MATHEMATICAL ACTIVITY IN A MULTI-SEMIOTIC ENVIRONMENT

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Abstract: Different semiotic systems provide different sets of resources for the construction of mathematical meanings. In this paper, we argue that a multi-semiotic environment not only affords rich potential for developing mathematical concepts but may also affect more fundamentally the goals of student activity. We present a multimodal analysis of an episode from a teaching experiment with software that allows students to construct animated models using equations. In the course of this short episode, the students made use of drawing and gesture as well as mathematical and everyday speech in ways that transformed the purpose of their activity from drawing a static pattern to constructing an animation, changing the mathematical problem from using velocities to determine the direction of motion to considering how to stop a moving object.

INTRODUCTION

The study of mathematical language and other sign systems has developed in recent years with increasing recognition of the importance of a variety of specialised mathematical systems, including graphical and diagrammatic forms as well as linguistic and symbolic (Alshwaikh, 2008; O'Halloran, 2005), and of interaction between the various systems (Duval, 2006) in the development of mathematical discourse. Moreover, where mathematical communication takes place in face-to-face contexts, body language and gesture also play a part (see, for example, Bjuland, Cestari, & Borgersen, 2007; Radford & Bardini, 2007). The development of new modes of representation through the medium of new technologies has generated further interest in this area by opening up possibilities for dynamic forms and for interactions between systems (such as graphs and algebraic equations) in ways that were previously inaccessible.

From a social semiotic perspective (see Morgan, 2006), each semiotic system provides a different range of meaning potentials (Kress & van Leeuwen, 2001). For example, as O'Halloran argues, visual modes such as graphs allow representation of 'graduations of different phenomena' rather than the limited categorical distinctions available through language or algebraic symbolism, while dynamic modes additionally allow the representation of temporal and spatial variation (2005, p.132). Such different potentials have been exploited in the design of interactive learning environments (for example, Yerushalmy, 2005) and research from various theoretical perspectives has focused on the kinds of mathematical meanings constructed by students working with such novel representations, especially in the contexts of use of dynamic geometry (for example, Falcade, Laborde, & Mariotti, 2007).

In this paper we report a teaching experiment, involving a multi-semiotic interactive learning environment, MoPiX, produced as part of the ReMath project [i]. This environment and the associated pedagogical plan were designed to provide multiple linked representations to support students' development of concepts of velocity and acceleration [ii] by allowing them to experience and connect formal symbolic definitions and dynamic animations. We report elsewhere how the semiotic resources provided by this environment appear to support students' development of ways of operating with velocity and acceleration compatible with their formal definitions and with Newtonian laws of motion (Morgan & Alshwaikh, 2008, 2009). Here, however, we discuss the influence of the multi-modal environment on the process of problem solving, presenting an example of an episode in which interaction with the various available semiotic systems transformed the goals of the activity.

A MULTI-SEMIOTIC ENVIRONMENT

The interactive learning environment of MoPiX allows users to construct animated models and investigate their behaviour. It is conceived as a constructionist toolkit (Strohecker & Slaughter, 2000), providing fundamental elements (in this case objects, represented by shapes such as squares or circles, and equations) with which students can build models, form and investigate hypotheses by activating their constructions and observing their behaviour. The environment of MoPiX is essentially multi-semiotic, linking symbolic representations (equations) using a variation of standard mathematical notation, with animated models and graphs. In addition, the planned pedagogy of the teaching experiment, the social environment of the classroom and the nature of the technology (individual tablet PCs) were intended to encourage use of a range of modes of communication, including talk, gesture, various paper-and-pencil representations and the electronic sharing of constructions through the ReMath portal [iii]. The variety of semiotic systems provides a range of meaning potentials and hence rich opportunities for users to construct meanings for the mathematical objects and concepts represented.

x(object_1,t)=x(object_1,t-1)+*Vx*(object_1,t)
x-coordinate of the circle (object_1) is augmented by the value of Vx as time (t) increases

Vx(object_1,0)=3
variable Vx, assigned an initial value of 3 (when time=0), may be considered the velocity of the circle

Vx(object_1,t)=*Vx*(object_1,t-1)+*Ax*(object_1,t)
Vx (velocity) is augmented by the value of Ax as time (t) increases

Ax(object_1,t)=-0.1
variable Ax, in this case assigned a value of -0.1, may be considered the acceleration of the circle

