THE IMPACT OF TECHNOLOGICAL TOOLS IN THE TEACHING AND LEARNING OF INTEGRAL CALCULUS

Lois, Alejandro* & Milevicich, Liliana*
Universidad Tecnológica Nacional (Argentina)

There is still a tendency to see that mathematics is not visual. At University education, it´s evident in several ways. One of them, is an algebraic and reductionist approach to the teaching of calculus.
In order to improve educational practices, we designed an empirical research for the teaching and learning of integral calculus whith technological tools as facilitator resources of the process of teaching and learning: the use of predesigned software that enables to get the conceptualization in a visual and numeric way, and the using of a virtual platform for complementary activities and new forms of collaboration between students, and between teachers and students.

KEY WORDS

Predesigned software – virtual enviroments – registers of representation - social infrastructure - epistemological infrastructure

INTRODUCTION

The ideas, concepts and methods of mathematics presents a visual content wealth, which can be geometrically and intuitively represented, and their use is very important, both in the tasks of filing and handling of such concepts and methods, and for the resolution of problems.
Experts have visual images, intuitive way of knowing the concepts and methods of great value and effectiveness in their creative work. Through them, experts are able to relate, most versatile and varied, often very complex, constellation of facts and results of their theory and, through such significant networks, they are able to choose from, so natural and effortless, most effective ways of solving the problems they face (Guzman, 1996). Viewing, in the context of teaching and learning of mathematics at the university, has to do with the ability to create wealthy images that individuals can handle mentally, can pass through different representations of the concept and, if necessary, can provide the mathematic ideas on a paper or computer screen (Duval, 2004). The creative work of mathematicians of all times has had “the visualization” as its main source of inspiration, and this has played an important role in the development of ideas and concepts of the infinitesimal calculus.
However, there is a tendency to believe that mathematics is not visual. At university education, it´s evident, particularly through an algebraic and reductionist approach of the teaching of calculus. One of the didactic phenomena which is considered essential in the teaching of Mathematical Analysis, is the “*algebrización*”, that is: the algebraic treatment of differential and integral calculation. Artigue (in Contreras, 2000) expresses this fact in terms of an algebraic and reductionist approach of the calculation which is based on the algebraic operations with limits, differential and integral calculus, but it treats the thinking and the specific techniques of analysis in a

simplistic way, such as the idea of instantaneous rate of change, or the study of the results of these reasons of change.

We believe that the problems with Mathematical Analysis learning, in the first year of college, have to do with this context. These difficulties are associated with the formalism in dealing with the concepts and the lack of association with a geometric approach. Anthony Orton has worked for a long time about the difficulties in learning calculus. His research work at the University of Leeds confirmed that students had difficulty in learning the concepts of calculus: the idea of exchange rate, the notion of a derivative as a limit, the idea of area as the limit of a sum (Orton, 1979). Cornu (1981) arrived at similar conclusions regarding the idea of "unattainable limit" and Schwarzenberger and Tall (1978) regarding the idea of "very near". Ervynck (1981) not only documented the difficulties of the students in understanding the concept of limit but he also remarked the importance of viewing the processes by successive approximations. In this sense, wue can see that usual graphs met in textbooks of calculus have two problems: they are static, which can not convey the dynamic nature of many of the concepts, and also they have a limited number of examples, usually one or two, which leads to develop, in students, a narrow image of the concept in question. (Tall and Sheath, 1983). In this sense, taking into account our previous exploratory research (Milevicich, 2008), we can say that students can not understand the concept of definite integral of a function as the area under the curve, because they do not visualize how to build this area as a sum, usually known as Riemann Sum.

In terms of the educational processes, it should be noted that teachers usually introduce the concept of integral in a narrative way, avoiding the real purpose, which is to obtain more precise approximations. A simplistic approach to the concept is usually done, disconnected from integral calculus applications, which hinders the understanding of students, and consequently, the resolution of problems relating to calculation of areas, length of curves, volume of solids of revolution, and those dealing with applications to the engineering work, pressure, hydrostatic force and center of mass.

JUSTIFICATION

Innovation in educational processes including th use of multimedia means demands not only on teachers´ professionalism but also new activity managing. Research work is currently being carried out at different universities aiming to find out what use teachers make of these tools and the specific competencies that they have to acquire for making effective use of them. From a didactic point of view, the usage of multimedia in teaching-learning process, presumably, should increase students motivation, and, in that sense, we ask ourselves: What should be the goals of education aimed at improving the university today? and How can we make it easier through the use of technological tools? The answers to these questions are not clear for us. Students, nowadays, have more and more information than they can process, so that one of the functions of the university education would be to provide them with cognitive and conceptual tools, to help them to select the most relevant information.

