ISSUES IN INTEGRATING CAS IN POST-SECONDARY EDUCATION: A LITERATURE REVIEW

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We discuss preliminary results of a literature review pilot study regarding the use of CAS in higher education. Several issues surrounding technology integration emerged from our review and are described in detail in this paper. The brief report on the type of analysis and the integration scope in curriculum suggest that the multi-dimensional theoretical framework proposed by Lagrange et al. (2003) needs to be adapted for our focus on systemic technology integration in tertiary education.

INTRODUCTION

A growing number of international studies have shown that Computer Algebra System (CAS-based) instruction has the potential to positively affect the teaching and learning of mathematics at various levels of the education system, even though this has not been widely realized in schools and institutions (Artigue, 2002; Lavicza, 2006; Pierce & Stacey, 2004). In contrast to the large body of research focusing on technology usage that exists at the secondary school level, there is a definite lack of parallel research at the tertiary level. However, Lavicza (2008) highlights that university mathematicians use technology at least as much as school teachers, and that the innovative teaching practices involving technology that are already being implemented by mathematicians in their courses should be researched and documented. Further, Lavicza (2008) found that within the research literature there existed only a small number of papers dealing with mathematicians and university-level, technology-assisted teaching. In addition, most of these papers are concerned with innovative teaching practices, whereas few deal with educational research on teaching with technology. These findings coincide with school-focused technology studies conducted by Lagrange et al. (2003) and Laborde (2008).

We aim to point out that it is particularly important to pay more attention to university-level teaching, because universities face new challenges such as increased student enrollment in higher education, decline in students’ mathematical preparedness, decreased interest toward STEM subjects, and the emergence of new technologies (Lavicza, 2008). Mathematicians must cope with these challenges on a daily basis and only a few studies have offered systematic review and developed recommendations in this area. Our project aims at both documenting university teaching practices involving technology, and formulating recommendations for individual and departmental change. Our research program also aims at raising the amount of attention paid to tertiary mathematics teaching from a research point of view and, from a more practical side, elaborating on specific issues and strategies for the systemic integration of technology in university mathematics courses.
METHOD DESIGN AND IMPLEMENTATION

Based on the above-mentioned Lavicza (2008) findings and recommendations, we designed a mixed methods research study which involves a systematic review of existing literature regarding CAS use at the tertiary level. The theoretical framework developed by Lagrange et al. (2003) involved several stages. They first reviewed a large number of papers in relevant journals and then categorized these papers into five “types.” Based on these types, they then selected a sub-corpus of papers dealing specifically with educational research papers focusing on technology use mainly in the secondary school. Through the careful analysis of this sub-corpus of papers, they further developed seven dimensions, each with key indicators, and then proceeded to identify and further analyze papers that best described each of these dimensions.

The theoretical framework of Lagrange et al. (2003) provided our research team with a helpful foundation from which to prepare for our own literature review which will involve approximately 1500 papers/theses. It was decided to implement a pilot study for this large literature review in order to begin to work with the Lagrange et al. framework and to determine if it would be sufficient for our purposes, or may be in need of certain modifications. In the summer of 2008, we therefore began our pilot study focusing on 326 contributions dealing with CAS use in secondary/tertiary education. These papers were drawn from two well-regarded journals, namely the *International Journal for Computers in Mathematical Learning* (issues since its beginning in 1996) and the *Educational Studies in Mathematics* (since 1990). We also selected proceedings from two technology-focused conferences, namely the *Computer Algebra in Mathematics Education* (since its first meeting in 1999) and the *International Conference on Technology in Collegiate Mathematics* (since 1994 with first electronic proceedings). A sub-corpus of 204 papers dealing specifically with CAS use at the post-secondary level was also identified to further focus the analysis.