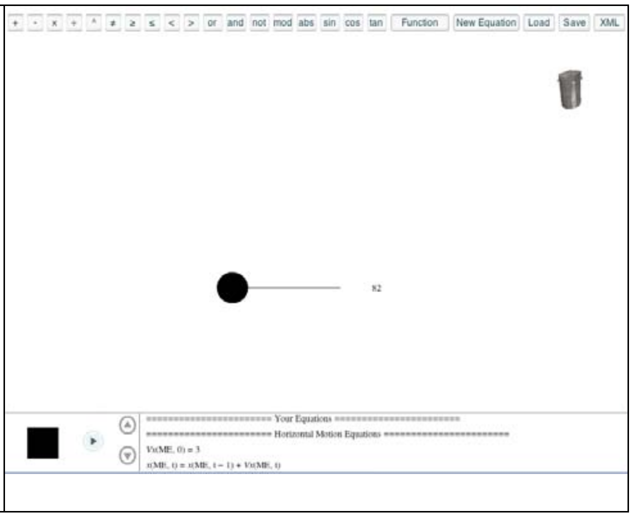


Figure 1: A set of equations defining horizontal motion

A MoPiX object is caused to move by applying a set of parametric equations defining how its position will change over time. For example, the set of equations shown in Figure 1 would cause object_1 (the circle in the screen shot) to move in the horizontal direction with an initial velocity of 3 and constant acceleration -0.1 [iv]. Horizontal and vertical components of motion are defined separately. The notation thus draws attention to vector concepts of velocity and acceleration, while the form of the equations embodies the definitions of velocity as change in position and acceleration as change in velocity. Equations may be taken from a library of basic equations, edited or authored directly and applied to objects. Once equations have been added to one or more objects, the model may be played and each object in the model will move according to its own set of equations. (It is also possible to apply equations defining interactions between two or more objects.) Visual feedback from the animated model allows students to test their hypotheses about the functioning of the equations they have used. They may then continue their investigations: editing the sets of equations and adding new objects to their model.

THE TEACHING EXPERIMENT

A pedagogic plan was devised, in collaboration with teachers in a London tertiary college, with the educational goal of developing understanding of ideas of velocity, acceleration and force. A group of seven students (aged 17-18 years) volunteered to participate in the study, which took place during 10 weekly one-and-a-half hour sessions outside the normal curriculum. The participants were all enrolled in an Advanced level mathematics course. They had not previously studied the mathematics of motion (though some had studied physics) and, though all were familiar with the formal definitions of velocity and acceleration as rates of change, a pre-course paper-and-pencil questionnaire revealed that they nevertheless relied on informal non-Newtonian intuitions in order to describe and explain motion. Participation in the study was presented to the students as extra preparation for the Applied Mathematics (Mechanics) module that they were to start the following term.

The intended pedagogy was founded on constructionist principles, providing students with access to the means of manipulating the elements of the MoPiX microworld while posing challenges that would encourage them to experiment, shaping their own goals and hypotheses. The episode we consider in this paper is taken from the second session. During the first half of this session, the students had been given a worksheet with a sequence of tasks introducing them to the equations needed to create straight line motion, to the idea that the direction of motion is determined by a combination of velocities in the horizontal and vertical directions and to the equations for drawing a trace of the motion of an object. Having done the set tasks, they experimented in a playful way with these and a range of other equations taken from the MoPiX equation library, creating multi-coloured objects moving in various ways, not only in straight lines. They then had their attention drawn to the next task on the worksheet: '*As a group, plan a design formed by several lines.*' In designing this challenge, it was anticipated that students would make use of the combination of horizontal and vertical motions to make objects move in different directions drawing straight lines with different gradients, thus developing their appreciation of relationships between components of motion in two dimensions.

DATA ANALYSIS

During the teaching experiment we gathered data in the form of video and audio records of pairs of students, together with any incidental paper-and-pencil work. In addition we administered paper-and-pencil pre- and post-questionnaires. Our broad research aim was to investigate how students would make use of the semiotic resources offered by MoPiX and the broader classroom environment in the course of their work on tasks related to motion. We were particularly interested to see what contribution the various resources might make to students ways speaking about and operating with ideas of velocity and acceleration.

Extracts of video were identified as ‘of interest’ and were transcribed. In accordance with our research focus on multiple semiotic resources, extracts chosen for transcription included, in particular, those where several modes of communication were being used together. We consider the form of transcription to be part of the analytic process as a preparation for the multi-semiotic analysis needed to address our research questions. The use made of each mode of communication was thus recorded in a separate column of a spreadsheet, allowing both horizontal (a snapshot of all simultaneous semiotic activity at each ‘moment’) and vertical (an overview of semiotic activity within a particular mode through the whole period of the extract) examination of the data. The transcript was divided into ‘moments’ of communication that were considered to have some meaningful coherence; this division was a pragmatic consideration with no explicit theoretical basis.

