

# INTERNET-BASED DIALOGUE: A BASIS FOR REFLECTION IN AN IN-SERVICE MATHEMATICS TEACHER EDUCATION PROGRAM<sup>[1]</sup>

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*In this paper, the asynchronous interactions of two groups of mathematics teachers in an internet-based in-service course are analyzed. During the interactions, teachers are solving a mathematical modeling activity designed to stimulate the teachers' reflections on the modeling process. In one of groups these kinds of reflections occurred frequently while they were absent in other group. The analyses reveal clear differences in the communicative characteristics of the interactions in the two groups. Some of the characteristics of the first group are argued to be important factors favoring the emergence of the teachers' reflections on the modeling process.*

## INTRODUCTION

In this work, the asynchronous interactions of two groups of mathematics teachers in an internet-based in-service course are analyzed. The teachers are involved in an internet-based mathematics education in-service program for teachers from different Latin American countries. The acronym for this program is PROME-CICATA, and this is an educational program sponsored by the Instituto Politécnico Nacional of México, one of the largest public universities in Mexico. I am interested in finding ways of encouraging “rich” interactions and reflections among the teachers enrolled in the PROME mathematics education program. That is why I am trying to determine when an interaction can be regarded as “rich” or not, and what characterise communication in such rich interactions.

## FRAMEWORK

The concept of communication is central for this work and particularly the computer-mediated communication (CMC). There are very clear differences between the everyday communication (or face-to-face) and the CMC. Although in both types of communication some kind of information (such as thoughts and feelings) is exchanged among individuals, the CMC does not require people staying in the same place or at the same moment of time. Communication may be atemporal to some extent and free of geographic barriers. Everyday communication is primarily verbal, but the CMC fosters written communication, which can be recorded, stored and accessed by people during conversation. This creates a record of ideas and comments that can serve as a reference or *collective memory* (de Vries, Lund & Baker, 2002) for the communication process. The expression and representation of ideas, and particularly mathematical ones, can be enhanced in CMC by the use of technological

tools such as software and video. The ideas can become entities with physical properties (such as a spreadsheet file in which somebody expresses a hypothesis based on graphical and arithmetical information represented in the file) which can be stored, handled and distributed.

The characteristics of the CMC influence the nature and dynamics of the interactions that I am analyzing in this study. The data analysis is based on the Inquiry co-operation model (IC-Model) of Alrø & Skovsmose (2002). This model was developed based on the observation of students, collectively solving mathematical open-ended activities. The model, strongly rooted in the critical mathematics education approach, argues that in order to have a fruitful interaction, it must be based on mutual respect, on the willingness to make public our ideas and subject them to scrutiny, as well as in a real interest to listen and analyse our interlocutor's ideas. The IC-Model is constituted by a set of communicative characteristics. According to this theoretical approach, an interaction as the previously described should have several of these communicative characteristics. In fact when these characteristics are present in an interaction, it is regarded as a special kind of interaction called *dialogue*, which possesses the potential to serve as a basis for critical learning and reflection. The communicative characteristics that define a dialogue are *getting in contact*, *locating*, *identifying*, *advocating*, *thinking aloud*, *reformulating*, *challenging* and *evaluating*; and they could be succinctly defined as follows:

*Getting in contact* basically refers to the act of paying attention to the ideas expressed by our partners in an interaction. The act of *locating* takes place when you discover an idea or a way of doing that you did not know or were not aware of before. It is a process of examining possibilities and trying things out. *Identifying* is a clarifier act in the sense that appears when you explore or try to explain an idea or perspective with the intention of making it clear to all the members of the interaction (including yourself). *Advocating* appears when you present your ideas or positions and you justify them with arguments. An advocating an also implies a willingness to revisit and discuss your own ideas or positions. To *think aloud* simply means to express in public your thoughts, ideas and feelings during the interaction process. *Reformulating* means repeating some idea but with different words or in other terms, usually to try to make it clear to your interlocutors. When we question a perspective or when we try to push it toward another direction to explore new possibilities, it is said that this is a *challenging* act. An *evaluative* act appears when we examine, criticize or correct an idea or proposal from others or ourselves.

In the communicative approach of Alrø & Skovsmose (2002), the concepts of dialogue and reflection are linked. First, reflection is defined as follows: "Reflection means considering at a conscious level one's thoughts, feelings and actions" (p. 184), but the dialogical interactions are also conceived as a basis for reflection: "We find that reflections are part of a dialogue. In particular we find elements of reflection in

dialogic acts like locating, thinking aloud, identifying, advocating, etc. This means that we do not follow the Piagetian line, seeing reflections as carried out by an individual. We consider reflections referring to ‘shared considerations’ and we see dialogue as including processes of reflection”

In the context of research on mathematics teacher education, reflection plays a key role. In her recent review, Judith T. Sowder says that several studies identify reflection as a crucial element in furthering teachers’ professional development (see Sowder, 2007, p. 198).

