ENGAGING EVERYDAY LANGUAGE TO ENHANCE COMPREHENSION OF FRACTION MULTIPLICATION

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Dedicated to the memory of the Cypriot teacher Georgia Kyriakidou

Using as analytic frames the Pirie-Kieren model and theoretical constructs on the role language and communication could play in the process of learning, I attempt to sketch the pathway of understanding of a sixth-grade student (Avgusta) while she is attempting to make sense of fraction multiplication. The viewing of mathematical understanding as a dynamic process proved supportive in enabling me to identify the role language could play both at any level and in the growth between levels of Avgusta's understanding. Occasioning learners to fold back to everyday language in order to collect the spontaneous interpretation of the word "of" and combine it with the scientific notation of multiplication could awaken learners' awareness that the interpretation of multiplication involves finding or taking a part of a part of a whole.

INTRODUCTION

The story to be recounted here evolves in a public elementary school in Cyprus, where I work as a full-time teacher. It is part of a two-year research studying the complexities of learning to compute fractions as revealed from the use of a novel peda-cultural tool. Though in Cypriot culture school mathematics textbooks introduce the concept of fraction with images of partitioned rectangles and circles, they make little or no use of diagrams when they show students the way to compute.

During the first year of the study I was the teacher of a fifth grade class (10 boys & 12 girls) and had to address all subjects' objectives set by the curriculum. Once a week, I took the role of a teacher-researcher and taught students how to learn fractions through manipulating diagrams. To be consistent and learn from my experiences I revisited my group of students a year later and conducted individual interviews in order to collect some retrospective evidence about the nature of their learning. It is the purpose of this paper to zoom in on one of those interviews and describe how one girl, Avgusta, could derive meaning in multiplication of fractions. Worthy of consideration is that in sixth grade my ex-students had been exposed to a different teacher's instructional mode which gave no emphasis on diagrams as a learning tool.

This study is of interest because it refers to an educational culture unused to use diagrams to compute fractions and more used to show and tell than to getting learners to make sense by using the diagrams as mediating tools. Its contribution lies in offering Avgusta's learning as grist for the learning and development of other pupils, beyond the local boundaries of the particular school.

THEORETICAL BACKGROUND

The role of language in learning and particularly the social role of other people in the development and use of language was explicitly stressed by Vygotsky when he emphasized the importance of getting students talking about their thinking in order to help them make sense of, or construct, mathematical meaning. Vygotsky also observed that there are differences between what pupils can achieve working alone and what they can achieve when assisted by someone more experienced, such as a teacher. He captured this in a phrase which in English is usually rendered by "zone of proximal development" (Vygotsky, 1978). This term suggests that the teacher wants to support awareness that is imminent but not yet available to learners and not do those things which learners can do, since this will only raise dependency. Bruner (as cited in Wood et al., 1976) while presenting Vygotsky's ideas in English, made use of the metaphor "scaffolding" to refer to the assistance that a teacher some time may offer, which can be gradually withdrawn as students are able to function independently. The critical part of scaffolding is its removal or fading because when the support has not been removed, pupils may become dependent upon the teacher or any employed pedagogical tool (Love & Mason, 1992).

Zack (2006) appears in synch with Vygotsky's and Bruner's observations when she claims that because "students use sophisticated reasoning but may not see the power in the reasoning they are doing", it might be useful if teachers could "revisit what students have said, and connect their talk with the ways in which a mathematician would express those ideas" (p. 211). Linking everyday and scientific ways of knowing in order to support learners' imminent awareness is, according to Zack (1999), a much more challenging task than most researchers have appreciated.

The Pirie-Kieren theory and its associated model [Figure 1] is a well-established and recognized tool for listening and looking at growing understanding as it is happening. Growth in understanding is seen as a dynamical and active process involving a continual movement between different layers or ways of thinking, with no implication of a linear ladder-like system. These layers, which are intentionally represented in the form of eight nested circles so that the accent is put on the embedded nature of understanding, are named Primitive Knowing, Image Making, Image Having, Property Noticing, Formalising, Observing, Structuring and Inventising. A critical feature of this theory is the act of returning to an inner layer, or re-visiting and re-working existing understandings and ideas for a mathematical concept. This act is called "folding back" (Pirie & Kieren, 1989). A slightly differentiated form but equally important to folding back is that of "collecting". Its major difference from folding back is that, in collecting, the inner level activity does not involve a modification (or thickening) of the individual's earlier understandings. Instead, learners' efforts are concentrated on finding and recalling what they know they need to solve a task. They are consciously aware that this knowledge exists but their understanding is not sufficient for the automatic recall of it (Pirie & Martin, 2000).