Our approach to analysis involved both the application of a priori categories and the iterative definition and refinement of categories derived from the data. In the episode discussed below, we discuss the interaction between mode of communication (an a priori categorisation) and the goal of the students’ activity (a strand of analysis arising from our exploration of the data). The episode is a five-minute extract from about half way through the second session, focusing on two male students, Baz and Vin as they start to work on the design task.

CM if two of you think about a pattern maybe with some parallel lines and perpendicular lines and a number of lines to make some sort of a pattern on the screen. Yeah? And design that in advance and then one of you does some of the lines, the other does the other set of lines and then you combine the two to make the whole pattern. Yeah? So you might want to do some pencil paper work first. think about your design

Vin Do you have a pen?

Baz Just use the computer

Vin Yeah.. in Paint [*this refers to the Paint drawing programme on the PC*]

Baz / Vin [*laughing together*] yeeaah.. Paint

Vin Bring it over

[… *about a minute trying to find the Paint programme on an unfamiliar PC*]

Baz Here we go. All right so we can do the horizontal lines and vertical lines.

Vin Can't we do the diagonal ones

Baz We can do squiggly lines, but

Vin Like in our thing, if it has a formula, then it's not going to be random is it

Baz Yeah exactly

Vin Do a log [*i.e. logarithmic function*] actually you can't do log because it'll get kind of mad because it'll go on for ever

Baz You can have different colours right [*both laugh*] so make it like a firework so it goes like that and then you could have vertical ones like that and diagonal ones and another horizontal, I mean vertical one going even further up

Vin like a sparkler

Baz yeah but we need it to start from here and then these start after this one and then .. I don't know how that'll work

We originally identified the extract for detailed transcription and analysis because it seemed interesting for two reasons. In the first place, the students chose to make use of the *Paint* programme on their PCs, thus providing us with an opportunity to consider how they were making use of the various modes of communication available to them. Secondly, the mathematical nature of the problem they were working on and the focus of their MoPiX programming task changed through the course of the episode.

Strand 1: Mode.

This strand of analysis was identified as a fundamental component of our social semiotic theoretical framework and of importance in addressing our research questions. It was initially defined by a priori categories. Each moment was first coded according to the mode or modes in use. The initial categories used were:

- spoken language (subdivided into everyday/ mathematics/ MoPiX registers)
- written language (natural language/ conventional mathematics notation/ MoPiX notation)
- drawing (outcome of MoPiX animation/ aid to problem solution)
- gesture (pointing/ mimicking MoPiX motion/ other)
- MoPiX equations (library/ authored/ complete models)

During the coding process, however, it became clear that this categorisation was not sufficient by itself to capture the ways in which the meanings produced during the extract were realised using the available semiotic resources. In particular, the functional relationship between the various modes used in any moment appeared significant. For example, Baz, creating the initial design, used simultaneous words and drawing (see Table 1). The initial causal connection 'so' made by Baz between the possibility of using *colours* and the decision to make the design ‘like a firework’ draws attention to the significance of the semiotic potential of the available technology. Both the *Paint* programme the students had chosen to use instead of paper-and-pencil and MoPiX itself afford easy application of a range of colours. It seems that the availability of colour as a resource suggests representational possibilities that might not have been chosen when working with traditional tools.

	spoken language	*drawing (in Paint)*
Baz	You can have different colours right [both laugh] so make it like a firework	
	so it goes like that	draws vertical bottom to middle twice
	and then you could have vertical ones like that	horizontal middle to left; horizontal middle to right twice
	and diagonal ones	3 diagonals: middle to NW; middle to NE; middle to SW
	and another horizontal, I mean vertical one going even further up	vertical middle to top

Table 1: Interaction of speech and drawing

There is a direct congruence between Baz’s words (*spoken -mathematics*) and his *drawing*; as he speaks the word ‘vertical’, he draws vertical lines (although he initially confuses vertical and horizontal). In addition, however, the motion of drawing (*gesture*) mimics the imagined motion of the firework (*spoken -everyday*) thus combining use of the static meaning potential of the descriptive language - vertical, horizontal, diagonal - and the completed drawing (displaying the outcome of the intended MoPiX animation) with the dynamic meaning potential of gesture.