University Students should try to get skill and develop attitudes that enable them to select, process, analyse and draw conclusions. This change in the goals represents a departure from traditional learning. In this sense, the use of a predesigned software in the classroom, designed within the group research, can be a teaching facilitator resource of the process of teaching and learning:

- to convey the dynamic nature of a concept from the visualization,
- to coordinate different registers of representation of a concept,
- for the creation of personalized media best suited to the pedagogical requirements of the proposal.

RESEARCH CHARACTERISTICS

Population and sample

The population is made up of Engineering students from Technological University and the specimen is a Electrical Engineering commission of about 30 students. Regarding the characteristics of the population, some considerations can be made about their previous knowledge of integral calculus. Some students come from the Mechanic School of a known automotive Company and others, from a technical electricians school. Based on a detailed analysis of library materials used by teachers in these institutions, and the students' writings, we infer that integrals are taught as the reverse process of derivation, with the focus on the algebraic aspects. These students study the concept of integral associated with a primitive, practice various methods of integration, transcribe or solve hundreds of exercises in order to calculate integrals, and some of them even achieve a considerable level of skill in the use of tricks and recipes that help to be more effective in getting results. Another group of students come from near schools where geometric concepts are little, essentially the calculating of areas studied during primary and middle school. However, the largest group, is made up of students studying Mathematical Analysis for the second or third time. Some of them have completed the course in previous years but failed in the exams. It may be that those students have some ideas about integral calculus and its applications, or not. It is possible that those ideas interfere with the getting of new knowledge or hinder it (Bachelard, 1938), primarily on those students who associate the integral exclusively to algebraic processes. That is why it was very important to carry out a diagnostic test (pretest) that would allow exploration on the previous skills and students ideas about definite integral and thus, categorize according to the following levels of the independent variable:

Level 1: associate the concept of integral to the primitive of a function and calculates easy integrals.

Level 2: associate the concept of integral to the primitive of a function, calculates easy integrals and links the concept with the area under the curve.

Level 3: associate the concept of integral to the primitive of a function and links the concept with the area under the curve.

Level 4: has no specific pre knowledge associated with the topic.

Focus

The general purposes of our research work were:

to determine if students understand the concept of integral through the implementation of a proposal that would allow its teaching in a approaching process, using different systems of representation, according to the processes man has followed in his establishment of mathematical ideas,

to analyze, in a reflective learning context, the ways in which students solve problems related to integral calculus,

and the specific purposes were:

to categorize the students, involved in the experience, according to his integral calculation preconceptions, at the beginning of the intervention,

to implement a proposal that provided, on the one hand, the use of different systems of representation in the development of individual and group activities, and on the other, to promote conjeturación, experiment, formalization, demonstration, synthesis, categorization, retrospective analysis , extrapolation and argumentation, with the help of specific software, and feedback on students' early productions so they could reflect on their own mistakes,

to review progress achieved after the implementation of the didactic proposal, to *analyze* the impact of using a virtual platform for complementary activities.

Methodology

The design is pre-experimental type of pretest - treatment - postest with a single group. The independent variables in this study are: the design of teaching and pre knowledge of students on the definite integral. The dependent variable is: the academic performance.

Regarding these previous knowledge, a pretest at the beginning of the intervention allowed to place each student in one of the preset categories. After 8 weeks of intervention, a postest allowed to determine the levels of progress made in learning the concepts of integral calculus in relation to the results obtained in the past three years cohorts (2003, 2004 and 2005). In addition, an interview at the end of the experience was implemented, in order to gather qualitative information.

In order to improve educational practices, we designed a proposal for teaching and learning integral calculus according to the proposal of using a pre designed software as indicated in the goals. In this sense:

We designed a software package allowing the boarding of integral calculus from the concept of definite integral associated with the area under the curve, from a geometric point of view.

We selected the problems students should solve, in a way, that their approach would allow to establish a bridge between conceptualization of integration and problems related to engineering. In that sense, the use of the computer allowed to have a very wide range of problems, where the choice was not conditioned by the difficulty of algebraic calculus.

