THE UTILIZATION OF MATHEMATICS TEXTBOOKS AS INSTRUMENTS FOR LEARNING

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The mathematics textbook is one of the most important resources for teaching and learning mathematics. Whereas a number of studies have examined the use of mathematics textbooks by teachers there is a dearth of research into the use of mathematics textbooks by students. In this paper results of an empirical investigation of the use of mathematics textbooks by students as an instrument for learning mathematics are presented. Firstly, a method to collect data on student's use of mathematics textbooks is introduced. It is explicated, that this method is capable to explore the actual use of the mathematics textbook by students, and a way of recording the use of the mathematics textbook whenever and wherever students use it. Secondly, results from the study are presented. The results outlined in this paper focus on typical self-directed uses of the mathematics textbook by students.

INTRODUCTION

Research in mathematics education has been concerned with the role of new technologies in the teaching and learning of mathematics from the very beginning computers and information technologies entered the mathematics classroom. In the first ICMI study the computer is even considered to be a new dimension in the mathematics classroom: "We now have a triangle, student-teacher-computer, where previously only a dual relationship existed" (Churchhouse et al., 1984). But, this perspective disregards the fact that tools have always been incorporated in teaching and learning mathematics and thus the relationship in the mathematics classroom has never actually been dual. The mathematics textbook was and still is considered to be one of the most important tools in this context. According to Howson, new technologies have not affected its outstanding role: "despite the obvious powers of the new technology it must be accepted that its role in the vast majority of the world's classrooms pales into insignificance when compared with that of textbooks and other written materials." (Howson, 1995)

Valverde et al. (2002) believe that the structure of mathematics textbooks is likely to have an impact on actual classroom instruction. They argue, that the form and structure of textbooks advance a distinct pedagogical model and thus embody a plan for the particular succession of educational opportunities (cf. Valverde et al., 2002). The pedagogical model only becomes effective when the textbook is actually used. Therefore, mathematics textbooks should not be a subject to analysis detached from its use. It is an interactive part within the activities of teaching and learning mathematics In order to develop a better understanding of the role of the mathematics textbooks within the activities of teaching and learning mathematics an activity theoretical model was developed (Rezat, 2006a):

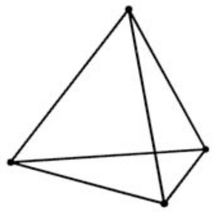


Fig. 1: Tetrahedron model of textbook use

This model is based on the fundamental model of didactical system: the ternary relationship between student, teacher, and mathematics (Chevallard, 1985). The mathematics textbook is implemented as an instrument at all three sides of the triangle: teachers use textbooks in the lesson and to prepare their lessons, by using the textbook in the lesson teachers also mediate textbook use to students, and finally students learn from textbooks. Thus, each triangle of the tetrahedron-model represents an activity system on its own. From an ergonomic perspective it is argued that artefacts have an impact on these activities, because on the one hand they offer particular ways of utilization and on the other hand the modalities of the artefacts impose constraints on their users (cf. Rabardel, 1995, 2002). Thus, the mathematics textbook has an impact on the activity of learning mathematics as a whole that is represented by the didactical triangle on the bottom of the tetrahedron.

Whereas a number of studies have examined the role of new technologies in terms of tool use (cf. Lerman, 2006) the role of the mathematics textbook as an instrument for teaching and learning has not gained much attention. So far, a number of studies have examined the use of mathematics textbooks by teachers (e.g. Bromme & Hömberg, 1981; Haggarty & Pepin, 2002; Hopf, 1980; Johansson, 2006; Pepin & Haggarty, 2001; Remillard, 2005; Woodward & Elliott, 1990) whereas there is a dearth of research into the use of mathematics textbooks by students (Love & Pimm, 1996). This is striking, because as pointed out by Kang and Kilpatrick (1992), textbook authors regard the student as the main reader of the textbook.

In order to develop a better understanding of the impact that textbooks have on learning mathematics a qualitative investigation was carried out in two German secondary schools that focused on how students use their mathematics textbooks.

METHOD AND RESEARCH DESIGN

The difficulty of obtaining data on students working from textbooks is one reason that Love and Pimm (1996) put forward in order to explain the dearth of research into student’s use of texts. Therefore, developing an appropriate methodology to collect data on student’s use of mathematics textbooks can be regarded as a major issue in this field.

First of all, the method of data-collection has to be in line with the situation of textbook use. In Germany, schools either provide mathematics textbooks to students for one year or students buy the books. Accordingly, students have access to their mathematics textbook at school and at home. From previous research there is evidence, that German teachers rely heavily on the textbook in the preparation of lessons and also during lessons. (Bromme & Hömberg, 1981; Hopf, 1980; Pepin & Haggarty, 2001).