While the descriptive categories found within the Lagrange et al. template were helpful, we began to notice that several other category/theme columns would be helpful at this stage of the instrument/template development (e.g., we added fields such as “computer/calculator,” “implementation scope,” and “implementation issues”). An important point to note here is that in contrast to the Lagrange study where the majority of papers were those describing educational research results, our selection of papers revealed a majority that focused on practitioner innovations with very few involving educational research. Thus, we realized that in order to develop our template for reviewing the large number (1500) of papers in the research study proper, we would have to separate the practitioner report type papers from the educational research papers, and further modify the template in both of these areas. In this paper we outline preliminary results of our ongoing pilot study, with a specific focus on a series of “issues of implementation” at the tertiary level of education.

RESULTS

The majority of the papers in the corpus are practice reports by practitioners (88%), whereas the remaining contributions are education research papers (10%) or letters to
Among the practice reports, different types of contributions become apparent. Some (94) are merely examples of CAS usage. Other papers (41) are mostly examples of CAS but feature reflections by the practitioner. A few (13) have the practitioners go further and include classroom data and perform some basic analysis. There are also papers (5) that focus on classroom surveys and a small set (7) that examines a specific issue in detail. The remaining contributions (23) are conference abstracts only. The analysis of the education research papers according to Lagrange et al.’s multi-dimensional framework (2003) is still in progress. In this paper, we focus our analysis mainly on practitioner reports.

In addition, nearly all papers are American (87%). The computer use is more evident (59%) than the use of graphical calculators (29%) or than the combined use of both computer and graphing calculators (10%). Furthermore, the most widely used CAS in the corpus is the graphing calculator (83 papers), followed by Maple (53) and Mathematica (43). Derive (21) and Matlab (11) are also common, as well as 27 papers dealing with other CAS. In what follows, we elaborate on one particular significant aspect of the study, namely “integration issues” that emerged from our review, and also briefly report on “integration scope.”

**ISSUES OF CAS INTEGRATION**

Education researchers and practitioners widely wrote about issues surrounding the use and implementation of CAS at post-secondary education (72 papers). With regard to practitioner reports, 56 papers identify some issues; of these there are 20 that go into considerable detail. These papers could be further divided into two categories: Seven of them deal with a specific problem relating to CAS (e.g., rounding error) and thirteen discuss various implementations of CAS while underlining the hurdles the authors encountered. Of the sixteen issues identified in the corpus and summarized in Figure 1, we divide them into three categories: Technical (first four columns), cost-related (fifth column), and pedagogical (last 11).

There are four issues discussed in the literature dealing specifically with technological aspects: Lab availability (Lab), reliability of technical support (Tec), system requirements (Sys) and troubleshooting (TrS). These issues may not be independent from each other. For example, May (1999, p. 4) urges instructors to test out their Maple worksheets on the lab computers rather than their own workstations due to such machines having less

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**Table 1: Type of Contribution**

<table>
<thead>
<tr>
<th>Type of Contribution</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presentation of Examples</td>
<td>46%</td>
</tr>
<tr>
<td>Examples with practitioner reflections</td>
<td>20%</td>
</tr>
<tr>
<td>Classroom Study</td>
<td>6%</td>
</tr>
<tr>
<td>Classroom Survey</td>
<td>3%</td>
</tr>
<tr>
<td>Examinations of a specific issue</td>
<td>3%</td>
</tr>
<tr>
<td>Abstract only</td>
<td>11%</td>
</tr>
<tr>
<td>Education research papers</td>
<td>10%</td>
</tr>
<tr>
<td>Letters</td>
<td>1%</td>
</tr>
</tbody>
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**Figure 1: Issues in integrating CAS in university education**
memory installed in them. Weida (1996, p. 3) notes that in troubleshooting, various hardware problems arise and his “experience and lots of calls to the Computer center” helps. An unexpected issue for him was the class interruption of students not enrolled in his class. While they would never think to disrupt a lecture, they would see nothing wrong with walking into his lab session to complete homework for other courses.

Many reports mention the issue of costs (Cost) incurred by integrating CAS into instructors’ courses, providing few further details beyond the existence of the financial obstacle. An exception occurs in one paper where the authors argue for a particular choice of open-source (free) technology (Hohenwarter et. al, 2007, p. 5).