METHOD AND METHODOLOGY

Avgusta, 12 years old when the interview was conducted, was one of the twenty two students participating in the study. I have chosen to present here selected pieces of her responses to a scenario on multiplication [Table 1], as well as explanations of these responses. By choosing particular moments and voicing them through a temporal sequence, I aim to convey not only a succession of Avgusta's learning experiences but also how she experienced this succession. What counts is not only the content and structure of the practice itself but also the ways in which it is talked about, perceived and assimilated by the learner.

When the principal of the school entered the classroom and asked the children what they were doing, they replied that they were learning how to multiply fractions. Then the principal asked who could come up to the board and show to her how to find the product 2/3 x 1/2 without performing any calculations but using only the area models. Orestes wrote the following on the board but the principal did not seem satisfied. If Orestes asked for your help, what would you say to him?

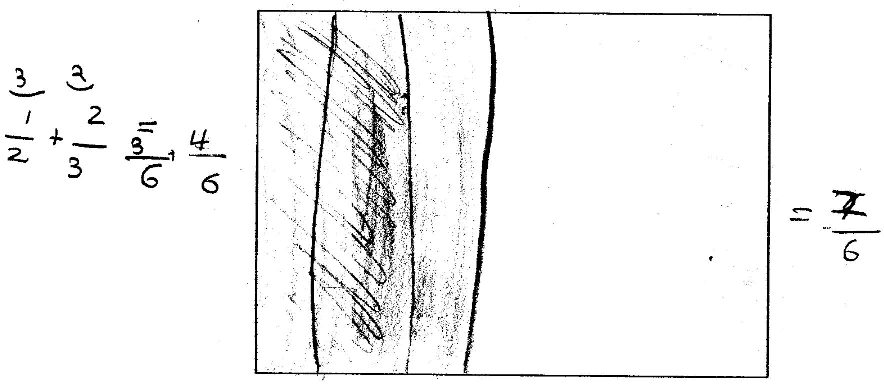


Table 1: Interview scenario

Using as analytic frames the Pirie-Kieren model for the growth of one's understanding, theoretical constructs on the role language and communication could play in the process of learning, as well as personal reflections on pedagogy, I shall attempt to map the growth of Avgusta's understanding. Throughout the analysis, my specific goal is to explore her thinking "in-change" and how this is accomplished and shared. In other words, how shifts in Avgusta's thinking occur and in what ways such shifts in thinking supported her understanding of the meaning of multiplication.

Taking the position with Doerr and Tripp (1999), I argue that shifts in thinking could be described in terms of an initial interpretation of the task situation and a later interpretation that stands in opposition to the initial interpretation. It is sensible to assume that somewhere between the two interpretations there will be evidence of what precipitated the change in Avgusta's thinking. For this reason, attention will be cautiously focused on the sequence of events between initial and later interpretations, as well as on identifying those characteristics that illuminate the growing understanding of Avgusta throughout the interview.

INTERVIEW FINDINGS

The conversation I had with Avgusta about the multiplication scenario [Table 1] is the focus of this section. The quoted transcript has been intentionally split into three parts each of which has a distinct subheading. This division is absolutely artificial and it does not imply any linearity in the girl's growth of understanding. Rather, it is meant simply to organize structurally the data and facilitate the development of discussion later on.

Avgusta's tenacious-but-futile struggle to recall and apply a half-remembered algorithm in order to shed meaning to the procedure of multiplying fractions

What really strikes me here is Avgusta's "trapped" awareness of the falsehood of her actions.

507 Interviewer: Would you like to write down what Orestes [Table 1 - scenario on multiplication] should have done?

508 Avgusta: Yes.

[Avgusta is drawing the first and second figure of sheet 5. See Table 2 below, read left to right, up to down direction].