The method to collect data on student's use of mathematics textbooks was developed within the framework of the activity theoretical model of textbook use. According to this model the use of mathematics textbooks is situated within an activity system constituted by the student, the teacher, the mathematics textbook, and mathematics itself. First of all, this implies that a method to investigate the use of mathematics textbooks by students has to take all four vertices of the tetrahedron-model into consideration.

In addition, three criteria were established for an appropriate methodology to collect data on student's use of mathematics textbooks:

1. The actual use of the mathematics textbook should be recorded in detail.
2. Biases caused by the researcher, by the situation or by social desirability should be minimized.
3. The use of the textbook should be recorded at any time and any place it is used.

Criterion 1 leads to the rejection of quantitative methods and of methods that are likely to reveal only verbalized uses of the textbook, e.g. interviews. Experimental settings and artificial situations are refused due to criterion 2. Approaches that are solely based on observation are discarded because of criterion 3.

The methodological framework that was developed according to the three criteria combines observation and a special type of questioning. First of all, the students were asked to highlight every part they used in the textbook. Additionally, they were asked to explain the reason why they used the part they highlighted in a small booklet by completing the sentence "I used the part I highlighted in the book, because …". By assigning more than one comment to a highlighted book section the reuse of book sections becomes apparent. This method of data-collection was developed in order to get the most precise information about what the students actually use and why they use it by keeping the situation of textbook use as natural as possible. Nevertheless, highlighting sections in a textbook is not the natural way to use the textbooks and therefore a bias on the data cannot be totally excluded.

Provided that the students take their task seriously, this method enables to collect data on the use of the textbooks whenever and wherever students use it and therefore meets criterion 3.

In addition, the lessons were observed and field notes were taken. On the one hand the overall structure of the lesson was recorded in the field notes using a table comprising three columns: time, activity/content and remarks. On the other hand all utterances concerning the textbook were transcribed literally. Furthermore, a focus was put on all utilizations of the textbook. Both, the use of the textbook by the students and by the teacher was taken into account. This is important for several reasons:

First of all, there is evidence from previous research that the teacher plays an integral part in mediating textbook use. Because of that, the teacher was included as a variable in the model of textbook use.

Secondly, the observation provides an insight into the way the teacher mediates textbook use in the classroom. It makes a difference if the students only use the textbook when they are told to by the teacher or if they use it of their own accord. This difference will become apparent through classroom observation.

Thirdly, the methodological triangulation provides a measure for the validity of the data. Collecting data on how the textbook has been used in the classroom makes it possible to compare the markings and comments of the students with the field notes. The degree of correspondence between these two sources relating to the use of the textbook in the classroom indicates how serious the students took their task.

While the method of highlighting and taking notes especially satisfies criterion 3 and at the same time aims at both, providing a precise record of the actual use of the textbook by students (criterion 1) as well as keeping biases low (criterion 2), the intention of the observation is threefold. On the one hand the idea is to lower biases that might be caused by the method of highlighting (criterion 2) and on the other the triangulation of two different data-sources provides a measure for the validity of the student's data.

In addition to the previously described methods interviews were conducted with selected students.

Data was collected for a period of three weeks in two 6th grade and two 12th grade classes in two German secondary schools. Within the German three partite school system, these schools are considered to be for high achieving students. All four classes were taught by different teachers.

The coding process followed the ideas of Grounded Theory by Strauss and Corbin (Strauss & Corbin, 1990). Categories were established in the process of analysing the data. Each highlighted section in the textbook was categorized according to the kind of block it belongs to (introductory tasks, exposition, worked example, kernels, exercises) (cf. Rezat, 2006b), the activity it was involved in, and finally whether the use of the section was mediated by the teacher or not.

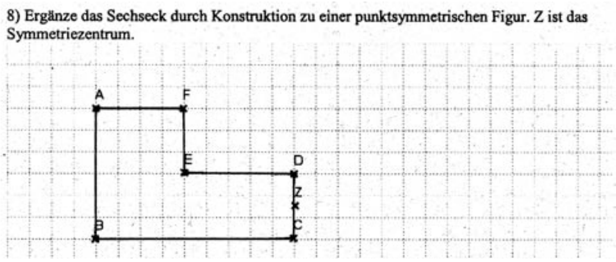
In order to understand the role of the mathematics textbook as an instrument within the activity system represented by the tetrahedron model Rabardel's (1995, 2002) theory of the instrument was used. As Monaghan (2007) points out, this theory has proven fruitful to provide insights into the use of new technologies as instruments for learning mathematics. According to Rabardel an instrument is a psychological entity that consists of an artefact component and a scheme component. In using the artefact with particular intentions the subject develops utilization schemes which are shaped by both, the artefact and the subject. Vergnaud (1998) suggests that schemes are characterized by two operational invariants: theorems-in-action and concepts-in-action. Since these two operational invariants are put forward in order to describe the representation of mathematical knowledge, it is not self-evident to apply them to knowledge related to the use of an artefact like the mathematics textbook. Therefore, it is suggested to generalize Vergnaud's notion of theorems-in-action and concepts-in-action to the notion of beliefs-in-action. As well as concepts-in-action beliefs are supposed to guide human behaviour by shaping what people perceive in any set of circumstances (Schoenfeld, 1998). Like theorems-in-action beliefs are propositions about the world that are thought to be true (Philipp, 2007). The appendix 'in-action' is supposed to underline that beliefs-in-action might be inferred from actions. They do not necessarily have to be expressed verbally. Because of its universality, the notion of beliefs-in-action offers an appropriate means to characterize operational invariants of utilization schemes linked to any artefact.

