the established university system and its mathematical community have expectations of how students will act in relation to the norms and expectations of learning mathematics in a university. They have goals for students' learning and their actions are a consequence of their goals.

For students however, the system looks different. They come from different traditions in school systems and wider society. They are used to the kinds of relationships with teachers and peers that are afforded by pre-university education. They are highly influenced by popular culture and their peers. Stepping into the university system requires a re-alignment in their engagement; imagination, relating to the various communities of which they are a part, inspires their re-alignment. Lave and Wenger (1991) have offered a theory of legitimate peripheral participation to account for the transition for a novice into a community of practice. Here, I draw rather on Wenger's tri-partite characterisation of *belonging* and to activity theory to account for the dichotomies that emerge from collision of communities. Engeström's (1998) use of the expanded mediational triangle shows recognition of tensions in and between activity systems which can help address dichotomies. I say more on this below.

The place of *inquiry* in these theories and systems is central to my arguments in the paper. I see inquiry first of all as a *tool* mediating mathematics learning, teaching and development and then as *a way of being* in practice (Jaworski, 2006). When we start to inquire, we can be seen to use inquiry as a tool. Through sustained use the tool becomes a part of our identity as well, possibly, as of the identity of our community. Concepts relating to inquiry in practice, and its relation to these two established areas of theory, have emerged from 5 years of research in Norway (Jaworski et al., 2007). Seeing inquiry first as a tool emphasises its mediational characteristics within an activity system. Teachers and students, inquiring into the processes of learning and teaching, achieve "metaknowing" (Wells, 1999, p. 65ff) through inquiry practice. Inquiry in mathematics involves asking questions and working on problems which engage participants and lead to new awareness and ultimately knowledge – we see this both in the activity of research mathematicians (e.g., Burton, 2004) and, where an inquiry pedagogy is in place, in classroom mathematics. Inquiry in teaching mathematics involves teachers in asking questions and working on problems in didactics and pedagogy; inquiring into ways in which opportunity can be created fruitfully for mathematical learning. Inquiry is also central to a developmental research process in which research into aspects of learning and teaching mathematics leads to enhanced knowledge in the academy and, importantly, to more *informed* practice (Goodchild, 2008; Jaworski 2008a).

Seeing inquiry as *a way of being* shifts inquiry from its status as a tool, to a more fundamental constituent of an activity system in which it becomes part of the "hidden curriculum", having a consequence of *making the hidden curriculum less hidden*. To manifest inquiry as a way of being requires inquiry to become part of the fabric of learning and teaching, what is taught and how it is approached, to such an extent that it permeates the rules, community and division of labour. It therefore offers a response to tensions and dichotomies that leads to metaknowing and possibilities for more knowledgeable practice. In order to explain this, I have introduced the concept, of *critical alignment*. Before discussing this in theory, I turn now to the context of university teaching and learning, and my own practice as a (novice) university teacher.

TEACHING MATHEMATICS TO FIRST YEAR ENGINEERING STUDENTS

At my university, the engineering faculty entrusts the mathematics teaching of its students to the *Mathematics Education Centre* which is the smaller of two parts of the *School of Mathematics*[1]. As I write this, I am currently in my second year of teaching a cohort of students in materials engineering some of whom have relatively low mathematical qualifications[2]. In the first year, I taught the weakest of these students (16 of them) separately from the rest and was able to develop good individual relationships. This year, all the students are together (around 70) and the approach to teaching is influenced strongly by this larger number. I want all students to be able to engage with mathematical concepts, to develop both conceptual understanding and procedural fluency and to be able to apply these to their engineering tasks. So, one area of inquiry is how I teach: what I do, how I do it, and what it achieves; included within this is encouraging students to inquire as part of their learning of mathematics. I bring an *inquiry way of being* as a result many years of experience, but nevertheless

[1] The other part is the Department of Mathematical Sciences. Members of both departments teach mathematics. Largely, those in the DMS do research in mathematics; those in the MEC do research in mathematics education.

[2] Some have not done mathematics beyond GCSE (the national examination at 16+). Others have very low grades in A level mathematics (the national examination at 18).

in this new arena I need to use inquiry overtly *as a tool*, both for myself and for my students. Methodologically, I engage in research and development cycles (Goodchild, 2008), planning, observing and analysing teaching and learning as it progresses; collecting data through teaching plans, reflective memos, student work, assessment tests, a student survey and student interviews.

Due to limitations of space here, I focus on just one aspect of teaching, for both year-groups of students. In the first year, to extend a more direct focus on curriculum topics, I offered a weekly investigative problem for students' exploration, requiring mathematical concepts with which students needed to develop strength and confidence[3]. It was introduced in a class session (we had two 50-minute sessions per week for 30 weeks); students were asked to continue to work on it in their own time, singly or in groups, and each one to give me some of their working and findings from the problem. Attendance at class sessions was very variable, but most of those who came handed in some work on which I wrote comments and returned to them. I learned about each student's mathematical skills and understanding from this activity. Observation over these weeks showed a willingness to engage with mathematics in non-routine ways on the part of more than half the students, and a classroom atmosphere in which questions could be asked and addressed and students mainly contributed actively (speaking up, asking questions, coming to the board) in class.