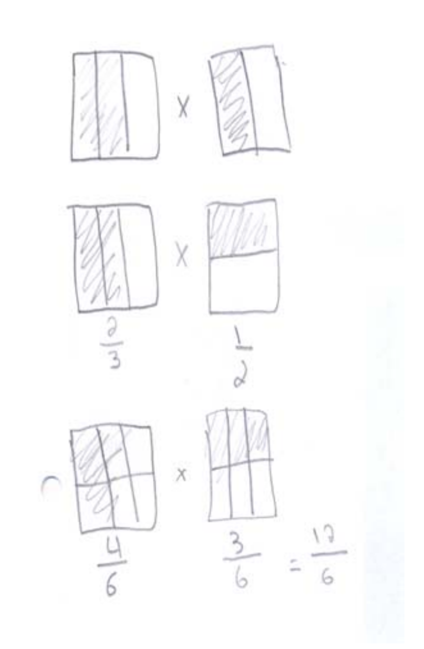
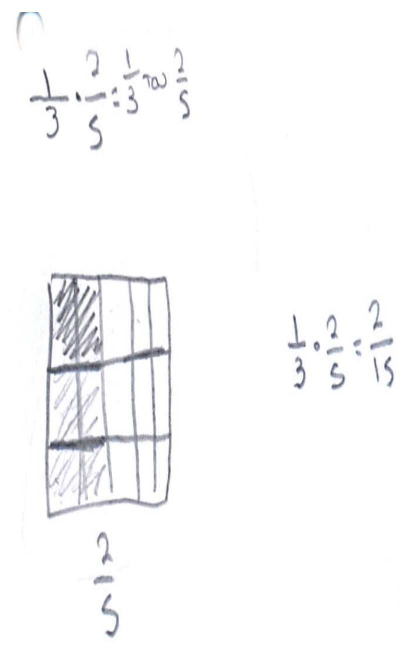
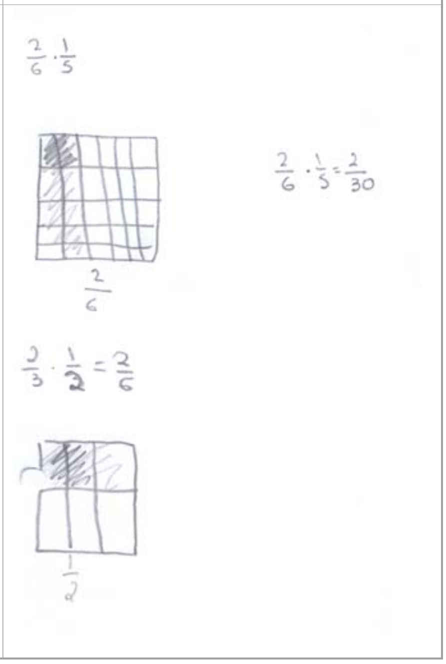
Sheet 5	Sheet 6	Sheet 7
		

Table 2: Avgusta's handwritten notes

509 Interviewer: What are you doing now?

510 Avgusta: The two thirds. He takes the two. Then… times one half. He takes the one and then we reverse them. No, I did it wrong.

511 Interviewer: Why?

512 Avgusta: I should have done it like that, a line.

513 Interviewer: How about doing it below to see what you mean?

[Avgusta is drawing the third and fourth figure of sheet 5 – Table 2]

514 Avgusta: Like this.

515 Interviewer: Yes?

516 Avgusta: We will reverse them and…we will reverse them.

517 Interviewer: Why?

518 Avgusta: To find…to find the same number of small boxes…to do them common fractions.

519 Interviewer: Okay, you could do whatever you think Avgusta and we will see.

[Avgusta is drawing the fifth and sixth fig of sheet 5 – Table 2]

520 Avgusta: We will reverse them.

521 Interviewer: Okay.

522 Avgusta: The two thirds…we will bring the one half…one minute…this one and then we will do times….We will reverse the one half and…

523 Interviewer: And what do we have now?

524 Avgusta: The small squares are now the same.

525 Interviewer: Yes?

526 Avgusta: But we have…

527 Interviewer: What do you have there?

528 Avgusta: Four sixths and here three sixths.

529 Interviewer: Yes.

530 Avgusta: And it becomes twelve sixths [She writes it at the bottom of sheet 5 – Table 2]

531 Interviewer: So is this your answer?

532 Avgusta: I think it is wrong.

533 Interviewer: Why do you think so?

534 Avgusta: [pause]

535 Interviewer: Would you like to tell me why do you think it is wrong?

536 Avgusta: But I don't know sir.

An invocative intervention aimed to occasion the link between everyday language and multiplication notation

The point that merits attention here is that Avgusta's folding back to everyday language could open the door for her to notice fractional symbols from a lens, which in turn could affect her way of thinking.