RESULTS

A first and a major result of the study is, that students do not only use the mathematics textbook when they are told to by the teacher. But, they also use the textbook self-directed. The following analysis focuses on utilizations of the mathematics textbook that students perform in addition to teacher mediated textbook use.

Students incorporate their mathematics textbook as an instrument into four activities: solving tasks and problems, consolidation, acquiring mathematical knowledge, and activities associated with interest in mathematics. From the data it was possible to reconstruct several individual utilization schemes of the mathematics textbook related to these activities. Comparing the individual schemes of different students related to the same activity revealed that some of the schemes were analogous in terms of the underlying beliefs-in-action. These schemes were generalized to utilization scheme types (UST). USTs are general in the way, that they allow to classify individual utilization schemes of the textbook into USTs and thus make individual utilizations comparable. Nevertheless, different students might show different USTs. The USTs are not general in the way that they are common to all students.

Solving tasks and problems is associated with activities where students utilize their mathematics textbook in order to get assistance with solving tasks and problems. Three different USTs were found related to this activity. It was observed that students repeatedly utilize specific blocks from the textbook as an assistance to solve tasks and problems. Worked examples and boxes with kernels were instrumentalized in most of the cases. This scheme could be traced back to the belief-in-action that a specific block from the textbook is useful in order to solve tasks and problems. It was also observed that students choose sections from the textbook that show similarities to the task. For example, Oliver is working on the following task that is not from the textbook:



He looks for assistance in the textbook and reads a task in the textbook that is located next to an image, which is identical to the image in the task. From this behaviour it can be inferred that Oliver expects information concerning the image next to it. In his case, the information is not useful for solving the task, because it is a task itself.

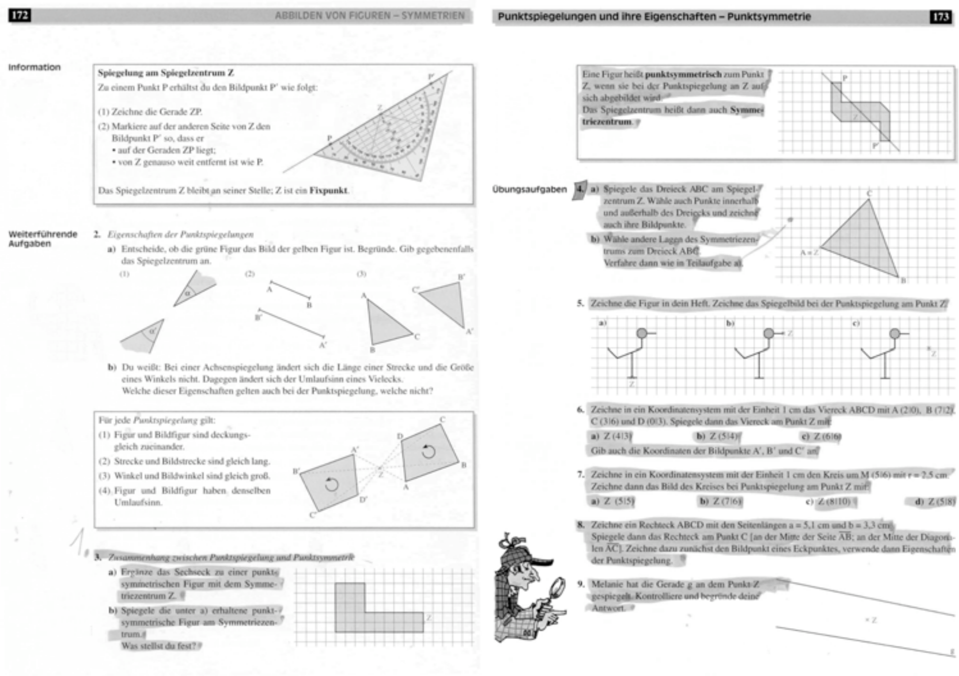


Fig. 2: Passage Oliver used from the textbook "because he was looking for something" (Griesel *et al.*, 2003)

In order to get assistance with solving tasks and problems it was also observed that students search an adequate heading in the book and start reading from there until they find useful information. From this behaviour it was inferred that these students expect useful information related to a subject at the beginning of a lesson in the textbook.

