INSTRUMENTAL ORCHESTRATION:
THEORY AND PRACTICE

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The paper concerns the way teachers use technological tools in their mathematics lessons. The aim is to investigate the explanatory power of the theory of instrumental orchestration through its confrontation with a teaching episode. An instrumental orchestration is defined through a didactical configuration, an exploitation mode and a didactical performance. This model is applied to a teaching episode on the concept of function, using an applet embedded in an electronic learning environment. The results suggest that the instrumental orchestration model is fruitful for analysing teacher behaviour, particularly in combination with additional theoretical perspectives.

INTRODUCTION

The integration of technological tools into mathematics education is a non-trivial issue. More and more, teachers, educators and researchers are aware of the complexity of the use of ICT, which affects all aspects of education, including the didactical contract, the working formats, the paper-and-pencil skills and the individual and whole-class conceptual development.

A theoretical framework that acknowledges this complexity is the instrumental approach (Artigue, 2002). According to this perspective, the use of a technological tool involves a process of *instrumental genesis*, during which the object or artefact is turned into an instrument. The instrument, then, is the psychological construct of the artefact together with the mental schemes the user develops for specific types of tasks. In such schemes, technical knowledge and domain-specific knowledge (in our case mathematical knowledge) are intertwined. Instrumental genesis, in short, involves the co-emergence of mental schemes and techniques for using the artefact, in which mathematical meanings and understandings are embedded.

Many studies focus on the students' instrumental genesis and its possible benefits for learning (e.g., see Kieran & Drijvers, 2006). However, it was acknowledged that instrumental genesis needs to be monitored by the teacher through the orchestration of mathematical situations. In order to describe the management by the teacher of the individual instruments in the collective learning process, Trouche (2004) introduced the metaphorical theory of instrumental orchestration.

Until today, however, the number of elaborated examples of instrumental orchestrations is limited. Therefore, the aim of this paper is to investigate the explanatory power of the theory of instrumental orchestration through its

confrontation with a teaching episode. As such, this contribution can be situated in the intersection of themes 2 and 3 of Cerme6 WG7: it concerns the interaction between resources or artefacts and teachers' professional practice, in which students use tools in their mathematical activity.

In the following, we first define instrumental orchestration. Then a description of a classroom teaching episode in which a technological tool plays an important role is provided. The episode is analysed in terms of the theory. This is followed by a reflection on the application and the conclusions which we have drawn.

INSTRUMENTAL ORCHESTRATION: A THEORETICAL MODEL

The theory of instrumental orchestration is meant to answer the question of how the teacher can fine-tune the students' instruments and compose coherent sets of instruments, thus enhancing both individual and collective instrumental genesis.

An *instrumental orchestration* is defined as the intentional and systematic organisation and use of the various artefacts available in an – in our case computerised – learning environment by the teacher in a given mathematical task situation, in order to guide students' instrumental genesis. An instrumental orchestration in our view consists of three elements: a didactic configuration, an exploitation mode and a didactical performance.

1. *A didactical configuration* is an arrangement of artefacts in the environment, or, in other words, a configuration of the teaching setting and the artefacts involved in it. These artefacts can be technological tools, but the tasks students work can be seen as artefacts as well.

   In the musical metaphor of orchestration, setting up the didactical configuration can be compared with choosing musical instruments to be included in the orchestra, and arranging them in space so that the different sounds result in the most beautiful harmony.

2. *An exploitation mode* of a didactical configuration is the way the teacher decides to exploit it for the benefit of his didactical intentions. This includes decisions on the way a task is introduced and is worked on, on the possible roles of the artefacts to be played, and on the schemes and techniques to be developed and established by the students.

   In the musical metaphor of orchestration, setting up the exploitation mode can be compared with determining the partition for each of the musical instruments involved, bearing in mind the anticipated harmonies to emerge.

3. *A didactical performance* involves the ad hoc decisions taken while teaching on how to actually perform the enacted teaching in the chosen didactic configuration and exploitation mode: what question to raise now, how to do justice to (or to set aside) any particular student input, how to deal with an unexpected aspect of the mathematical task or the technological tool?

> In the musical metaphor of orchestration, the didactical performance can be compared with a musical performance, in which the actual inspiration and the interplay between conductor and musicians reveal the feasibility of the intentions and the success of their realization.

