TENSIONS BETWEEN AN EVERYDAY SOLUTION AND A SCHOOL SOLUTION TO A MEASURING PROBLEM

Frode Rønning

Sør-Trøndelag University College
N-7004 Trondheim
Norway

This paper reports on an empirical study from a mathematics lesson in a Norwegian 4th grade classroom. The pupils are making batter for waffles, and the mathematical challenges are mainly connected to measuring. The paper will focus on the process of determining the correct amount of milk for the batter and furthermore on the tension that can be observed in the discursive practice as a result of the pupils' and the teacher's conflicting goals.

THE CLASSROOM SITUATION

This study is done in a group of 20 4th grade pupils in a Norwegian primary school in a mathematics lesson. During the lesson the pupils come in groups of five to the kitchen area in the back of the classroom where they make batter for waffles that are going to be prepared later the same day and eaten by themselves and the rest of the 4th graders at the school. Each group is supposed to make an equal amount of batter based on a recipe that is written on a poster. Before starting the actual work with the batter each group had a discussion where the task was to find out how much of each ingredient they would need in order to make three times as much as indicated on the recipe. The teacher expressed to me that her main mathematical focus with the waffle making was the discussion about the three folding. I will not report on this discussion but I will go into the part of the working process where the pupils are actually going to measure out 15 dl of milk. The milk comes in boxes marked "1/4 liter", and the pupils have measuring beakers available that can take 1 litre. The beakers are transparent, with a scale reading "1 dl, 2 dl, …. 9 dl, 1 lit" from bottom to top. Each group has to determine the number of boxes needed to get the correct amount of milk.

THEORETICAL BACKGROUND

The notion of a complex mediated act goes back to Vygotsky (e.g. 1978) and has led to the idea of sociocultural artefacts that mediate between stimulus and response. Such artefacts can take many forms and they shape the action in essential ways (Wertsch, 1991). In mathematics the tools are often signs and symbols that represent an abstract mathematical concept, and the signs and symbols also often refer to a context or a specific object. A sign typically has two functions, a semiotic function – something that stands for something else – and an epistemologic function as the sign contains knowledge about that what it stands for (Steinbring, 2005).

One of the pioneers of semiotics is the American mathematician and philosopher Charles Sanders Peirce (1839-1914). He defines the terms involved in his triadic model of semiosis in the following way.

> A *sign* is a thing which serves to convey knowledge of some other thing, which it is said to *stand for* or *represent*. This thing is called the *object* of the sign; the idea in the mind that the sign excites, which is a mental sign of the same object, is called an *interpretant* of the sign. (Peirce, 1998, p. 13, emphasis in original)

Peirce describes three kinds of signs (or representamens), *icons*, *indices* and *symbols* referring to three ways the representamen is related to its object. An icon stands for its object by likeness, an index stands for its object by some real connection with it or because it makes one think about the object, whereas a symbol is only connected to the object it represents by habit or by convention (Peirce, 1998, pp. 13-17, 272-275).

Presmeg (2005) turns the triadic model of semiosis into a nested model. This nestedness is based on the idea that the totality of the triad (representamen, object and interpretant) becomes reified (Sfard, 1991) as a new object to which one can assign a representamen and an interpretant. This gives a nested chaining of signs which can serve as a model to describe processes leading to more general or more abstract situations.

An important justification for mathematics in school is often the alleged usefulness of mathematics in other subjects and in situations outside of the school. It has been questioned whether it is possible to use a school subject such as mathematics outside of its own domain, and in this context it has been found fruitful to investigate the boundaries between the in-school and out-of school practices (Evans, 1999).

On areas where an overlap between in-school and out-of-school practices occurs it could be expected that there is some tension between the motives and goals lying in the school mathematics and the specific out-of school activity. To analyse this tension I will use the framework from activity theory. Leont'ev writes that *activity* is energised by a *motive*, and that "[t]here can be no activity without a motive" (Leont'tev, 1979, p. 59). Further he talks about the components of the activity as *actions* – processes that are subordinated to certain *goals*. On the third level there are the *operations* – the means by which the action is carried out. It is possible to carry out the same action by means of various operations, which means that the chosen operation "is defined not by the goal itself, but by the objective circumstances under which it is carried out" (Leont'ev, p. 63). Hence, the choice of operation may depend on the specific *conditions* in the given situation. It is henceforth possible to envisage one particular action but different operations that may be chosen depending on whether one is situated within a school practice or within an out-of school practice. According to Leont'ev the activity is driven by a motive, and the actions are directed towards certain goals. An important point is that each activity answers to a specific need of the active agent. "It moves towards the object of this need, and it terminates when it satisfies it" (Leont'ev, p. 59).