All three USTs reveal that students are looking for information in the book that can be directly applied to the task. The only difference is the way they are approaching the information. Hardly ever does it seem like students want to understand the mathematics first and then apply it to the task.

Consolidation is associated with all activities that students perform in order to improve their mathematical abilities related to subject matters that were already dealt with in the mathematics class. One UST of students using their mathematics textbook for consolidation is strongly related to teacher mediated exercises from the textbook. They either recapitulate tasks and exercises from the book that the teacher mediated or they pick tasks and exercises that are adjacent to teacher-mediated exercises. This was traced back to the belief-in-action that effective practising means to do tasks and exercises that are similar to teacher-mediated exercises. If students pick tasks that are adjacent to teacher mediated tasks this is also supported by the belief-in-action that adjacent tasks in the textbook are similar. The use of specific blocks for consolidation was also observed. One UST is that students only read the boxes with the kernels of several lessons in the textbook.

So far, consolidation seems to comprise learning rules, recapitulating teacher mediated tasks and solving tasks that are similar to teacher mediated tasks respectively. But, it was also observed that students either utilize special parts at the end of a unit that are designed especially for recalling and practising the main issues of the unit or they scan the section in the book relating to the actual topic in the mathematics class and read different parts of it in order to consolidate their understanding of the topic. Both UST are less dependent on teacher mediation and show more proficiency in the utilization of the textbook.

Whereas consolidation related to previously treated topics, acquisition of knowledge is associated with activities where students use parts of the book that have not been a matter in the mathematics class so far. The UST identified in this context is that students use parts from the proximate lesson in the textbook. This is supported by the belief-in-action that the chronological succession of topics in the mathematics class will follow the order of the textbook.

Students also used parts of their textbook because they thought they were interesting. These utilizations are associated with activities related to interest in mathematics. In this case the UST is connected to the use of images and other salient elements from the book. Students either only look at the images or they read passages that are next to images or other salient elements. Looking just at the pictures does not seem to be associated with learning mathematics though. This UST usually is observed in the context of other utilizations of the textbooks. It seems like this UST is not based on a belief-in-action, but that salient elements in the textbook catch the attention of the students while there utilizing it for another purpose.

CONCLUSIONS

The activities the mathematics textbook is involved in do not only give an insight into student’s utilizations of mathematics textbooks, but they also give an idea of what learning mathematics is about for students. The USTs show that learning mathematics with the mathematics textbook comprises activities as solving tasks and problems, consolidating mathematical knowledge and skills, acquiring new contents.

The USTs show how the textbook is used as an instrument within these activities. Furthermore, these USTs reveal interesting insights into student's dispositions towards mathematics. Learning mathematics comprises mainly learning rules, applying rules and worked examples to tasks, and developing proficiency in tasks that are similar to teacher mediated tasks.

Consciousness about student's USTs could affect teacher's ways of implementing the mathematics textbook in the teaching process. Some USTs show that the use of mathematics textbooks by teachers in the classroom is an important reference for student's utilizations of the textbook. For example, the UST that is characterized by the utilization of tasks that are adjacent to teacher mediated tasks for consolidation is dependent on the mediation of tasks from the textbook by the teacher. Therefore, it is important that the teacher uses tasks from the textbook in order to support student's individual learning of mathematics. Another example is the anticipation of the next topic in the mathematics class by reading parts of the proximate lesson in the textbook. This UST shows that students belief that the course of the mathematics lessons will follow the order in the book. Accordingly, the textbook provides orientation for students, and it can therefore be considered important that teachers follow the succession of the topics in the book.

It was pointed out, that Valverde et al. (2002) argue that the structure of mathematics textbooks advances a distinct pedagogical model and is likely to have an impact on actual classroom instruction. From an ergonomical perspective it can be argued that the structure of the book also has an impact on the USTs of the students. This raises the question of how a textbook must be structured in order to promote desirable USTs.

Furthermore, this study provides evidence that Rabardels theory of the instrument is not only capable of conceptualizing human-computer-interaction, but is also applicable to non technological resources. The conceptualization of student-textbook-interaction on the basis of this theoretical framework provides interesting insights into different aspects of learning mathematics. The UST do not only provide a better understanding of student's utilizations of mathematics textbooks, but also reflect student's ways of learning mathematics. Furthermore, it can be inferred from student's USTs how the textbook is effectively used in the classroom by the teacher. Accordingly, a better understanding of student's utilizations of mathematics textbooks is a prerequisite for effective implementation of mathematics textbooks into teaching.

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