569 Interviewer: Okay. Now I would like to ask you something else. What does “times” mean? For instance, when we say one half times one hundred, what does that mean? You may write it down if you want.

[Avgusta is writing on the top of sheet 6 – Table 2]

570 Avgusta: We will multiply one half times one hundred.

571 Interviewer: Yes. Could you not say “we multiply”? How about our everyday language? Will you say one half times? Or, do we use any other word?

572 Avgusta: The word of?

573 Interviewer: How about saying it to see what you mean?

574 Avgusta: One half of one hundred.

575 Interviewer: That is? What does it mean? One half of one hundred is what?

576 Avgusta: Fifty.

577 Interviewer: Could you tell me Avgusta what does one half mean?

578 Avgusta: They are two and we are taking the one.

579 Interviewer: Nice. If I had one fourth, what does that mean?

580 Avgusta: There are four and I take one of them.

Educating awareness through encountering conflicting results and detecting the origin of the conflict

After Avgusta had been exposed to the foregoing intervention, she worked on the examples 1/3 x 2/5 [Table 2 – sheet 6] and 2/6 x 1/5 [Table 2 – sheet 7]. Lines 720-759 are indicative of what had been exchanged between me and Avgusta later on. Of great importance here is the gradual refinement of the girl’s awareness of what it means to multiply two fractions, and the restructuring of ill-defined algorithmic knowledge.

720 Interviewer: Which way from the two, do you think, could help a child to understand what multiplication means? If you show him that you should multiply the… But, first, Avgusta do you know how we could multiply two fractions?

721 Avgusta: Yes, don’t we do them common fractions?

722 Interviewer: Could you show me the example two thirds of one half, with the way of area models?

[Avgusta is drawing the second figure of sheet 7 – Table 2]

723 Avgusta: We will do the one half, we will take the one and then we will divide it in three…vertical ones and we will take the two.

724 Interviewer: Would you like to shade again what are you going to take?

725 Avgusta: These here [She shades again the two left small squares of the top row of the second figure of sheet 7 – Table 2].

726 Interviewer: Could you now tell me which your result is?

727 Avgusta: Two sixths.

728 Interviewer: Right. Earlier Avgusta we had this example again, it was on sheet 5 [Table 2]…and you found what?
729 Avgusta: Twelve sixths.
730 Interviewer: You found twelve sixths and now you found two sixths. Which of the two is the correct one? Earlier you said that when we multiply we do the fractions common ones, didn't you?
731 Avgusta: Yes.
732 Interviewer: Here [He points to sheet 5 – Table 2] you did common fractions, didn't you?
733 Avgusta: Yes.
734 Interviewer: You did two thirds, four sixths, and one half, three sixths. And what did you do then?
735 Avgusta: I did it times.
736 Interviewer: Could you explain a bit more?
737 Avgusta: I did four sixths times three sixths.
738 Interviewer: And how much did you find?
739 Avgusta: Twelve sixths.
740 Interviewer: How did you find twelve?
741 Avgusta: Four times three.
742 Interviewer: And how about six?
743 Avgusta: Because the denominators are…
744 Interviewer: But here [He points to sheet 7 – Table 2] how much did you find?
745 Avgusta: Two sixths.
746 Interviewer: Which of the two is the correct one?
747 Avgusta: This one, the two sixths.
748 Interviewer: Could you tell me why?
749 Avgusta: [pause]
750 Interviewer: You saw it here Avgusta, didn't you? Whereas there [He points to sheet 5 – Table 2]?
751 Avgusta: I didn't see it.
752 Interviewer: What should you have done here [He refers to sheet 5 – Table 2], do you think?
753 Avgusta: The same with this one [She points to sheet 7 – Table 2].
754 Interviewer: So, how do we multiply Avgusta? Do you see here [He points to sheet 5 – Table 2]? There was something wrong. When we multiply two fractions, we multiply the numerators…
755 Avgusta: And the denominators.