The students used pre designed software for:

a) The successive approximations to the area under a curve, considering left and right points on each of the subintervals. The software allows to select the function, the interval and the number of subdivisions. (See Figure 1).
b) The successive approximations to the area under a curve through the graph of the series which represents the sum of the approach rectangles (See Graphic 1) and the table of values (See Table 1).
c) The visualization of the area between two curves, it also allows to determine the points of intersection.
d) The representation of the solid of revolution on different axes when rotating a predetermined area. (See Figure 2)
e) The numerical and graphical representation (through table of values) of the area under the curve of an improper integral.

It was designed a set of activities with the purpose students conjecture, experience, analyze retrospectively, extrapolate, argue, ask their peers and their teachers, discuss their own mistakes and evaluate their performance. Assessment techniques were redesigned, so that the analysis of students productions would provide feedback about their mistakes.

We incorporated a Virtual Campus using Moodle supporting design, as an additional element, in order to keep continuity between two spaced weekly meetings. According to Misfeldt and Sanne (2007), communication on mathematical issues is difficult using computers and a weekly meeting is insufficient. In response to this problem, we used the virtual campus for communication, flexibility and cooperation, but the use of it was not a learning objective in itself. Instead, we used it to publish texts and exercises guides and also, students made active use of the forum for discussion groups.

We also had in mind that the challenges in creating an online learning environment might be different when working with mathematics than in other topics (see also: Misfeldt et. al, 2007 & Duval, 2006). Many of the signs that goes into building mathematical discourse is not available on a standard keyboard, and the way that mathematical communication often is supported by many registers and modalities that are used simultaneously, as writing and drawing various representations on the blackboard or paper is also not avalilable. Students, using the Virtual Campus, had the possibility to upload files showing the solving process and using every symbol they needed.

Implementation of the proposal

Students were distributed in small groups no more than three, who worked in several sub-projects. Each of them included a significant number of problems.
Subproject No. 1: The concept of integral.
Subproject No. 2: Fundamental theorem of Calculus.
Subproject No. 3: Improper integrals.
Subproject No. 4: Area between curves.
Subproject No. 5: Applications of Integral Calculus.

Guidelines for systematic work for each of the meetings were made. In the first part, it was discussed the progress and difficulties of the previous practice, where the essential purpose was to ensure that students analyze their own mistakes, and the second part, teachers and students worked on new concepts at the computer laboratory. The first part of each meeting was guided by the teacher, but a assistant teaching and a observer teacher were present in the class. The second half had the same staff and an extra assistant teaching.

The assesment took place during the whole experience through:

- weekly productions of students reflected in their electronic folders and notebooks. These ones allow cells to keep comments, observations, etc.; very valuable material in assessing the level of understanding achieved by students.
- students interaction in classes and into working groups.
- Students participation in the discussion forums of the virtual campus.

In that sense, spreadsheets were used for monitoring activities, which proved to be an effective tool to assess different aspects relevant to student´s performance. Summary notes taken by the observer teacher along the 8 weeks allowed us to infer the change of attitude in an important group of these students. From the initial population, made up of 30 students, 24 of them showed increased commitment to the development of activities.

Some of these activities were:

Subproject 1: Evaluate the following integrals by interpreting each in terms of areas

a) $\int_1^3 e^x dx$ *b)* $\int_0^3 (x-1)dx$

Case a: because f(x)=e^x is positive the integral represents the area. It ca be calculated as a limit of sums and a computed algebra system can be used to evaluate the expression.

Case b: The integral cannot be interpreted as an area because f takes in both positive and negative values. But students should realize that the difference of areas works.

Subproject 3:Sketch the region and find its area (if it is possible)

a) $S=\{(x,y)/\ 0 \le x \le \pi,\ 0\le y \le Tan(x)Sec(x)\}$
b) $S=\{(x,y)/\ x \ge 0,\ 0\le y \le e^{-x^2}\}$

Case a: Probably students confuse the integral with an ordinary one. They should warn that there is an asymptote at x= π/2 and it must be calculated in terms of limits. At this point students must bear in mind that whenever they meet the symbol $\int_a^b f(x)dx$ they must decide, by looking at the function f on [a,b], whether it is an ordinary definite integral or an improper integral.

Case b: The integral is convergent but it cannot be evaluated directly because the antiderivative is not an elementary function. It is important students look for a way to solve the problem and although it is impossible to find the exact value, they can know whether it is convergent or divergent using the Comparition Test for Improper Integrals.

