INTEGRATING DIFFERENT PERSPECTIVES TO SEE THE FRONT AND THE BACK: THE CASE OF EXPLICITNESS

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The paper contributes to the ongoing discussion on ways to connect theoretical perspectives. It draws explicitly on the introductory article and the concluding article of the Theory Working Group publication ZDM – The International Journal on Mathematics Education 40(2)*, particularly on the strategy of local theory integration. In the first part of the paper, a classroom scene is presented to provide some footing in empirical data. This data is used to illustrate the theoretical propositions, made from two theoretical perspectives, on the topos of explicitness in mathematics teaching and learning. In the second part, the two theoretical accounts are locally integrated resulting in a deepened and more balanced understanding of the role of explicitness. In the last part, this example is used to differentiate three modes of local theory integration: bricolage, recontextualisation and metaphorical structuring.*

PRELIMINARY REMARKS

According to Lakoff and Johnson (1980), the attribution of a front and a backside to something is metaphorical in nature and depending on the experience and interest of the attributor. A front-back orientation, they cogently argue, is not an inherent property of objects but a property that we project onto them relative to our cultural functioning. The front is what we see. If we want to see the back of it, we need to walk around it or to turn it round. This is quite clear for concrete objects like, say, mountains and fruits. Attributing a front-back orientation to the abstract concept of explicitness is different because there is no cultural agreement about what the front and the back of it may be. By projecting categories that emerge from direct physical experience onto non-physical constructs, a metaphorical structuring occurs which transmits the connotations of the former to the latter. It is thus no value-neutral endeavour to discuss the concept of explicitness in terms of its front and its back. In many cultures the front of something is regarded as being more important than its back, but otherwise the front may be taken as just a surface and you need to look at the back of it to see the 'real thing'. I will come back to some consequences of this issue, in terms of Radford's (2008) conceptions of theories, at the end of the paper.

In the paper, I present empirical data from a 5th grade mathematics classroom for looking at the degree of explicitness in a case of mathematics teaching. I draw on the consequences of this teaching practice for the students' learning of mathematics from two theoretical perspectives, a semiotic ("the front") and a structuralist ("the back") one. While arguing that both perspectives connect fruitfully, I use this example for taking on the ongoing discussion of the challenges and possibilities of connecting theories in mathematics education (Prediger, Arzarello, Bosch & Lenfant, 2008).

THE EMPIRICAL DATA

In most federal states in Germany, primary school ends after 4th grade. From 5th grade on, the students are grouped according to achievement and assumed capacity. Those students, who achieved best in primary school, attend the *Gymnasium* (about 40% in urban settings). The data I am drawing on in this paper is the videotape of the first lesson of a new *Gymnasium* class, which consists of 5th graders from different primary schools. The teacher and the students do not know each other. It is the very first lesson after the summer holidays. The teacher starts the lesson by immediately introducing a strategic game known as “the race to 20” (Brousseau, 1975, p. 3). [1]

Teacher: Well, you are the infamous class 5b, I have heard a lot about you and, now, want to test you a little bit, that’s what I always do, whether you really can count till 20. [*Students’ laughter*.] Thus it is a basic condition to be able to count till 20, so I want to ask, who has the heart to count till 20? [*Students’ laughter*.] Okay, you are?

Nicole: Nicole.

Teacher: Nicole, okay. So you think you can count till 20. Then I would like to hear that.

>[2] Nicole: Okay, one two thr …
>Teacher: Two, oh sorry, I have forgotten to say that we alternate, okay?

Nicole: Okay.

Teacher: Yes? Do we start again?

Nicole: Yes. One.

Teacher: Two.

Nicole: Three.

Teacher: Five, oops, I’ve also forgotten another thing. [*Students’ laughter*.] You are allowed to skip one number. If you say three, then I can skip four and directly say five.

Nicole: Okay.

Teacher: Uhm, do we start again?

Nicole: Yeah, one.

Teacher: Two.