DISCUSSION

Avgusta's main difficulty seems to be a dependence on a half remembered algorithm. The way she manipulates the rectangles she drew [Table 2 – sheet 5], her rapid but purposeful shift from solely vertical to both vertical and horizontal type of partitioning [lines 507-518], as well as the multiplying of the numerators of the newly formed common fractions [lines 527-530], all could suggest that her understanding of multiplication is compartmentally drawn upon a vague memory of the standard change-into-common-denominators rule.

The ability to produce a partition of a partition in the service of finding the product of 2/3 x 1/2 might not be straightforward to Avgusta because it entails the composition of the operator “2/3 of” and the operator “1/2 of”. This idea is complex because it is removed from the whole number knowledge that learners could employ when first introduced to a single operator, such as “1/2 of”.

In lines 532-536 Avgusta is observed to express concerns about the correctness of her actions but is failing to exemplify the origin of this uncertainty, at least in the short term. This could indicate that after using diagrams, Avgusta pauses and reflects by considering what it is that the results tell her. It is possible that while checking against her intuitions that the results seem to be reasonable and roughly what she expects, the girl encountered an internal conflict which, in turn, generated doubt. Avgusta’s assertion that she knows that something went wrong [line 532] but does not know what [line 536], catches my attention and opens the possibility that I could provide for her some cognitive “scaffolding” (Wood et al., 1976) to support, and perhaps transform that state. There was a sense of her having, and being aware that she has the necessary understandings but that these are just not immediately accessible.

One of my enduring questions, thus, while interviewing Avgusta [lines 569-580] was in regard to the role I could play in pulling to the forefront of her mind the “Primitive Knowing” (Pirie & Kieren, 1989) that was going to be the basis for locating the source of perplexity. My intention was to encourage the girl to keep in touch with her personal way of knowing mathematics and sustain a back and forth movement, not unidirectional, between that understanding and the conventions of the culture. It is for this reason I occasioned [lines 569-580] Avgusta to “fold back” (Pirie & Kieren, 1989) to everyday language, “collect” (Pirie & Martin, 2000) the spontaneous interpretation of the word “of” and combine it with the scientific notation of multiplication. This invocative intervention resulted in the student returning to an inner, more localized layer of understanding, which, in turn, seems to have given rise to a succession of “Image Making” activities (Martin, 2008). The handwritten notes on sheets 6 and 7 [Table 2] are indicative of the replacement of faded images of multiplication by meaningful diagrammatic illustrations linking recursive area partitioning with the respective symbolic notation.

It is of great importance to stress here that it is the response of Avgusta to the particular intervention that determined the actual nature of it, namely, to occasion folding back to existing understanding, searching for, finding and then remembering this understanding (Martin, 2008). If the girl did not assign herself the everyday meaning of the word “of” to “x” or “times” [lines 569-576], it is ambiguous whether Avgusta would awaken her awareness that the interpretation of multiplication involves finding or taking a part of a part of a whole. Standard multiplication symbols appear, hence, not mere marks on paper for her but become manageable and confidence-inspiring so as to be used in further manipulation.

After successfully re-collecting the image she needed and through experiencing a series of Image Making activities [Table 2, sheets 5-7], the last of which was centered on the same example she worked on at the very beginning, Avgusta noticed a conflict between the two images she had constructed for the product of 2/3 x 1/2. This discerned contradiction [lines 728-747] between 12/6 [Table 2 – sheet 5] and 2/6 [Table 2 – sheet 7] is likely what occasioned Avgusta to reject her initial way of using diagrams and revise her existing Formalizing level of understanding by re-structuring the procedure of multiplying two fractions [lines 748-755]. Figure 1 is an attempt to illustrate by means of the Pirie-Kieren onion model (Pirie & Kieren, 1989) the pathway of Avgusta's growth of understanding. Based on my observations, this is seen to grow in a non-linear way: from the Primitive Knowing layer to the Image Making and Image Having layers. Then, evidence exists of folding back to the Primitive Knowing in order to collect an earlier understanding to use it anew at the Image Making layer. Avgusta seems to reach the Formalizing layer having first gone through the Image Having and Property Noticing layers.