Both examples above show activities where students need to find out solutions and get conclutions without teacher telling them.

RESULTS

The pretest was done by 30 students, the results allowed us to locate them as follows: 15 at Level 1, 1 at Level 2 and 14 at level 4. It should be noted that those who came from technical schools had achieved a considerable level of skill in the calculation of integrals but they didn´t know about the links with the concept of the area. The postest consisted of 6 problems related to the sub projects students had worked on, each of which was formed by several items. It was provided to the 24 students remaining at the end of the experience, and took place at the computer laboratory, where students usually worked. In general, the level of effectiveness was above 50%, except in the case where they were asked to determine the area between two curves and then the volume to rotate around different axes. The difficulty was to get the solid of revolution from a shift in the rotation axis. Although the students had no difficulty in getting the solid geometrically, they could not get an algebraic expression for it.

In a comparison with the three previous year cohorts, it was possible to emphasize the following differences:

a) There were no important difficulties in linking the concepts of derivative and integral.
b) An important group of students (83% of them) successfully used Fundamental Theorem of Calculus.
c) In general, there were no difficulties in algebraic developments, however it is possible to associate the lack of such obstacles to the use of the computer. All of students tested, could associate the concept of solid revolution with the concept of integral, and even more, they were able to correctly identify the area to rotate.
d) The 74% of the students tested could identify improper integrals, but only 43% of them, correctly, applied the properties.
e) Most of the students tested succeeded in establishing a bridge between the conceptualization of integration and problems related to engineering: 89% of them correctly solved problems relating to applications for work, hydrostatic pressure and force.

The written interviews at the close of the experience reflects the importance that students attribute to the use of virtual campus as an additional resource: most of students were very keen on having prompt responses from the teacher when asking questions in the forum and the help offered by other students.

One of the questions was:

"How did teachers interventions at the forum helped, when you had difficulties in the development of practices? (A: they were decisive, B: they helped me to understand, C: they were not decisive. I managed without them, D: they did not contribute at all. Please explain your choice)."

12 students selected A , 8 puplis selected B, 4 students selected C and D was not selected.

Some of the explanations given by students were:

Student a: *"…They helped me because teachers answered quickly and clearly"*

Student b*: "…Excellent, clear and concise answers that helped with the resolution of the problems."*

Student c:*"…There were many situations where I managed to solve a problem just reading the doubts of my fellow students. I have not done a lot of questions at the forum because someone asked my doubt before me…"*

It is worth mentioning that there were no substantial differences between the students belonging to different categories, according to the pretest. An analysis of results in relation to the initial categorization, suggests that pre conditioned ideas did not influenced the acquisition of new knowledge. There were no significant differences among the largest groups of students ranked in levels 1 and 4.

CONCLUTIONS

The failure of the students in understanding the concepts of calculation, more generally, and the definite integral, in particular, is one of the most worrying problems in the learning of Mathematical Analysis, in the first year of Engineering, as this hinders the understanding and resolution of problems of application. The way to search for the causality of this failure led us to raise the need for a change in the point of view. This is a change in the processes and representations through which students learn, in this case, the concept of integral.

Focusing our attention on the problem how students can understand more deeply the concepts using tools and technology, we can conclude that the recent evolution of digital materials leads to devote a specific interest to the change of activities induced by virtual learning environments which allow new forms of collaboration between students, and between teachers and students. Besides, the use of the computer is a valuable strategy with the aim of achieving significant learning. While learning the concept of definite integral, the computer facilitates making the important amount of calculations and displays the successive approximations, contributing to the concept of area under the curve. In that sense, the use of a predesigned package software allowed students to view the alignment between the smaller and smaller geometric rectangles and curvilinear area to be determined.

The carrying out of the activities required the use of the predesigned package software, specifically adapted to the needs of the experience. Students had to make numerous graphs, edit their guesses, propose new solutions, test, and analyze retrospectively the achieved results. Dynamic graph was valued for making student work with figures easier, faster and more accurate, and consequently for removing drawing demands which distract them from the key point of a problem. Various aspects of making properties apprehensible to students through dynamic manipulation were expressed in CERME V Plenaries: "*When a dynamic figure is dragged, students can see it changing and see what happens, so that properties become obvious and students see them immediately*" (Ruthven,2007: 56). In that sense, technology is seen as supporting teaching approaches based on guiding students to discover properties for themselves. We agree on suggesting that teachers might guide students towards an intended mathematical conclusion, but students could find out how it works without us telling them so that they could feel they are discovering for themselves and could get a better undestanding.