Both continue ‘counting’ according to the teacher’s rules. In the end, the teacher states “20” and says that Nicole was not able to count till 20. Then he asks if there were other students who really can count till 20. During the next 7 min. of the lesson, eight other students try and lose against the teacher whilst an atmosphere of students-against-the-teacher competition is developing. While ‘counting’ against the teacher, the tenth student (Hannes) draws on notes that he has written in a kind of notebook –

and he is winning against the teacher. After Hannes has stated “20”, the following conversation emerges:

Teacher: Yeah, well done. [*Students applaud*.] Did you just write this up or did you bring it to the lesson? Did you know that today …

Hannes: I have observed the numbers you always take.

Teacher: Uhm. You have recorded it, yeah. Did you [*directing his voice to the class*] notice, or, what was his trick now?

Torsten: Yes, your trick.

Teacher: But what is exactly the trick?

During the next 5:30 minutes the teacher guides the mathematical analysis of the race to 20. In form of a teacher-student dialogue, he calls 17, 14, 11, 8, 5 and 2 the “most important numbers” and writes theses numbers on the blackboard. He makes no attempt of checking whether the students understand the strategy for winning the race. Instead, he introduces a variation of the race: you are allowed to skip one number and you are also allowed to skip two numbers. The students are asked to find the winning strategy by working in pairs. After 10 minutes, the teacher stops the activity and prompts for volunteers to ‘count’ against the teacher. The first six students lose, but the seventh student (Lena) succeeds. After Lena has stated “20”, the following conversation emerges:

Teacher: Okay, good. [*Students applaud*.] Well, don’t let us keep the others in suspense, Lena, please tell us how you’ve figured out what matters in this game?

Lena: Well, we’ve figured it out as a pair.

Teacher: Yes.

Lena: We have found out the four most important numbers and, in addition, the other must start if you want to win.

Teacher: Do you want to start from the behind?

Lena: From behind? No.

Teacher: No? Okay, then go on.

Lena: Okay, well if the other starts then he must say one, two or three. Then you can always say four. [*Teacher writes* 4 *on the blackboard*.] When the other says five, six or seven, then you can say eight. [*Teacher writes* 8 *on the blackboard*.] And when the other says nine, ten or eleven, then you can say twelve. [*Teacher writes* 12 *on the blackboard*.] And when the other says thirteen, fourteen or fifteen, then you can say sixteen. [*Teacher writes* 16 *on the blackboard*.] And then the other can say seventeen, eighteen or nineteen and then I can say twenty.

Teacher: Yeah, great. What I appreciate particularly is that you have not only told us the important numbers, but also have explained it perfectly and automatically. Yes, this is really great. Often, students just say the result, they haven't the heart, but you have explained it voluntarily. That's how I want you to answer.

In the next two paragraphs the focus is on the theoretical issue of explicitness. First, it is argued from a semiotic perspective that implicitness is a precondition for learning and that an exaggerated explicitness counteracts mathematical learning in school. Second, the structuralist argument that students benefit differently from invisible pedagogies is explored. The data is used to illustrate the theoretical propositions. [3]

THE FRONT: IMPLICITNESS AS A PRECONDITION OF LEARNING

From a theory of semiotic systems, Ernest (2006, 2008) explores the social uses and functions of mathematical texts in the context of schooling, where the term 'text' may refer to any written, spoken and multi-modally presented mathematical text. He defines a semiotic system in terms of three components (Ernest, 2008, p. 68):

1. A set o signs;
2. A set of rules for sign use and production;
3. An underlying meaning structure, incorporating a set of relationships between these signs.

According to this perspective, the learning of mathematics in school presupposes the induction of the students into a particular discursive practice, which involves the signs and rules of school mathematics. Whereas signs are commonly introduced explicitly, the rules for sign use and production are often brought in through worked examples and particular instances of rule application. The working of the tasks, the reception of corrective feedback, and the internalisation gradually enrich the students' personal meaning structures. It is only at the end when the underlying mathematical meaning structure is made explicit.

By referring to Ernest's semiotic system, we can make sense of the 5th grade teacher's actions: First, he is explicitly stating that counting the normal way till 20 is well-known for all students and he is playfully introducing a (growing) set of rules for sign use. Second, the strategies for winning the different races to 20 remain on an exemplary level and are not transformed into a general rule. Third, he leaves any exploration of the underlying meaning structure completely to the students.