Wu (1995) notes that besides the cost aspect, enacting calculus reform “requires more talent and training” (p. 1). This need for trained staff (staf) is mentioned in seven papers, often in conjunction with other issues. For example, to deal with technical difficulties during labs, Weida relies on his own experience to assist in troubleshooting (1996, p. 3). At the beginning of an attempt at CAS integration, Schurrer and Mitchell (1994, p. 1) wondered, “how they could go about motivating [sceptical mature faculty] to consider introducing the available technology and making the curricular changes this would require?”

Schurrer and Mitchell (pp. 1-2) further discuss the need for time for the faculty (TimF) to design courses and meaningful activities with technology. Their department required decisions on types of technology used and on what technology curriculum package had a “right mix.” They note that program-wide integration takes time. In their case at University of Iowa, it took seven years to implement (p. 3). Even after a curriculum change, additional time demands on faculty are reported by practitioners. Wrangler (1995, p. 8) notes that near constant improvement is needed in lab experiments and stresses that for faculty there is “no resting on laurels.” A closely related issue is the problem of time management in courses (TimC). Wrangler (p. 8) remarks that besides the time he spent outside of class, he had to take his students into the lab and walk them through basic commands. Many other practitioners, such as May (1999, p. 4), express similar sentiments. While this issue is discussed less frequently than time spent outside the classroom, practitioners report about both issues in conjunction (e.g., Wrangler p. 8).

CAS integration also affects classroom time management with respect to course content. Dogan-Dunlop (2003, p. 4) remarks that, “since class time was allocated for in-class demonstrations and discussions, detailed coverage of all the topics that were included in the syllabus was not possible.”

Another source of pressure on time management is the failure of students to achieve learning objectives (Obj). Krishanamani and Kimmons (1994, p. 4) note that students failed to learn material assigned in labs and they had to include it in later lectures.

One particular type of student error that clashes with learning objectives is the assumption on the part of students that their methodology is correct if their paper-and-pencil calculations match up with results obtained from the computer. As Cazes
et. al. (2006 p. 342) write, “a correct answer does not mean the method is correct or is the best one. Teachers and students must be aware of such… pitfalls.” Often students engaged in trial and error strategies, with students guessing the answer from feedback without making a proper mathematical argument (p. 347). Instructors sometimes failed to ensure that students found an “optimal” solution to a particular problem rather than just having a “correct” answer (pp. 342-343).

Pedagogical difficulties with learning objectives can place demands on faculty time not only inside but also outside of the lecture hall. Dogan-Dunlap (2003 p. 4) had to redesign his course and the use of CAS within it three different times because of such concerns. As previously discussed, there is an ongoing time commitment by faculty to improve their lecture and laboratory instruction and Dogan-Dunlap’s experiences show that student difficulties may greatly influence the nature of those changes.

Related to the learning objectives issue, that of guidance (Gui) also emerges from the review. Often practitioners show concerns as to how much help they should give their students without compromising learning objectives. Westhoff (1997) designed a student project for Multivariate Calculus on the lighting and shading of a 3-dimensional surface. He found that the difficulty in the project, due to its complexity, lays in determining how much he could tell his students (p. 6). Another area in which guidance becomes an issue is mentioned by Weida (1996). Noting that there is a “fine line between helping students… and ‘giving away’ the answers,” he remarks that such a problem is “particularly exacerbated at the end of a lab when the slower workers are running out of time” (pp. 3-4). Weida further presents the idea that careful scheduling could help alleviate this by ensuring that there isn’t a need to leave immediately after the lab.