REFERENCES

Bachelard, G. (1938). *La formation de l'esprit scientifique*, Paris, Librairie philosophique J. Vrin.

Contreras de la Fuente, A (2000). La enseñanza del Análisis Matemático en el Bachillerato y primer curso de universidad. Una perspectiva desde la teoría de los obstáculos epistemológicos y los actos de comprensión. *Cuarto Simposio de la Sociedad Española de Investigación en Educación Matemática*. España, Huelva. pp. 71-94

Cornu, B.(1981). Apprentissage de la notion de limite: modèles spontanés et modèles propres. En *Actes du Cinquième Colloque du Groupe Internationale PME,* (pp. 322-326). Grenoble, France.

Duval, R. (2004). A crucial issue in mathematics education: The ability to change representation register, Regular Lecture en el *10th International Conference on Mathematics Education* (ICME10), Dinamarca.

Duval, R (2006). A Cognitive Analysis of Problems of Comprehension in a Learning of Mathematics, Educational Studies in Mathematics 61 (1-2), 103-131.

Ervynck, G.(1981). Conceptual difficulties for first year university students in the acquisition of the notion of limit of a function. En *Actes du Cinquieme Colloque du Groupe Internationale PME,* (pp. 330-333). Grenoble, France.

Guzmán, M (1996). El rincón de la pizarra. Ensayos de Visualización en Análisis Matemático. *Elementos básicos del Análisis*. España. Madrid: Pirámide.

Milevicich, L (2008) Las ideas previas sobre el calculo integral en los alumnos de primer año de la universidad. *Acta Latinoamericana de Matemática Educativa*, 21, 329-338. México, Colegio Mexicano de Matemática Educativa A.C. y Comité Latinoamericano de Matemática Educativa A.C.

Misfeldt, M., M &, Sanne, A. (2007). Flexibility and cooperation: Virtual learning environments in online undergraduate mathematics. In Pitta-Pantazi,D., Philippou, G. (eds.) *Proceedings of the V Congress of the European Society for Research in Mathematics Education. CERME V*, Department od Education, University of Cyprus, ISBN: 978-9963-671-25-0, pg. 1470-1478.

Orton, A.(1979). An investigation into the understanding of elementary calculus in adolescents and young adults. En *Cognitive Development. Research in Science and Mathematics*, Universidad de Leeds (pp.201–215). Gran Bretaña.

Ruthven, K. (2007). Teachers, technologies and the structures of schooling. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of the fifth congress of the European Society for Research in Mathematics Education*, CERME 5, Larnaca, Cyprus, http://ermeweb.free.fr/CERME5b/.

Schwarzenberger, R., Tall, D.(1978). Conflicts in the learning of real numbers and limits. *Mathematics Teaching*, 82,44–49.

Tall, D., Sheath, G. (1983). Visualizing Higher Level Mathematical Concepts Using Computer Graphics. En *Proceedings of the Seventh International Conference for the Psychology of Mathematics Education*, (pp. 357–362). Israel.