Regarded from the adopted semiotic perspective, the teacher is inviting the students to a very open and not much routed search for regularities and more general relationships between signs. This way of teaching avoids what Ernest calls the "General-Specific paradox" (Ernest, 2008, p. 70):

> If a teacher presents a rule explicitly as a general statement, often what is learned is precisely this specific statement, such as a definition or descriptive sentence, rather than

> what it is meant to embody: the ability to apply the rule to a range of signs. Thus teaching the general leads to learning the specific, and in this form it does not lead to increased generality and functional power. Whereas if the rule is embodied in specific and exemplified terms, such as in a sequence of relatively concrete examples, the learner can construct and observe the pattern and incorporate it as a rule, possibly implicit, as part of their own appropriate meaning structure.

Apparently the teacher is introducing his mathematics class as a kind of heuristic problem solving. He is giving no hints for finding a route through the mathematical problem of the race to 20. When Hannes has succeeded in the race, the teacher is explicitly framing the solution as a "trick" that is useful in the particular task under study. He then continues by modifying the rules. This may allow the students to come closer to a general heuristic insight: It may be an appropriate strategy to work the solution back from 20. However, the teacher is not insisting upon Lena explaining backwards. The 'official' underlying (heuristic) meaning structure of the race to 20 is not made explicit during the lesson, though the students are gradually inducted into the generals of heuristic mathematical problem solving.

THE BACK: EXPLICITNESS AS A PRECONDITION OF ACCESS FOR ALL

From a structuralist position, Bernstein (1990, 1996) polarises two basic principles of pedagogic practice: visible and invisible. A pedagogic practice is called visible "when the hierarchical relations between teacher and pupils, the rules of organization (sequence, pace) and the criteria were explicit" (Bernstein, 1996, p. 112). In the case of implicit hierarchical and organisational rules and criteria, the practice is called invisible. He argues that in invisible pedagogic practice access to the vertical discourses, on which the development of subject knowledge concepts ultimately depends, is not given to all children. Instead, evaluation criteria remain covert thus producing learners at different levels of competence and achievement.

In terms of Bernstein's differentiation of pedagogic practices, invisible practice dominates the 5th class' first mathematics lesson. When comparing the teacher's talk with Hannes and with Lena, it can be seen that the teacher keeps the students in the dark about some essential aspects of the mathematical teaching that is going on. Although students, who read between the lines of the teacher's talk, may well identify some characteristics and criteria of the pedagogic practice they are participating in, the teacher transmits these characteristics and criteria only implicitly. All those students who do not notice these implicit hints, or cannot decode them, remain in uncertainty about:

… if the race to 20 is meant as a social activity of getting to know each other (It is the very first lesson!) or as a mathematical problem disguised as a students-teacher competition,

… if thus students should fish for "the trick" or heuristically develop a mathematical strategy and

… if thus successful participation in this classroom activity is granted when the race has been won or when a strategy has been established by mathematical substantiation.

Only at the end of Lena's explanation, the teacher makes the criteria for successful participation in 'his' mathematics class explicit. As a consequence, students' successful learning has been contingent on their abilities to guess the teacher's didactic intentions. Recording the numbers the teacher always takes (Hannes) without transcending the number pattern for a mathematical rule, is only legitimate to a certain extend. As long as the hierarchical and organisational rules and the criteria (which Bernstein (1996, p. 42) calls respectively the "distributive rules", the "recontextualizing rules" and the "evaluative rules") remain implicit, students are intentionally kept unconscious about the very practice they are participating in. Only visible pedagogic practices facilitate that students collectively access, and participate in, academically valued social practices and the discourses by which these practices are constituted (cf. Bourne, 2004; Gellert & Jablonka, in press).