The model for instrumental orchestration initially was developed by Trouche (Trouche 2004) and included the first and the second points above, i.e. the didactical configuration and the exploitation mode. As an instrumental orchestration is partially prepared beforehand and partially created 'on the spot' while teaching, we felt the need for a third component reflecting the actual performance. Establishing the didactical configuration has a strong preparatory aspect: often, didactical configurations need to be thought of before the lesson and cannot easily be changed during the teaching. Exploitation modes may be more flexible, whereas didactical performance has a strong ad hoc aspect. Our threefold model thus has a time dimension.

The model also has a structural dimension: an instrumental orchestration on the one hand has a structural, global component in that it is part of the teacher's repertoire of teaching techniques (in the sense of Sensevy *et al*. 2005) and can be reflected in operational invariants of teacher behaviour. On the other hand, an instrumental orchestration has an incidental, local actualisation appropriate for the specific didactical context and adapted to the target group and the didactical intentions.

The instrumental orchestration model brings about a double-layered view on instrumental genesis. At the first level, instrumental orchestration aims at enhancing the students' instrumental genesis. At the second level, the orchestration is instrumented by artefacts for the teachers, which may not necessarily be the same artefacts as the students use. As such, the teacher himself is also involved in a process of instrumental genesis for accomplishing his teaching tasks (Bueno-Ravel & Gueudet, 2007).

In literature, the number of elaborated examples of instrumental orchestrations is limited. Trouche (2004) and Drijvers & Trouche (2008) describe a so-called Sherpa orchestration. Kieran & Drijvers (2006), without mentioning this orchestration explicitly, describe an instrumental orchestration of short cycles of individual work with the artefact and whole-class discussion of results.

## THE CASE OF TWO VERTICALLY ALIGNED POINTS

The case we describe here stems from a research project on an innovative technology-rich learning arrangement for the concept of function[1]. In this project, a learning arrangement for students in grade 8 was developed, aiming at the development of a rich function concept. This includes viewing functions as input-output assignments, as dynamic processes of co-variation and as mathematical

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[1] For further information on the project see Drijvers, Doorman, Boon, Van Gisbergen & Gravemeijer (2007) and the project website www.fi.uu.nl/tooluse/en/.

objects with different representations. The main technological artefact is an applet called AlgebraArrows embedded in an electronic learning environment (ELO). The applet allows for the construction and use of chains of operations, and options for creating tables, graphs and formulae and for scrolling and tracing. A hypothetical learning trajectory, in which the expected instrumental genesis is sketched, guided the design of the student materials. An accompanying teacher guide contained suggestions for orchestrations.

After group work on diverse problem situations involving dependency and co-variation, the notion of arrow chains is introduced to the students. In the third and fourth lessons, students work with arrow chains in the ELO. One of the tasks of the fourth lesson, which some of the students did at home, is task 8, shown in Figure 1.
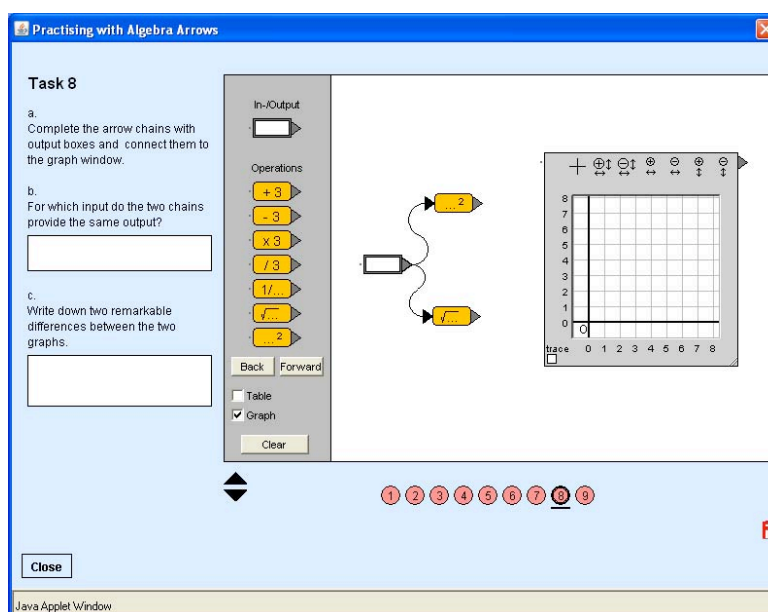
Figure 1 Computer task 8

At the right of Figure 1 is the applet window, which in this task contains the start of the square and the square root chain, and an empty graph window. At the left you see the tasks and two boxes in which the students type their answers. The numbered circles at the bottom allow for navigation through the tasks.