APPENDICES

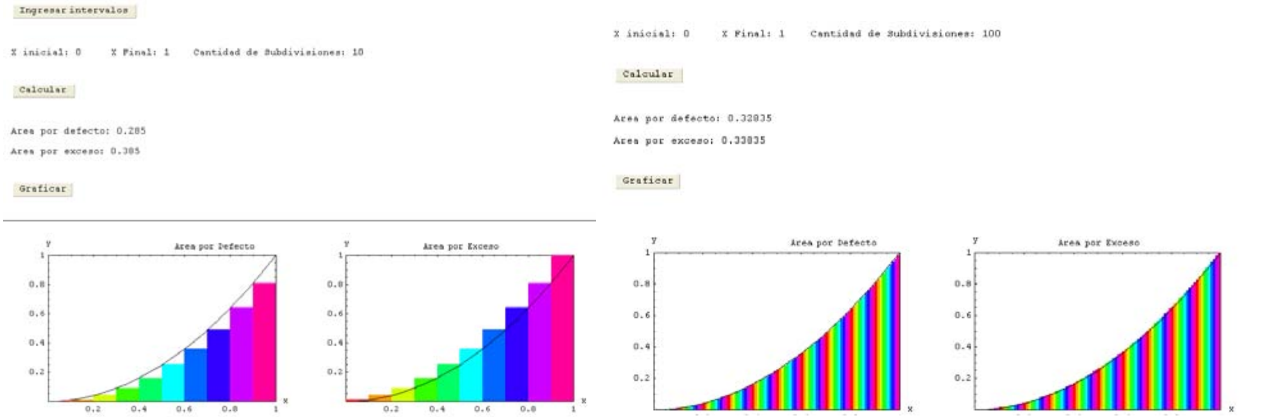

Figure 1. Capture screen from the predesigned software about conceptualization of definite integral. Estimation of the area of $y=x^2$ using 10 subdivisions and 100 subdivisions, $0 \leq x \leq 1$

number of subdivisions	default sums	excess sums
4	0,219	0,467
10	0,285	0,385
20	0,308	0,358
30	0,316	0,35
40	0,321	0,346
50	0,323	0,343
60	0,325	0,342
70	0,326	0,34
80	0,327	0,339
90	0,328	0,339
100	0,327	0,337

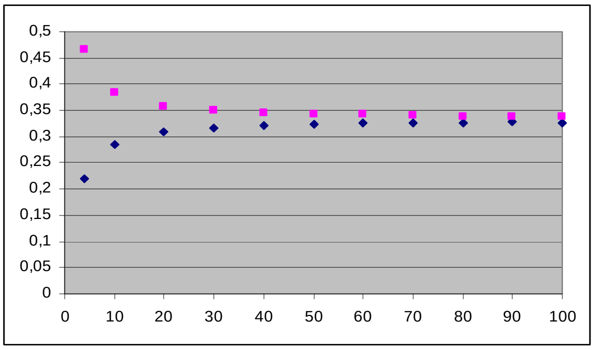

Table 1. Sums for different subintervals increasingly small under the curve $y= x^2$ on the interval [0,1]

Graphic 1. the series which represents the sum of the approach rectangles, default sums are in blue and excess sums are in pink.

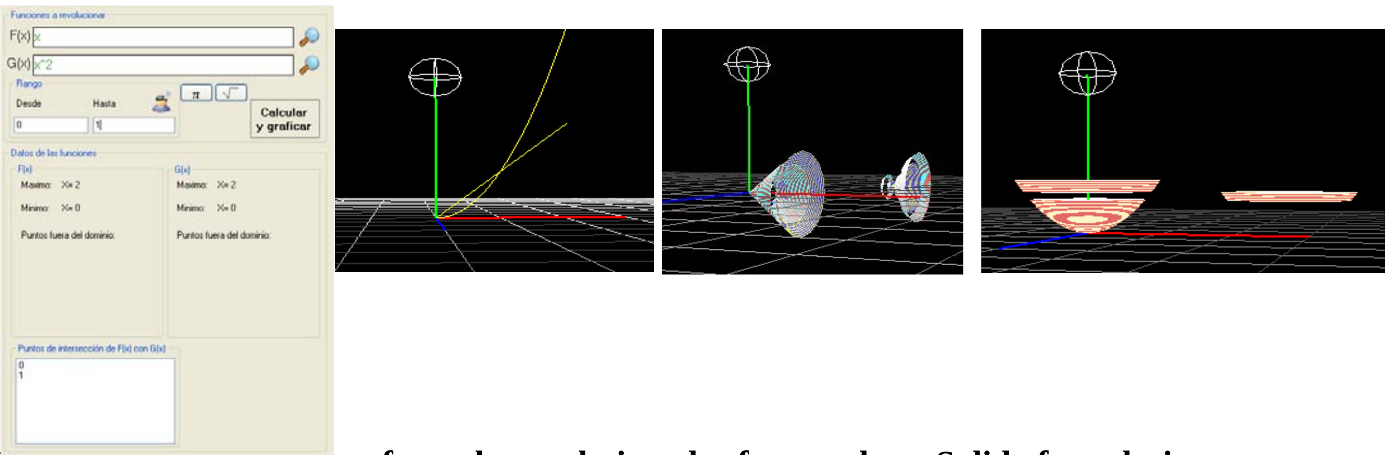

Figure 3. Captured screen from the predesigned software about Solid of revolution. Area between the functions y=x and $y=x^2$, and the solid of revolution that is generated to rotate on the x-axis and the y- axis.