CONNECTION: INTEGRATING THE TWO PERSPECTIVES

The contrasting perspectives on explicitness reveal that the rules and criteria of mathematics education practice remain – in part as a matter of principle – implicit. On the one hand, the need for implicitness is due to the very character of the learning process: whoever strives for whatever insight cannot say ex ante what this insight exactly will be. Ernest's "General-Specific paradox" is an interpretation of this issue. On the other hand, the principles that structure the practice of mathematics education remain implicit to the participants of this practice, without any imperative to do so for facilitating successful learning processes.

However, for that the general can be fully acquired, the students indeed need to understand that the specific examples and applications have to be interpreted as the teacher's means to organise the learning of the general. Successful learning in school requires the capacity to decode some of the implicit principles of the teacher's practice. The structuralist perspective supports the argument that the students actually benefit more from teaching-the-general-by-teaching-the-specific if they are conscious about the organising principle that is behind this teaching practice. By making the organisational and hierarchical rules and the criteria of the teaching and learning practice explicit, the teacher provides the basis for that all students can participate successfully in the learning process.

It is quite clear from the empirical data presented above that the teacher is partly aware of this relation: In the end of the passage, he explicitly explains to the students the characteristics of legitimate participation in 'his' classroom. However, as this explanation is given retrospectively and in a relatively late moment of the lesson it seems that some of the pitfalls of the implicit-explicit relation have not been avoided:

(1) It is neither obvious from their behaviour nor does the teacher check whether this very important statement is captured by all students. Particularly those students, who

did lose interest in the mathematical activity because they do not know where it can lead to, might not pay attention. (The fact that some students do not listen to the teacher's statement can be observed in the videotape.)

(2) By giving the explanation retrospectively, the teacher has already executed a hierarchical ordering of the students. Although no criteria for legitimate participation in the mathematical activity of the race to 20 has explicitly been given in advance of the activity, the teacher favours Lena's over Hannes' participation: Hannes is offering a "trick" (which might be more appropriate for playing outside school) while Lena is giving a mathematically substantiated explanation of her strategy. Apparently, Lena demonstrates more capacity of decoding the teacher's actions than Hannes does.

(3) It might be difficult for many students to transfer the teacher's statement to their mathematical behaviour during the next classroom activity. Indeed, the teacher is giving another specific statement, which the students gradually need to include in their meaning structure. This is another case of teaching-the-general-by-teaching-the-explicit: a general expectation ("students explain voluntarily") is transmitted by focussing on a specific example (Lena's explanation). Again, and on a different level, the students need to decode the teacher's teaching strategy: the teacher's statement is not only about legitimate participation in the race to 20, but also about participation in 'his' mathematics class in general.

Particularly the point (3) shows how the local integration of two theories may lead to a deepened and more balanced understanding of the issue of explicitness and its role within the teaching and learning of mathematics.

REFLECTIONS ON THE 'GENERAL': CONNECTING THEORIES

The connection of the two perspectives has structurally woven the front ("learning requires implicitness") into the back ("making hierarchical and organisational principles of classroom practice explicit"). A structuring of theoretical perspectives has thus taken place. But what is the nature of the new structure, and what are the characteristics of the process that has taken place?

Radford (2008) develops a conceptual language for talking about connectivity of theories in mathematics education. He takes theories as triples $\tau = (P, M, Q)$ of principles, methodologies and paradigmatic research questions. For questions about connectivity of theories, he argues that the principles seem to play a crucial role as "divergences between theories are accounted for not by their methodologies or research questions but by their principles" (Radford, 2008, p. 325). Indeed, at first glance, Ernest's semiotic perspective and Bernstein's structuralist perspective share an attention to the explicitness and implicitness of rules. The divergence of the two perspectives becomes apparent when the mode of these rules and their status is considered. Whereas from the semiotic perspective rules are rules for sign use and sign production and thus closely linked to the individual student's capacity of using and producing mathematical signs (P_1), the structuralist perspective takes rules as the constitutive elements of classroom practice (P_2). Ernest's semiotics is concerned with text-based activities where the texts are mathematical texts and the semiotic system is school knowledge. Bernstein's set of rules is the mechanism that provides an intrinsic grammar of pedagogic discourse. Although this looks like a fairly different understanding of rules and their respective theoretical status, the principles P_1 and P_2 of the two theories seem to be "'close enough' to each other" (Radford, 2008, p. 325) to allow for integrative connections.