The following verbatim extract describes the way the teacher discusses this task during the fifth lesson.

> Using a data projector, the ELO with the list of student pairs is projected on the wall above the blackboard. The teacher T navigates within this list to Tim and Kay's solution for task 8.
>
> T: It says here [*referring to question c*]: what do you notice? Oh yes, I actually wanted to see quite a different one, because they had …
>
> T navigates to Florence and her classmate's work. The Table option is checked. That leads to 'point graphs' on the screen. The students' answer to question c reads:

"For the square they are all whole numbers, and for the square root they are whole numbers and fractions. And the square of a number is always right above the root. ?"

T:	Look here, what this says. [*indicates the students' answer of question c on the screen with the mouse*] For the square they are all whole numbers, okay, and for the square root they aren't whole numbers, we agree with that too, and the square of a number is always right above the square root.
F(lorence):	Was that right?
T:	I'm not saying.
St1[2]:	Yes, I had that too.
T:	What they say, then, is that every time there is…if I've got something *here*, there is something above it, and if I've got something *there*, there is also something above it. [*points vertically in the graph with the mouse*] Why is that, that these things are right above each other?
F:	Well, because it…the square root is just…no the square is just, um, twice the root, or something.
St2:	No.
T:	Kay?
Kay:	That's because the line underneath, that's got a number on it, which you take the square root of and square, so on the same line anyway.
T:	What are those numbers called that are on the horizontal line then?
St3:	The input numbers.
T:	The input numbers.
T:	Ehm, Florence, did you follow what Kay said?
F:	No, but I […]. It was about numbers and about square roots and about…
Sts:	[*laughs*]
St:	It was about numbers!
T:	Kay said: these are the input numbers, here on the horizontal line. [*indicates the points on the horizontal axis with the mouse*] And for an input number you get an output number. And that is right above it. So if you take the same input number for two functions… [*indicates the two arrow chains with the mouse*]
F:	Oh yes.

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[2] St1, St2, .. stands for one of the students

T:	… then you also get…then you get points above it. So that's got nothing at all to do with the functions. It's just got to do with from which number you are going to calculate the output value. Now, if for both of them you calculate what the output value is for 10, they both get a point above the 10 [indicates on the screen with the mouse]. Do you understand that?
F:	Oh yes, I didn't know that.
T navigates back to the list of student pairs.	

Figure 2 shows the work of Florence and her classmate on this task in Dutch at the end of the teaching sequence. They changed their answer to question c into: "for the square they are always whole numbers, and for the square root they are whole numbers and fractions. The squares get higher with much bigger steps."
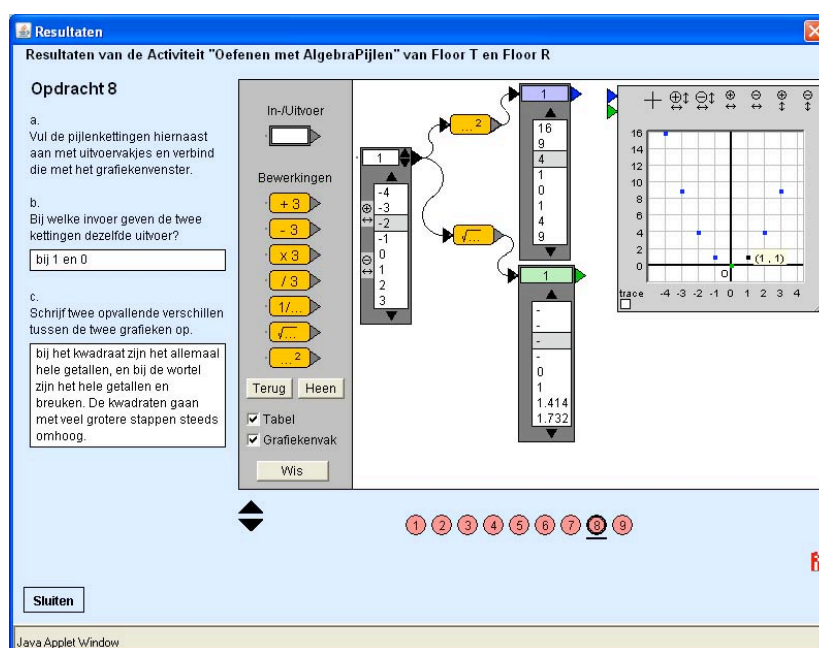

Figure 2 Revision of the answer after whole class discussion

APPLYING THEORY TO PRACTICE

In this section we apply the theory of instrumental orchestration to the above teaching episode, which essentially reflects the teacher's way to treat a misconception of (at least) one of the students, whose use of the Table-Graph technique leads to thinking it is 'special' that two points reflecting function values for the same input value are vertically aligned.