Student frustration (Frus) is another issue related to learning objectives. Cazes et. al. (2006, p. 344) note that students would often seek help either online or via the instructor “after having encountered the first difficulty” rather than attempting to solve the problem on their own. Krishahamani and Kimmons (1994) took steps to reduce anxiety both in course design and in providing additional help for students. Several measures, including reduced expectations, more time for tests, increased extra credit problems and a homework hotline were implemented (p. 2). Clark and Hammer (2003, p. 3) had a project for first year calculus modeling a rollercoaster. They found that “students who were not as “good” at Maple struggled, found the project (and Maple syntax) frustrating and were just happy to produce one mathematical model.” This suggests possible relationship between student frustration and failure regarding activity learning objectives, and the CAS syntax issue.

Syntax (Synt) is the second most frequent concern for both practitioners and students. Cherkas (2003) found this to be a source of student dissatisfaction. He quotes a student complaining, “Mathematica would cause a lot of problems. If I make a mistake in the syntax, I couldn’t do my work” (p. 31).
Tiffany and Farley (2004) exclusively focus on common mistakes in *Maple*, emphasizing the hurdle for practitioners caused by syntax. Practitioners employ various schemes attempting to minimize this difficulty. Some such as May (1999) design interactive workbooks that eliminate the need for teaching syntax entirely. Others like Herwaarden and Gielen (2001, p. 2) provide *Maple* handouts with expected output to their students. Some emphasize a pallet-based CAS such as *Derive* (Weida, 1996, p. 1) because it is easier to learn and has, according to them, a more straightforward notation.

Another source of student frustration is the unexpected behaviour of CAS (UnExp) even when their reasoning is syntactically and mathematically correct. Sometimes this is merely the case of paper-and-pencil calculations not easily matching up with CAS output. CAS may employ an algorithm efficient for computation and not necessarily one that matches a hand technique. For example, Holm (2003, p. 2) found that an online integral calculator would (rather than using the substitution method for $\int (3x^2 - 1)^7 \, dx$) simply expand the product and use the power rule. He notes that such cases provide an opportunity for learning, and that, referring to another classroom assignment, the more “savvy student would… expand $\sum_1^8 (3x^2 - 1)^8$.” Unexpected behaviour of CAS also takes the form of errors by the computers themselves. Due to the nature of floating point arithmetic and in spite of correct input by the user, roundoff error can cause the output to be wrong (Leclerc, 1994, p. 1). To encourage her students to adapt, Wu (1995, p. 2) purposely designed a lab with roundoff error. LeClerc urges students to be instructed in the nature of floating point arithmetic so that they “will be able to detect when roundoff has corrupted a result and hopefully find better ways to formulate or evaluate the computation” (1994, p. 4).

The concept of the “black box” (bbox) is examined in seven papers. Though this issue tends to be explored in more detail in education research papers, practitioners comment on it as well. O’ Callaghan (1997, p. 3) writes that faculty at Southeastern Louisiana University expressed concern that “students would become button pushers rather than problem solvers.” The managed used of the black box as an opportunity for students to explore complex mathematics beyond their level is discussed in great detail in education research papers (e.g., Winsløw, 2003, p. 283). Practitioners do not emphasize this potential as much. However, Cherkas (2003, p. 234) notes that CAS allows practitioners “to teach at a higher level of mathematical sophistication than is possible without such technology.”

Closely related to the “black box” issue, is the fear that students become too reliant on the technology (rely). This, along with student frustration, is the least mentioned pedagogical issue. Cherkas reports on a student complaint that s/he could not do questions on tests because “Mathematica usually did them for me” (pp. 231-232). An over-reliance on technology may interfere with learning objectives. Considering this, Shelton (1995, p. 1) emphasizes her “top-down” approach and writes that “students can avoid the technology crutch and approach the goal of developing determination and mathematical maturity to perform mathematics without the technology.”
The last and most commonly examined issue encountered in the literature is that of assessment (Ass). Practitioners encounter problems in evaluation. Schlatter (1999) allowed for CAS use during his exam for his multivariate calculus course. Unfortunately, in a question designed to test student understanding of the divergence theorem, several students simply used the CAS capabilities to solve the integral in a “brute force” approach (pp. 8-9). A poorly designed assessment thus leads to a failure in learning objectives. Schlatter further writes that he expected “to spend more time during this semester... more carefully designing exam questions” (p. 8), pointing again to the issue of faculty time.