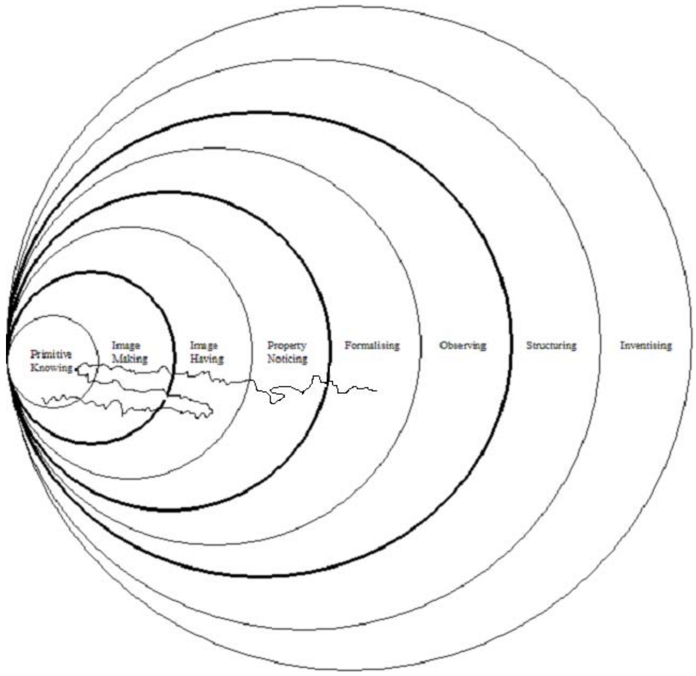


Figure 1: Avgusta's growth of understanding

The case of Avgusta comes to question the generalization of the assumption that once the meaning of a mathematical concept has been discussed, explained, formally articulated in class and students have at one time proven fluent with the corresponding algorithm, then the learning of this concept has been accomplished and a degree of readiness has been achieved for more sophisticated ones (Rasmussen et al., 2004). The fact that Avgusta struggled with the idea of fraction multiplication that had been taught to it while in fifth grade, neither speaks of a teacher’s nor of a learner’s failure per se. Rather, it points to the need for teachers to occasion students to re-encounter ideas that they already have, in a different light or in relation to unfamiliar circumstances.

The viewing of mathematical understanding as a dynamic process proved in the current study supportive in enabling me as a teacher-researcher to identify the roles language and thought could play both at any level and in the growth between levels of Avgusta’s understanding. If, as in the case of Avgusta, the student needs to activate a link between everyday language and mathematical notation, then in order to allow that student to progress in making sense, occasioning –not imposing- an awareness as to what to collect could be of assistance.

REFERENCES

Love, E., & Mason, J. (1992). Interlude on readiness and fading. In E. Love & J. Mason (Eds.), Teaching mathematics: Action and awareness (pp. 54-58). Southampton, UK: Hobbs the Printers Ltd.

Martin, L. C. (2008). Folding back and the dynamical growth of mathematical understanding: Elaborating the Pirie-Kieren Theory. The Journal of Mathematical Behavior, 27(2008), 64-85.

Pirie, S. E. B., & Kieren, T. E. (1989). A recursive theory of mathematical understanding. For the Learning of Mathematics, 9(3), 7-11.

Pirie, S. E. B., & Martin, L. (2000). The role of collecting in the growth of mathematical understanding. Mathematics Education Research Journal, 12(2), 127-146.

Rasmussen, C., Nemirovsky, R., Olszewski, J., Dost, K., & Johnson, J. L. (2004). On forms of knowing: The role of bodily activity and tools in mathematical learning. Educational Studies in Mathematics, 57, 313-316.

Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. Cambridge, MA: Harvard University Press. Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. Cambridge, MA: Harvard University Press.

Wood, D. G., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. Journal of Child Pyschology and Psychiatry, 17(2), 89-100.

Zack, V. (1999). Everyday and mathematical language in children's argumentation about proof. Educational Review, 51(2), 129-146.

Zack, V. (2006). What's a literature person like you doing, teaching and researching in elementary level mathematics? In C. Langrall (Ed.), Teachers engaged in research: Inquiry into mathematics classrooms in grades 3-5 (pp. 201-223). Greenwich, Connecticut: NCTM.