Let us call the instrumental orchestration the teacher puts into action the 'spot and show orchestration'. By 'spot' we mean that the teacher, while preparing the lesson, spotted the students' work in the ELO and thus came across Florence's misconception. The 'show' refers to the teacher's decision to display Florence's results as a starting point for the whole-class discussion of item 8c. The teacher's

phrase "Oh yes, I actually wanted to see quite a different one" and her straight navigation to Florence's work reveal her deliberate intention to act the way she does.

The *didactical configuration* for the preparatory phase consists of the ELO's option for teachers to look at the students' work at any time. As a result, the teacher notices the misconception and decides to deal with it in her lesson. This preparation is instrumented by ELO-facilities that are not available for students. In this sense, the teacher's artefact is different from the students' artefact. For the classroom teaching, the configuration includes a regular classroom with a PC with ELO access, connected to a data projector. Apparently, the teacher finds the computer lab not appropriate for whole-class teaching. The configuration includes putting the computer with the data projector in the centre of the classroom. This choice is driven by the constraints of one of the artefacts: if the projector was at the front, the projection would get too small for the students to read. The screen is projected on the wall above the blackboard, thus enabling the teacher to write on the blackboard, which she regularly does – though not in the episode presented here. Both the way of preparing the lessons and the setting in the classroom are observed more often in this teacher's lessons.

The *exploitation mode* of this configuration includes that teacher's choice to operate the PC herself. These two aspects of the exploitation mode result in the teacher standing in the centre of the classroom, with the students closely around her, all focused on the screen on the wall. From these and other observations, we conjecture that this exploitation mode enhances classroom discussion and student involvement. Observations of another teacher using the same orchestration in a less convenient setting support this conjecture.

The *didactical performance* starts with the teacher reading the student's answer with some minor comments ("Look here, …"). Then she reformulates the answer and asks Florence for an explanation ("What they say…"). When the explanation turns out to be inappropriate, she makes Kay give his explanation, and checks whether Florence understands it. When this is not the case, the teacher rephrases Kay's explanation and once more checks it with Florence, who now says she understands. Of course, this didactical performance might be different a next time. For example, Florence could be asked to explain her understanding in her own words.

Now how about the link between instrumental orchestration and instrumental genesis? As the episode does not show students using the artefact, we do not see direct traces of the students' instrumental genesis. We do claim, however, that Florence's idea of two vertically aligned points being special is part of her scheme of using the TableGraph technique to produce point graphs. Even though this is a misconception, the episode shows that the teacher can exploit the students' experiences, and those of Florence and Kay in particular, for the purpose of attaching mathematical meaning to the technique they used, which leads to a convergence in a

shared function conception in class. We see the development of mathematical meanings of techniques as an important aspect of instrumental genesis.

This 'spot&show' orchestration was one of the options suggested in the teacher guide accompanying the teaching sequence. This teacher used it quite often, whereas she felt free to neglect other suggestions made in the teacher guide. In the post-experiment interview, she indicated to really appreciate the possibility to get an overview of students' results while preparing the lesson: "The ELO is practical to see what students do, you can adapt your lesson to that." She seemed to see this 'spot&show' orchestration as a means to enhance student involvement and discussion, which she believed to be relevant and seem to be part of her operational invariants . We do not have data, however, that confirm such operational invariants across other teaching settings.

Finally, an interesting aspect of the teacher's own instrumental genesis is worth discussing. The teacher points with her mouse on the screen, but does not really make changes in the students' work. Other observations suggest that she doesn't do so because she is afraid that such changes will be saved and thus affect the students' work. When she learns that this is not the case as long she uses her teacher login, she benefits from the freedom to demonstrate other options and to investigate the consequences of changes. This behaviour is instrumented by the facilities of the artefact that she initially was not aware of.

REFLECTION ON THE THEORY AND THE CASE

Let us briefly reflect on the application of the theory of instrumental orchestration to the data presented above. A first remark is that the three elements of the model – didactical configuration, exploitation mode and didactical performance – allow for a distinction and an analysis of the relevant issues within the orchestration, and their interplay. As such, the model offers a useful framework for describing the orchestration by the teacher.