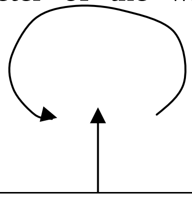
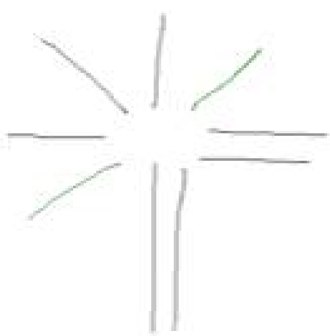
	spoken language	*gesture*	*drawing*
Vin	like a sparkler		
Baz	yeah but we need it to start from here and then these start after this one and then I don't know how that’ll work	slide-pointing bottom to middle, then slide-pointing anticlockwise circle around the perimeter of the whole shape 	

Table 2: Interaction of speech, gesture and drawing

In the next moment (see Table 2), Vin echoes Baz's original everyday discourse identification of the design as a firework, now specifying it more concretely as a sparkler, then Baz uses *gesture* to interact with the now complete *drawing*, simultaneously verbalising the process needed to construct the design with moving objects (*spoken -MoPiX*). In this case, the students use the drawing mode as readers, producing new meanings for the drawing through their use of spoken language and gesture. The spoken language naming of the design as firework/ sparkler here provides a holistic (everyday) image of the outcome of the design, while Baz's simultaneous use of language and gesture affords a dynamic representation of the development of the animated design over time.

Strand 2 Goal of the design activity: static versus dynamic outcome

In order to capture the complexity of the relationships between modes in use in any moment, the coding was developed to take account of the changing nature of the design activity. This strand of analysis was developed after initial examination of the whole extract, emerging as a theme from the data. It was observed that the ways in which the participants talked about their pattern included attending both to the properties of the lines drawn as traces of the MoPiX animation (a static outcome) and to the properties of the motion itself (a dynamic outcome). At the beginning of the chosen extract, the task is introduced by the teacher/researcher, using what we have now characterised as a static representation of the goal of the task:

> think about a pattern maybe with some parallel lines and perpendicular lines and a number of lines to make some sort of a pattern on the screen.

This static goal is taken up initially as the students discuss the types of lines they might make using MoPiX (horizontal, vertical, squiggly, defined by a formula). By the end of the episode, however, the focus of the activity is related to the motion of objects needed to construct the pattern. This focus was not the anticipated task of coordinating horizontal and vertical components of motion in order to draw lines with particular gradients. Rather, the students identified an important new goal that influenced the progress of their work through the remainder of the session: to find a way of stopping a moving object. This proved a substantial problem for them as its solution demanded a more analytic use of MoPiX equations than they had developed up to that point, in particular the use of equations specifying values of velocity or acceleration at a given time.

The question thus arises as to why this change from static to dynamic goal may have occurred. We coded references in any mode to the pattern or to components of the pattern as static or dynamic, identifying for each reference the mode and the indicators used to apply the code. Through this process of coding it became apparent that significant moments in the students' developing image of their pattern occurred as they moved between different modes of representation (see Table 3). In particular, the naming of the pattern as a 'firework' (apparently influenced by the articulated recognition of the possibility of using colour in their design), and interaction using gesture with the drawing of their design introduced new semiotic resources with meaning potentials that highlighted dynamic aspects of the design.

(i)	The original MoPiX programming challenge focuses on the direction of lines: "parallel", "perpendicular".	written and spoken language - mathematics	static
(ii)	Vin discusses the need for mathematical formulae to define MoPiX motion.	spoken language - mathematics; MoPiX programming	static
(iii)	Vin introduces of the idea of using a formula involving 'log' and the dynamic idea that it will 'go on forever', perhaps invoked by a concept image of a logarithmic graph (note O'Halloran's (2005) identification of the dynamic meaning potential of mathematical graphs).	spoken language - mathematics; imagined graph?	static - dynamic
(iv)	The use of Paint or perhaps the use of MoPiX enables the suggestion to use different colours.		
(v)	This suggestion then seems to trigger the naming of the design as a "firework".	spoken language - everyday; imagined dynamic object	dynamic
(vi)	The firework idea is realised in Paint.		
(vii)	Interaction with this drawing through gesture introduces a temporal aspect.	drawing; gesture	dynamic
(viii)	This temporal aspect is taken up immediately by Baz's verbal description of the motion "we need to start from here and then these start after this one"	drawing; gesture; spoken language - MoPiX	dynamic
(ix)	The MoPiX programming challenge then becomes the problem of how to make motion stop.	MoPiX programming	dynamic

Table 3: Change from static to dynamic

CONCLUSIONS AND DISCUSSION

The analysis we have offered here has focused on the multiple modes of communication used by these two students. Not only does each mode have its own set of meaning potentials but the different modes also interact, providing further potential. The complex interaction of use of language, drawing, gesture and MoPiX programming thus contributes to the construction of new meanings in the communication between the two students. The new semiotic resources provided by

MoPiX play relatively little explicit part in the episode we have considered. Nevertheless, we would argue that they play an influential role in shaping the students' activity, not only because the overt goal of the task involved use of MoPiX but also because the students were influenced by their recent use of MoPiX and their awareness of its potential. Moreover, the technological environment and the students' familiarity with its capabilities enabled them to choose to use Paint and its colour resources rather than traditional monochrome paper-and-pencil.