It became clear that some students had very weak mathematical skills, especially relating to algebra. When we came to the topic of exponential and logarithmic functions, I anticipated the difficulties that this topic would present. It seemed necessary to put all time and energy into the topic, and this halted the weekly problems. While maintaining an active questioning approach, I moved into a more direct approach to the topic: involving the class in sketching graphs, noting functional characteristics and relationships, expressing meanings aloud and addressing fundamental questions, and a strong emphasis on the rules of exponents and logs and their use in solving equations. Two outcomes were (a) in the related class test, several students achieved more highly than in two previous tests; (b) in a questionnaire in which I asked students to comment on their participation in the course, the level at which they rated their understanding of this material seemed more realistic and accurate than in relation to earlier topics. In my own reflections, while I was regretful of the demise of the weekly problem (it was not reinstated), I recognised that the teaching approach to *exp* and *log* had also achieved significant outcomes. I then had to rethink the objectives of my approach overall and their practical interpretation within constraints of time, curriculum and so on (Jaworski, 2008b). This has had implications for the current teaching. With a cohort of 70 the investigative problems with quick feedback would not be possible. The more direct approach has been maintained to a strong degree, and liai-

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[3] For example, the *painted cube* problem which affords experience with algebraic formulation and manipulation – a wooden cube is painted on the outside and then sliced into smaller cubes all the same size; how many cubes have paint on one face, two faces, three faces?

son with the engineering department has started to produce problems relevant to the study of the particular students. An investigative element has been included using a *GeoGebra* medium.

The activity outlined above incorporated an *inquiry cycle* (plan → act and observe → reflect and analyse → feedback to planning) which led to growth and recognition of knowledge which should feed back into planning for teaching both locally and globally. Issues addressed included problems of variable attendance, a wide range of mathematical experience within the class, the time factor in focusing on a problem of the week, the demands of concepts that students found difficult and so on. *Aligning* within the university system was and is a necessity, but the element of inquiry has allowed a questioning of what is possible, experimentation and critical review of outcomes, and modification according to observation and analysis. This shows *critical alignment* in practice with related growth of knowledge and understanding.

An activity theory analysis shows some conflicts/tensions in these issues. For example, the problem of the week afforded development of confident mathematical participation and opportunity to work algebraically. The more direct addressing of mathematical concepts and associated skills afforded a greater achievement in curriculum-related summative assessment. Time and other factors militated against inclusion of both of these approaches. These issues can be seen as breaks in the mediating links in Engeström's triangle and highlight areas where the system is in conflict. Such conflict fosters the meta-knowledge that is needed to move forwards productively (e.g., Engeström 1998, p. 101; Jaworski & Goodchild, 2006).

I contrast here the two ways of theorising teaching development. Seeing *critical alignment in practice* emphasises *the inquiry process* in belonging to the community of practice which allows modification and change within engagement, imagination and alignment. The practitioner here brings an overtly critical eye to the practice and finds ways of adjusting her alignment. An activity theory analysis allows juxtapositioning of key elements of the activity system and examination of their relationships. Tools (e.g., the investigative problems), rules (e.g., lecture timetables), community norms (e.g., students who do not attend lectures) and division of labour (e.g., the expected roles of students and lecturers) can be seen to be in tension. Thus the analyst finds here a valuable tool in revealing the issues, their nature and relationship. This is both explanatory and predictive: it offers ways of seeing the status quo and reveals possibilities for consequent activity.

I see these two theoretical frames to have rather different functions. The first is closely related to action in practice: recognising where alignment is required and where it can be adjusted. It offers a practical interpretation in the use of inquiry as a tool, and aids development of an analytical awareness of how the inquiry cycle can both raise and address issues. The second allows a more holistic vision of the various factors and issues with a framework, a set of constructs, with which to characterise and link, and through which to see where the tensions lie. This allows further activity to be planned from the outside. Seen in these ways, the two frames offer complementary insights to the developmental process and the hidden curriculum.

THEORETICAL FRAMES AND ONGOING PRACTICE/ACTIVITY

One reviewer of this paper asked why students' goals had not been taken into account. This is an important question. With the first cohort of students, a questionnaire was completed asking about their course participation, understanding and achievement and some interviews were conducted (Jaworski, 2008b). Both cohorts completed the standard university evaluation of the course. In another research project into university teaching we have tried to organise focus groups with students to discern their perspectives. A discussion of analysis of these sources is beyond the scope of this paper. However, a future study would valuably bring students' goals to centre stage, particularly in an activity theory analysis in juxtaposition with teachers' goals. For example, in the use of GeoGebra as an exploratory tool, indications are that students do not so far see what the teacher perceives as value in its use. An activity theory analysis suggests that we have here tensions between the teacher's goals for creating conceptual understanding and students' goals for instrumental success. This could be shown by juxtapositioning of two activity systems, one for the students and one for the teacher. However, stronger data is needed before this would make sense. Critical inquiry into how GeoGebra can be used by students to achieve conceptual understanding is proposed as action.

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