As a second remark, however, we notice that it is not always easy to decide in which category something that is considered relevant should be placed. For example, does the fact that the teacher operates the computer herself belong to the didactical configuration or to the exploitation mode? This probably is a matter of granularity: if we study the 'spot & show' orchestration, this is part of the exploitation mode. If the focus of the analysis is on students' activity, it might be identified as a didactical configuration issue.

A third reflection is that the model has the advantage of fitting with the instrumental approach of students learning while using tools. This has proved to be a powerful approach (Artigue, 2002; Kieran & Drijvers, 2006), and it is therefore of great value having a framework for analysing teaching practices that is consistent with it. As such, instrumentation and orchestration form a coherent pair. In terms of instrumentation, we notice that the teachers' tasks, artefacts and techniques are not

the same as those of the students; still, we can use a similar framework for analysis and interpretation.

The time dimension in the model – ranging from the didactical configuration having a strong preparatory character to the didactical performance with its strong ad hoc character – comes out clearly in the model. For the structural dimension, this is not as straightforward. As a fourth remark, therefore, we notice that operational invariants of the teacher are not limited to the preparatory phases, but also emerge in the performance. For example, the wish to have students explain their reasoning to each other appears as an operational invariant for this teacher, which is more explicit in the performance than in the configuration or in the exploitation mode. As an aside, we are aware that the data presented here do not allow for full identification of the teacher's operational invariants. More observations over time need to be included.

CONCLUSION

As far as this is possible from the one single, specific exemplary case study presented in this paper, we conclude that the model of instrumental orchestration can be a fruitful framework for analysing teachers' practices when teaching mathematics with technological tools. As it is important for teachers to develop a repertoire of instrumental orchestrations, more elaborated examples are needed. Such examples could not only help us to better understand teaching practices, but could also enhance teachers' professional development.

In addition to the need to find and elaborate exemplary orchestrations, a second challenge is to link the theory of instrumental orchestration with complementary approaches. Lagrange (2008), for example, uses additional models provided by Saxe (1991) and Ruthven and Hennessey (2002) to identify and understand teaching techniques. Another interesting perspective concerns the alternation of teacher guidance and student construction, as described by Sherin (2002). In short, the instrumental orchestration approach is promising, but needs elaboration and integration with other perspectives. For the moment, its descriptive power seems to be more important than its explanatory power.

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REFERENCES

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning,7*, 245–274.

Bueno-Ravel, L. & Gueudet, G. (2007). Online resources in mathematics: teachers' genesis of use? In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of the V Congress of the European Society for Research in Mathematics Education CERME5* (pp.1369-1378). Cyprus: Larnaca.

Drijvers, P., Doorman, M., Boon, P., Van Gisbergen, S., & Gravemeijer, K. (2007). Tool use in a technology-rich learning arrangement for the concept of function. In D. Pitta-Pantazi & G. Philipou (Eds.), *Proceedings of the V Congress of the European Society for Research in Mathematics Education CERME5* (pp. 1389 - 1398). Cyprus: Larnaca.

Drijvers, P., & Trouche, L. (2008). From artifacts to instruments: A theoretical framework behind the orchestra metaphor. In G. W. Blume & M. K. Heid (Eds.), *Research on technology and the teaching and learning of mathematics: Vol. 2. Cases and perspectives* (pp. 363-392). Charlotte, NC: Information Age.

Kieran, C., & Drijvers, P. (2006). The Co-Emergence of Machine Techniques, Paper-and-Pencil Techniques, and Theoretical Reflection: A Study of CAS use in Secondary School Algebra. *International Journal of Computers for Mathematical Learning, 11*(2), 205-263.

Lagrange, J.-B. (2008). *Teachers using technology, expectations and classroom activity.* Retrieved on 14 September 2008 from http://jb.lagrange.free.fr.

Ruthven, K., & Hennessy, S. (2002). A practitioner model of the use of computer-based tools and resources to support mathematics teaching and learning. *Educational Studies in Mathematics, 49*, 47–88.

Saxe, G.B. (1991). *Culture and Cognitive Development: Studies in Mathematical Understanding.* Hillsdale NJ: Laurence Erlbaum Associates.

Sensevy, G., Schubauer-Leoni, M.L., Mercier, A., Ligozat, F., & Perrot, G. (2005). An Attempt to Model the Teacher's Action in the Mathematics Class. *Educational Studies in Mathematics*, 59(1-3), 153-181.

Sherin, M.G. (2002). A balancing act: developing a discourse community in a mathematics community. *Journal of Mathematics Teachers Education, 5*, 205-233.

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning, 9*, 281-307.