METHOD

I have been collaborating with all the teachers in grades 1-4 at this particular school for two years. This collaboration has involved working with the teachers in workshop activities, discussing in small groups and observing in classroom situations. When observing in the classrooms I have videotaped the activities going on. On some occasions parts of the videotapes have been shown and discussed with the teachers afterwards. Prior to the episode reported on here the teachers and I had been working with aspects of multiplication and division in a sequence of several workshops. We had agreed that on two given days in February I was going to videotape a session from each of the four grades 1-4. Each teacher, or group of teachers, was free to design the activities in accordance with the normal progression in the class. The only constraint was that it should have something to do with multiplication and division, or preliminary work leading up to these concepts. I did not partake in designing the lessons.

In the grade four class, which is the focus of this paper, the mathematics lesson was scheduled for two hours. I stayed in the kitchen area all the time, and with a hand held video camera I tried to capture as much as possible of the activity going on. During the lesson I was mostly passive but as can be seen from the excerpts of the dialogue I sometimes posed questions to the pupils.

THE HANDLING OF THE MEASURING PROBLEM IN EACH GROUP

Group 1

One measuring beaker is filled with flour, and Ellie is mixing flour and eggs. Lucy (the teacher) asks what they think is a good idea to do to avoid lumps, and they agree to start adding milk. James and Jessica fetch one box of milk each, and they agree that altogether they need 15 dl. Jessica looks at the box on which is written “1/4 liter”.

1.1	Jessica:	This is one four litre
1.2	James:	One four litre
1.3	Jessica:	Yes, so we take one of these first. One whole of these
1.4	Lucy:	How are you thinking now?
1.5	James:	Have no idea
1.6	Jessica:	Yes, it should be five
1.7	James:	Yes, fifteen so now you must. We just say that this is one and a half
1.8	Jessica:	It is one comma[1] five. No, we are supposed to take … like this
1.9	Lucy:	Emily, what do you think?
1.10	James:	Now it will be two comma eight, now it is two comma eight if we take
1.11	Ellie:	You are supposed to measure in the other decilitre measure

Jessica starts by looking at the text “1/4 liter” on the box but she and James do not have a clear sense of what this means and how it relates to the 15 dl that they know they are supposed to have. In utterance 1.10 James states that the two boxes they have will be “two comma eight” which indicates that one box would be “one comma four”. It is not clear which unit this relates to, and it is also not clear what is the meaning of the words (two comma eight) that are spoken out. The teacher perceives what the pupils are saying as not correct and asks them what they are thinking. When they do not give a satisfactory answer she turns to Emily (#1.9) but she does not react to the question. Ellie comes to rescue by pointing to the existence of one more measuring beaker (#1.11). The existence of the second measuring beaker makes the meaning of “two comma eight” or “1/4 liter” redundant. After this Jessica and James are no longer interested in how much there is in one box, and the conversation that follows is about practical solutions, for example how to avoid lumps. The teacher also seems to be mainly interested in the practical solutions at this point.

After having put in the first litre of milk Jessica and James start to measure out another 5 dl. Jessica pours in one box, looks at the scale and says “three decilitres”. She does not seem to make any connection between the sign on the scale (level of milk being close to 3 dl) and the sign 1/4 liter on the box. Then she gets another box and gives it to Emily who asks “How much is it we need?” Jessica answers: “We had ten before and then we need fifteen.” Up to now I have not contributed to the discussion at all but at this point I ask a question which seems to shift the focus somewhat for the rest of the lesson.

1.12	Frode:	How many decilitres are there in one of these? (Jessica looks at the box)
1.13	Lucy:	How many decilitres are there in one box?
1.14	Jessica:	It is one comma four litres. (Emily pours in the content of the box. Jessica looks at the scale.)

I suggest that they keep track of how many boxes they have used. They figure this out by counting the empty boxes but make no connection to the number of decilitres. I do not push this any further but Lucy repeats the question about how many decilitres there are in one box, and James answers:

1.15	James:	One comma four
1.16	Lucy:	One comma four?
1.17	James:	One comma four litres.
1.18	Jessica:	Yes, but she asked about decilitres.
1.19	Lucy:	Is it more than one litre?
1.20	Ellie:	No, it isn’t. It is less. This isn’t even half a litre.