The resources afforded by gesture have been identified as significant in the move from a static to a dynamic goal. We consider here not only the pointing gestures accompanying the deictic spoken language seen in Table 2 but also the bodily movement implicit in the act of drawing in Table 1. This draws attention to the duality of the *drawing* mode: it is both a product - the outcome or picture - and the process by which the outcome is produced. In different moments it thus has both static and dynamic meaning potential and may play an important part in shifting focus between the two types of meaning.

However, the change from a static to a dynamic focus for the students' problem solving activity was not solely a product of the multi-semiotic environment. The nature of the pedagogic discourse of the classroom also played an important role. In particular, the students had enough agency within the classroom to enable them to make decisions about their own activity. In the first place, they were able to decide to ignore the teacher/researcher's suggestion to use paper-and-pencil, choosing to use Paint instead. Further, they were able to follow their own interests in designing their firework, thus enabling the change in the focus of their attention. Indeed, at a later stage in the same lesson, the teacher/researcher worked with this pair to help them solve the MoPiX programming problem of making a moving object stop, using techniques whose introduction had been planned for a later lesson.

Our analysis of this episode illustrates the very complex space of communication and learning and, we hope, contributes to Kress's call for development of theory of learning from a social semiotic perspective (Kress, 2008). The focus of students' attention and the direction of their learning are shaped by the multi-modal resources available and the interactions between them. However, this takes place within a learning environment that affords and/or constrains students' agency and their ability to change the direction of their activity in ways that will be considered legitimate.

NOTES

[i] ReMath (Representing Mathematics with Digital Technologies) funded by the European Commission FP6, project no. IST4-26751.

[ii] MoPiX also has potential to be used in many other areas of mathematical modelling.

[iii] MoPiX version 1 is available at http://remath.cti.gr; version 2.0 is under development at http://modelling4all.nsms.ox.ac.uk/

[iv] Units are non-standard and not identified explicitly in the notation.

REFERENCES

Alshwaikh, J. (2008). 'Reading' geometrical diagrams: a suggested framework. In M. Joubert (Ed.), *Proceedings of the British Society for Research in Mathematics Education* (Vol. 28, pp. 1-6).

Bjuland, R., Cestari, M. L., & Borgersen, H. E. (2007). Pupils' mathematical reasoning expressed through gesture and discourse: A case study from a sixth-grade lesson. In D. Pitta-Pantazi & G. Philippou (Eds.), *European Research in Mathematics Education V: Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 1129-1139). Larnaca, Cyprus: Department of Education, University of Cyprus.

Duval, R. (2006). A Cognitive Analysis of Problems of Comprehension in the Learning of Mathematics. *Educational Studies in Mathematics, 61*(1 - 2), 103-131.

Falcade, R., Laborde, C., & Mariotti, M. (2007). Approaching functions: Cabri tools as instruments of semiotic mediation. *Educational Studies in Mathematics, 66*(3), 317-333.

Kress, G. (2008). *A social semiotic theory of learning: multimodality, provisionality and design*. Paper presented at the Multimodality and learning.

Kress, G., & van Leeuwen, T. (2001). *Multimodal Discourse: The modes and media of contemporary communication*. London: Arnold.

Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational Studies in Mathematics, 61*(1/2), 219-245.

Morgan, C., & Alshwaikh, J. (2008). Learning about motion in a multi-semiotic environment. *Proceedings of the British Society for Research into Learning Mathematics, 28*(2).

Morgan, C., & Alshwaikh, J. (2009). Learning about motion in a multi-semiotic environment. *Research in Mathematics Education, 11*(1), 79-80.

O'Halloran, K. L. (2005). *Mathematical Discourse: Language, Symbolism and Visual images*. London: Continuum.

Radford, L., & Bardini, C. (2007). Perceiving the general: The multisemiotic dimension of students' algebraic activity. *Journal for Research in Mathematics Education, 38*(5), 507-530.

Strohecker, C., & Slaughter, A. (2000). Kits for learning and a kit for kitmaking. *CHI '00 Extended Abstracts on Human Factors in Computing Systems*, 149-150.

Yerushalmy, M. (2005). Functions of interactive visual representations in interactive mathematical textbooks. *International Journal of Computers for Mathematics Learning, 10*, 217-249.