## **METHODOLOGY**

In this section I refer to different aspects of the production and collection process of data, namely, the mathematical activity applied, the selected population, and the collection and presentation of data.

### **The selected population and the research goal.**

The data that I will present were taken from one of the courses of the PROME program. The course was taught between March and April 2008. The course was an introduction to the teaching and learning of mathematical modeling. The teachers who participated in this course are in-service teachers working in different educational levels, from elementary to university level. This course was part of their academic obligations in order to get a master’s degree in mathematics education.

I present here the analysis of the asynchronous interactions produced in two groups of teachers while working collectively with a mathematical modeling task. I use the term ‘asynchronous interactions’ to specify that the sort of communication that takes place into this interaction is asynchronous. An asynchronous communication is the one that is carried out mainly by means of an exchange of written messages between two or more people (very often located in different geographical positions), but the answers or reactions that the participants get are not immediate, for example, you can raise a question or an observation and get the feedback or reactions to it several minutes or hours after. The asynchronous discussions usually last several days, allowing the participants to have more time to formulate their opinions and to reflect on comments and opinions expressed by the other participants. It is even possible to consult external sources in order to enrich and clarify a discussion in an asynchronous communication. The email messages and the discussion forums are some examples of asynchronous communication.

The activity lasted six days and although both groups of teachers solved the mathematical activity, only in one group emerged some meta-reflections about the modeling process, which were expected to be produced through the activity and the interaction. In other words, I will show an interaction that is “rich” in terms of the reflections produced and another that it is not rich, and, through the application of IC-

Model, I will try to identify the differences in the communicative characteristics that are present in each of those interactions. That is the purpose of the research.

### The mathematical activity

The mathematical activity was taken from Lesh & Caylor (2007), but it was slightly modified to fit the purposes of the course. The context of the activity is a paper airplane contest in which four planes were involved, and where each of these planes were thrown by three different pilots five times each. The activity includes two tables (see tables 1 and 2) containing numerical values generated during one of the tests. Table 1 shows the landing points for each launch, represented by ordered pairs  $(x, y)$ ; Table 2 shows data such as distance from target, length of throw and air time for those launches. In this test the three pilots flew the four paper planes. Each time the pilot was placed at the point  $(0, -80)$  on the floor, and their aim was to launch the planes so that the plane come as close as possible to the point  $(0, 0)$ , which was marked with an X.

A non-explicit purpose of this activity was that teachers will experience a portion of a mathematical modeling process, enabling them to see that in an mathematical activity as such, it is possible to have several possible and valid answers (or models), depending on the assumptions and considerations in which the model is based. To support the emergence of multiple approaches and answers to the activity, I decided to replace the original request “[to explain] how they could use this data and data from future contests to measure and make judgments about the accuracy of the paper airplanes”, for a more general question, namely: “Which one is the best airplane?”. Any model that answered the previous question should be based on the definition or concept that the modeler holds about what does it means to be “the best airplane”. This is where I expected to have a variety of definitions/concepts, and as a consequence, a variety of possible answers to the question.

|         | Plane 1 |     |     | Plane 2 |     |     | Plane 3 |     |     | Plane 4 |     |     |
|---------|---------|-----|-----|---------|-----|-----|---------|-----|-----|---------|-----|-----|
|         | Flight  | X   | Y   |
| Pilot 1 | 1       | 45  | -78 | 1       | -37 | -30 | 1       | -42 | -4  | 1       | 15  | 66  |
|         | 2       | 7   | -78 | 2       | -48 | -9  | 2       | 45  | 5   | 2       | 32  | -46 |
|         | 3       | 55  | -42 | 3       | 28  | 26  | 3       | 35  | -7  | 3       | 5   | 32  |
|         | 4       | -14 | -46 | 4       | 12  | -35 | 4       | -13 | 5   | 4       | 14  | -21 |
|         | 5       | 21  | -29 | 5       | -19 | 26  | 5       | 4   | -11 | 5       | 27  | -5  |
| Pilot 2 | 1       | -12 | 26  | 1       | -11 | 13  | 1       | -4  | 12  | 1       | -13 | 22  |
|         | 2       | -40 | -20 | 2       | -14 | -36 | 2       | 24  | 9   | 2       | -3  | -21 |
|         | 3       | -38 | 22  | 3       | 31  | 13  | 3       | -9  | 34  | 3       | -11 | -51 |
|         | 4       | -61 | 15  | 4       | -43 | 10  | 4       | -39 | 29  | 4       | -12 | 56  |
|         | 5       | -48 | 61  | 5       | -14 | 50  | 5       | 40  | 21  | 5       | 5   | -69 |
| Pilot 3 | 1       | 42  | 71  | 1       | -9  | 6   | 1       | 24  | 41  | 1       | 53  | -62 |
|         | 2       | 61  | 38  | 2       | -27 | -13 | 2       | -3  | 52  | 2       | 52  | 54  |
|         | 3       | 43  | 27  | 3       | 17  | -17 | 3       | -31 | 54  | 3       | 48  | -26 |
|         | 4       | 18  | 50  | 4       | 14  | 47  | 4       | 5   | 64  | 4       | 45  | 25  |
|         | 5       | 15  | 6   | 5       | -36 | 41  | 5       | 36  | 63  | 5       | 25  | 40  |