As in the beginning of the episode 1/4 is read as “one comma four”, this time with the emphasis “litres”. Jessica realises that the question was about decilitres, and on Lucy’s expressed doubt whether it could be more than one litre (#1.19), Ellie gives a practical estimate, stating that it is indeed less than half a litre (#1.20). After this I end the conversation on this topic suggesting that it might be better that they work on the batter.

The pupils in Group 1 make notice of the sign 1/4 liter but they never develop a meaning of it. They also have no real need to find out what the sign means because they solve the practical task using the measuring beaker. The pupils answer the question about how many boxes they have used but they do not make any connection between the number of boxes and the number of decilitres.

Group 2

Also this group starts by looking at the milk box and the pupils pay attention to the text 1/4 liter.

2.1	Chloe:	One (looking at the box)
2.2	Chris:	slash four, what does that mean?
2.3	Chloe:	Four and a half
2.4	Chris:	Four and a half
2.5	Chloe:	And we need fifteen.

The teacher asks the same question as to the previous group about how much is in one box.

2.6	Chris:	Four and a half
2.7	Lucy:	Four and a half?
2.8	Chris:	Decilitres. No, litres.
2.9	Lucy:	Is it four and a half litres in here?
2.10	Chris:	No, decilitres.

The answer is first given in terms of the number words only (four and a half), and when Lucy wants them to be more precise they hesitate a little between decilitres and litres but stick to litres (#2.8). To this Lucy expresses astonishment (#2.9), and Chris changes to decilitres. Lucy is still not satisfied, and she takes Chris and Matthew to the board at the other end of the room. Lucy writes $\frac{1}{4}$ on the board. She also draws a circle that she partitions into four equal sectors, and she fills one of the sectors. This evokes the concept "one fourth" in the children. Lucy links this to "one fourth of a litre" and asks how many of these go into one litre. This evolves into a discussion that moves between various issues; how many decilitres in one litre, how many boxes in one litre, how many decilitres in total, and how many boxes in total.

Group 3

Joseph and Thomas find the crate with the milkboxes and Joseph starts by asking how much one box is. Thomas says that it is a quarter of a litre. At first Thomas will not engage in Joseph's thinking when he wants to find out how many boxes they need. Joseph asks Lucy if he may use the measuring beaker. Lucy encourages him to try without it and after a brief discussion he accepts this.

3.1	Joseph:	Ohh. A quarter of a litre, that is … a quarter … ten decilitres is one litre. We have to have three of these then, then it will be. Five of these I think … no not five. How much should we, Thomas, if we take three

		of these, no four, then it is one litre and we want fifteen decilitres, and that is, and ten decilitres that is one litre. But how many more than four do we have to take then?
3.2	Thomas:	Then we have to take four, and then we have to take … two
3.3	Joseph:	Then we have two, and ten decilitres here. And then it is fifteen.
3.4	Thomas:	Yes.
3.5	Joseph:	Lucy, is this correct?

In turn 3.5 Joseph asks the teacher for reassurance of the solution, and then she makes him explain his reasoning. Joseph explains that four boxes equal one litre, and that two more boxes are two quarters which is equal to a half. Joseph and Thomas now state that they have one and a half litre which is the same as fifteen decilitres.

Group 4

Group 4 starts in the same way as Group 1 by pouring milk into the beaker. When they cannot find 15 on the beaker they decide that they have to split, and they choose to measure 9 dl first and 6 dl afterwards. They do not pay any attention to the number of boxes they use or to what is written on the boxes. When fetching the sixth box Katie says “it could be that it will be enough”. Grace looks at the scale saying “no, it is … it is exactly enough”. Katie replies “yes, exactly. Good.” Lucy asks how many boxes they have used. Katie counts them and answers “six”. Again Lucy asks the pupils to figure out how many boxes they need without using the measuring beaker. The following dialogue takes place.

4.1	Grace:	Put in three milkboxes … no six
4.2	Lucy:	Yes, but why?
4.3	Grace:	(…)
4.4	Lucy:	Yes, because you know that now
4.5	Grace:	Yes.
4.6	Lucy:	Yes, but if you hadn’t known
4.7	Adam:	Then we could have imagined having one like this (pointing to the measuring beaker)
4.8	Grace:	Then I could have walked home to get one

Lucy pushes them further and Katie asks how much is in one box. They come up with some suggestions, and I suggest that maybe something is written on it. They look at the box.