Interpreting CAS output is discussed frequently. Quesada and Maxwell (1994, p.207) never accept a decimal answer (even if correct) if there is a proper algebraic expression. Many papers that discuss mathematical projects stress the use of written reports (e.g. Westhoff, 1997, p. 1). Lehmann (2006, p. 3) writes in his assignment “the important part of this assignment is the thought you put into it, the analysis you do and the presentation of your solution, not the answers themselves.” Xu (1995, p. 1) found that students were finding derivatives of easy functions by hand on assignments, but using graphing calculators to solve the more difficult questions. To show students “that the calculator could not do everything for them” he found functions in the textbook that “were easy to handle by hand but could not be done easily on the calculator.”

CAS INTEGRATION SCOPE
Policy making regarding the curriculum in tertiary education is rather different than in school education. Hodgson and Muller (1992) mention that school mathematics curricula are in general developed by Ministries or Boards and implemented in the classroom by teachers, whereas tertiary mathematics curricula are developed and implemented by the same actors, i.e., faculty in departments of mathematics. However, change involving technology in tertiary curriculum, like in its secondary school counterpart, seems to remain very slow (Ruthven & Hennesssy, 2002). Lavicza (2006) argues that due to academic freedom, "Mathematicians have better opportunities than school teachers to experiment with technology integration in their teaching". This ad hoc basis is strongly reflected in our literature review. A large majority (67%) of the corpus restricted to practice reports discusses CAS usage with regards to one course, or in other words, CAS integration by one practitioner. While 16% has a scope that reaches across a series of courses (e.g. calculus courses), 11% discusses a CAS implementation with a grouping of courses (e.g. all first year courses). Only 6% discusses a program-wide implementation within a department.

CONCLUSIONS
There is a need to develop a framework for the review of literature on the use of CAS at tertiary education that will integrate specificities of university-level education and technology integration. A significantly stronger majority of papers in our study stemmed from practitioner use (88%) than in Lagrange et al.‘s (2003) study (60%)
which stated, ”Most of the [practitioner] papers lack sufficient data and analysis and we could not integrate them into the [detailed (statistical) analysis]” (p.242). Our selection of journals and conferences for our pilot study may have influenced the above percentage. Nevertheless, this reality will clearly influence the development of our analytical framework henceforth. Lagrange et al. (2003) further state,

[Practitioner] papers offer a wealth of ideas and propositions that are stimulating, but diffusion is problematic because they give little consideration to possible difficulties. Didactical research has to deal with more established uses of technology in order to gain insights that are better supported by experimentation and reflection. We have then to think of these two trends as complementary rather than in opposition. (p.256)

We aim at elaborating upon these complementary trends at the post-secondary level by both analyzing existing instructional practices and scrutinizing problematic issues within implementation. Lagrange et al. (2003) further state that the “integration into school institutions progresses very slowly compared with what could be expected from the literature” (pp. 237-8). This might be the case for school education, but apparently less so for tertiary education (Lavicza, 2008). The research literature about school mathematics and technology seems to pay less than adequate attention to the actual classroom implementation piece. The literature about tertiary mathematics and technology tends to inform us more about (individual) implementation than its didactical issues and benefits. This suggests that there may be a need for more education research focusing on the integration of technology in tertiary education. It also points, as suggested by Table 2, to the need of resources for departments of mathematics for systemic integration of technology in curriculum. At the recent ICME 11 conference, the results of a special survey highlighted concerns about the international trend of disinterest in university mathematics (ICME 11, n.d.). Departments of mathematics have a responsibility to question the current curriculum. We contend that part of this responsibility includes the careful consideration of the role and relevance of technology within that 21st-century curriculum and classroom.

REFERENCES


1 *http://archives.math.utk.edu/ICTCM