Prediger, Bikner-Ahsbahs and Arzarello (2008, p. 173) describe "local integration" as one of the strategies for connecting theories. Acknowledging that the development of theories is often not symmetric, the strategy of local integration aims at an integrated theoretical account of a local theoretical question (e.g., Should rules be made explicit?). As a matter of fact, the principles P_i and P_j of two theories τ_i and τ_j deserve closer attention: How get P_i and P_j connected, what modes of mediating their divergence exist?

Bricolage. The mode of integration of theories Prediger et al. refer to is Cobb's notion of "theorizing as bricolage" (Cobb, 2007, p. 28). Cobb describes a process of adaptation of conceptual tools from the grand theories of cognitive psychology, sociocultural theory and distributed cognition. His goal is to "craft a tool that would enable us to make sense of what is happening in mathematics classrooms" (Cobb, 2007, p. 31). Here, the mode of mediation between theoretical principles is essentially pragmatic: Non-conflicting principles P_{g1}, P_{g2}, P_{g3}, … of the grand theories τ_{g1}, τ_{g2}, τ_{g3}, … are adapted for fit into the bricolage theory τ_b. As the goal of the integration is the development of a tool, τ_b is essentially an externally oriented language of description of empirical phenomena. Cobb's theorizing as bricolage is reminiscent of Prediger et al.'s (2008, p. 172) "coordinating" strategy. As the bricolage theory τ_b is a theory *en construction*, it is problematic to make the criteria for the selection of non-conflicting principles explicit.

Recontextualisation. Another mode of integration of theories is recontextualisation, "the subordination of the practices of one activity to the principles of another" (Dowling, in press, ch. 4). This is the case when the principles P_i of the theory τ_i dominate the principles P_j of the theory τ_j. An example of theory recontextualisation can be found in Gellert (2008) where an interactionist methodology M_i is subordinated to structuralist conceptual principles P_s. This process results in an asymmetrical role played by the methodologies M_i and M_s as a consequence of a hierarchical ordering of the principles of the corresponding theories (P_s over P_i; cf. Radford, 2008, p. 322f.). Hierarchical organisation of theories in the mode of recontextualisation is a device for avoiding theoretical inconsistencies.

Metaphorical structuring. A third mode of integration of theories is mutual metaphorical structuring. As Lakoff and Johnson (1980, p. 18f.) remark, "so-called purely intellectual concepts […] are often – perhaps always – based on metaphors". Since metaphors aim at "understanding and experiencing one kind of thing in terms of another" (Lakoff & Johnson, 1980, p. 5), this is again a case of subordination: metaphorical structuring. If we talk about the learning of mathematics in terms of rules, then the learning of mathematics is partially structured and understood in these terms, and other meanings of mathematics learning are suppressed. Similar things occur when concepts from one theory are infused into another theory. For an example see the infusion of the General-Specific paradox into the principles of a visible pedagogy. The argument that the advantage of a visible pedagogy relies on the explicitness of its criteria becomes differently structured when understood in terms of the General-Specific paradox: How can criteria be made explicit without producing blind rule-following and a formal meeting of expectations only? Infusing the term decoding capacity into the components of the semiotic system has produced a mutual effect: The teacher's strategy of teaching-the-general-by-teaching-the-specific is effective only if the students are able to decode the respective activities.

CONCLUSION

Bricolage, recontextualisation and mutual metaphorical structuring show different effects on the theoretical components that become locally integrated. This is still a complex issue and it might be very useful to further develop a meta-language for the connection of theoretical perspectives. I am convinced that a systematic description of the organising principles of local theory integration is an essential part of this developing language.

NOTES

1. The transcript presented, here, is my translation from the German original. Students' names are pseudonyms.
2. The sign > indicates overlapping of speech.
3. For a detailed analysis of what these passages can tell us about the exigencies that students face in mathematics classes, see Gellert and Hümmer (2008).

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