4.9	Hollie:	There, one comma five.
4.10	Katie:	No, one comma ….
4.11	Grace:	Comma, this is a slash. One slash four litres.
4.12	Lucy:	What does that mean?
4.13	Hollie:	Haven’t a clue.

Adam suggests “one fourth”, Lucy completes this to “one fourth of a litre” and goes on to ask how many they would need to get one litre. The pupils suggest that they need four fourths, and Lucy asks how many boxes that will be. They agree that this will be four, and Lucy points to the original problem to explain why they need two more to get the correct amount of milk.

4.14	Lucy:	Why do you need two more then?
4.15	Grace:	To get six, no
4.16	Adam:	To get three times as much
4.17	Grace:	To get fifteen – fifteen decilitres
4.18	Lucy:	Mmmm
4.19	Adam:	Can we put in the flour now?

Lucy is pushing the issue further and wants to know how many decilitres there are in four boxes which she states to be equal to one litre. In the dialogue that follows answers like “four fourths”, “four decilitres”, and “four litres” can be heard. At the end Lucy holds up one box at a time and they count one fourth, two fourths, three fourths and four fourths. Lucy states that four fourths is one whole. The pupils add “litre” and Katie says “plus two more is one half”.

DISCUSSION OF THE EPISODES

The semiotic issues

Central to the task is the sign or symbol 1/4 liter printed on the milk boxes. The pupils read the sign in various ways (one comma four, one slash four, four and a half) but many of them do not have a clear meaning linked to it. Groups 1 and 4 solve the measuring task completely by using a measuring beaker holding 1 litre. For these groups it is irrelevant to know the meaning of 1/4 liter to solve the task. They relate to the fact that they need 15 dl of milk and by using the measuring beaker as a mediating tool (Vygotsky, 1978) they are able to get the correct quantity. When the teacher asks these two groups to figure out how many boxes they would need without using the measuring beaker they are facing a difficult problem. I interpret the teacher here to be working with 1/4 liter as the representamen and the amount of milk in the box as the object. The teacher’s interpretant is that this is a fourth of a litre and that four boxes are needed to get one litre. The pupils are working within another triad where the representamen is the scale on the measuring beaker, an indexical sign pointing to the quantity of milk in the beaker as the object. The interpretant is the concept “fifteen decilitres” or “one and a half litre”, which they know that they need. I see the problem as having to do with creating a link between these two semiotic triads. As it is the symbolic sign 1/4 liter is not seen as a representamen for the semiotic triad involving the measuring beaker. Since the pupils do not have a clear meaning of what 1/4 liter means, the sign might just be an index connected to the box. In Group 3 the situation is quite different. The pupils make the connection between the sign 1/4 liter and the amount of milk, and as a result they are able to identify 4 + 2 boxes with one and a half litre.

In Group 2 the teacher physically moves from the kitchen part of the classroom to the opposite end where the blackboard is. She writes $\frac{1}{4}$ on the blackboard and also draws a circle partitioned in four sectors, filling one of them. Here the interpretant 'one fourth' is evoked in the pupils, and the teacher and the pupils seem to be working within the same semiotic triad, situated in a school practice. However, the sign $\frac{1}{4}$ is not seen as a representamen for the triad in which 1/4 liter is the sign, and therefore the link to the actual measuring problem is also missing in this case.

The sign $\frac{1}{4}$ is a symbol, clearly embedded in the school practice. The scale on the measuring beaker is an index, firmly based in the everyday practice. The sign 1/4 liter could be seen as a symbol representing the amount of milk in one box but for some of the pupils it might seem as if it is an index by its connection to the box, or a symbol with no interpretant. Based on this I identify three semiotic triads; the first where the scale is the sign, the second where 1/4 liter is the sign, and the third where $\frac{1}{4}$ is the sign. The everyday solution to the measuring problem is to pour milk into the measuring beaker until the indexical sign (the scale) points to 15 dl (seen as 1 litre + 5 dl, or 9 dl + 6 dl). The school solution could for example be to establish the relation $6 \cdot 1/4 = 1{,}5$ (litres) or $6 \cdot 2{,}5 = 15$ (decilitres). I have showed various attempts to create connections between these two practices. Based on Presmeg's (2005) model I suggest that a nested chaining of the semiotic triads described above could establish a connection between the practices, and I have showed that lack of connection can be explained by lack of connection between the semiotic triads.

The discursive practice

Seen as a task from school mathematics the measuring problem could be formulated as follows. "Each milk box holds ¼ litre of milk. How many boxes are needed to get 15 decilitres of milk?" All four groups were able to find a solution to the practical problem of getting the right amount of milk, so indirectly they also know how many boxes of milk they need. Therefore they have all found the solution to the question in the imaginary school task, albeit not in a school like manner. I perceive the main *motive* for this lesson to be to produce batter for the waffles, and this determines the direction of the *activity* in the lesson. The activity consists of a number of different *actions* that can be linked to specific *goals.* Some of these actions can be carried out in a number of different ways, using different *operations.* The choice of operations depends on the *conditions* that are there at any given time (Leont'ev, 1979). My main objective in this section is to analyse the teacher's and the pupils' goals and actions in the lesson. My interpretation is that there is some tension between the teacher's and the pupils' goals, and that this tension is due to the fact that the lesson is operating on the border between a school practice and an everyday practice.

In Group 1 it seems that both teacher and pupils share the same goals in the beginning. The pupils (Jessica and James) have the idea to try to figure out how many boxes of milk they will need (#1.1-1.11). The teacher sees that their idea will not work and she tries to guide them or bring in Emily to help (#1.4 and 1.9) but when Ellie (#1.11) points to the fact that there is one more measuring beaker the teacher just lets them go on with the measuring without going any further into their interpretation of 1/4 liter. The measuring beaker is the only tool they rely on to get the correct amount of milk. When I pose the question about how many decilitres there are in one box (#1.12), the situation changes somewhat. This question seems to bring in new goals that guide the teacher's action and in turn influences the pupils' goals. The teacher becomes more concerned about the mathematical content of the situation (e.g. #1.13). The fact that her attention to the mathematics appears after my question leads me to characterise her new goals as 'seeing the mathematics' and 'satisfying me'. The pupils do not relate this question to the work they are doing so their new goal can be expressed as 'answering the questions' or maybe 'satisfying the teacher'. They stick to reading 1/4 as "one comma four" (#1.15), emphasising "litres" (#1.17). Ellie is aware that there is not more than one litre in one box, "[t]his isn't even half a litre" (#1.20), indicating a lack of meaning to "one comma four".

In Group 2 the process with the milk starts with the pupils reading on the box "one slash four" (#2.1-2.2) which they suggest means "four and a half" (#2.3), but they are not quite sure whether it is litres or decilitres (#2.8). With this group the teacher to a much larger extent goes into the role of the mathematics teacher, and she literally crosses the boundaries between practices by walking over to the blackboard at the other end of the room. In a funnelling pattern of interaction (Bauersfeld, 1988, p. 36) the teacher leads the group to a conclusion about how many boxes are needed.

Group 4 solves the whole measuring problem using the measuring beaker, thereby reaching their goal. It is only on the teacher's request that the number of boxes being used is brought into the picture. The pupils give an answer, because that is what is expected of them as pupils, but without enthusiasm. They have reached their goal, and they have no need to use any more energy on this. Each activity, here the measuring of the milk, answers to a specific need of the active agent, here getting the correct amount of milk for the batter, and when this need is satisfied the activity stops (Leont'ev, 1979). The answers of the pupils (some examples are shown in turns 4.14 to 4.19) indicate little interest. The numbers that come up can be connected to certain incidents throughout the process but not necessarily corresponding to the questions that the teacher asks. For example in turn 4.15 when Grace answers "to get six", she applies the fact that they used six boxes, which she already knows, but this is not in line with the hypothetical situation that the teacher has constructed. Towards the end the teacher leads the pupils via the question about how many boxes they need to get one litre. Even this evokes answers that indicate that the pupils do not engage in the problem.

I have shown that by operating on the border between practices, the mediating tools from the non-mathematical practice offer alternative possibilities for solving a task. The teacher, being pulled between the two practices, is seen to struggle in order to keep the pupils' motivation to solve the task in the mathematical context when they already have solved it in the practical context.

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[1] In Norwegian the sign for the decimal point is a comma. Since this sign is central in the interpretation of the dialogues I am using, I will keep the word 'comma', and I will also for example use the notation 1,5 instead of 1.5 which would be the standard English notation. Also when I directly refer to the text on the milk box I will use the Norwegian word 'liter' instead of 'litre'.