**Table 1: Where did the plane land?**

| Flight  | Plane 1                       |                          |                    | Plane 2                       |                          |                    | Plane 3                       |                          |                    | Plane 4                       |                          |                    |      |
|---------|-------------------------------|--------------------------|--------------------|-------------------------------|--------------------------|--------------------|-------------------------------|--------------------------|--------------------|-------------------------------|--------------------------|--------------------|------|
|         | Distance from target (inches) | Lenght of throw (inches) | Air time (seconds) | Distance from target (inches) | Lenght of throw (inches) | Air time (seconds) | Distance from target (inches) | Lenght of throw (inches) | Air time (seconds) | Distance from target (inches) | Lenght of throw (inches) | Air time (seconds) |      |
| Pilot 1 | 1                             | 90                       | 45                 | 0.66                          | 47.6                     | 62.2               | 0.52                          | 42.2                     | 86.8               | 0.59                          | 67.7                     | 146.8              | 2.22 |
|         | 2                             | 78.3                     | 7.2                | 0.58                          | 48.8                     | 85.7               | 0.91                          | 45.3                     | 96.2               | 0.7                           | 56                       | 47.7               | 1.42 |
|         | 3                             | 69.2                     | 66.9               | 0.76                          | 38.2                     | 109.6              | 1.05                          | 35.7                     | 81                 | 0.55                          | 32.4                     | 112.1              | 1.91 |
|         | 4                             | 48.1                     | 36.8               | 0.51                          | 37                       | 46.6               | 0.51                          | 13.9                     | 86                 | 0.61                          | 25.2                     | 60.6               | 1.51 |
|         | 5                             | 35.8                     | 55.2               | 0.65                          | 32.2                     | 107.7              | 0.99                          | 11.7                     | 69.1               | 0.54                          | 27.5                     | 79.7               | 1.58 |
| Pilot 2 | 1                             | 28.6                     | 106.7              | 0.94                          | 17                       | 93.6               | 0.95                          | 12.6                     | 92.1               | 0.73                          | 25.6                     | 102.8              | 1.88 |
|         | 2                             | 44.7                     | 72.1               | 0.81                          | 38.6                     | 46.2               | 0.54                          | 25.6                     | 92.2               | 0.7                           | 21.2                     | 59.1               | 1.4  |
|         | 3                             | 43.9                     | 108.8              | 0.93                          | 33.6                     | 98                 | 0.92                          | 35.2                     | 114.4              | 0.77                          | 52.2                     | 31                 | 1.35 |
|         | 4                             | 62.8                     | 112.9              | 0.9                           | 44.1                     | 99.7               | 0.99                          | 48.6                     | 115.8              | 0.8                           | 57.3                     | 136.5              | 2.35 |
|         | 5                             | 77.6                     | 148.9              | 1.28                          | 51.9                     | 130.8              | 1.1                           | 45.2                     | 108.6              | 0.78                          | 69.2                     | 12.1               | 0.97 |
| Pilot 3 | 1                             | 82.5                     | 156.7              | 1.34                          | 10.8                     | 86.5               | 0.88                          | 47.5                     | 123.4              | 0.8                           | 81.6                     | 56                 | 1.36 |
|         | 2                             | 71.9                     | 132.8              | 1.14                          | 30                       | 72.2               | 0.73                          | 52.1                     | 132                | 0.88                          | 75                       | 143.7              | 2.39 |
|         | 3                             | 50.8                     | 115.3              | 1.01                          | 24                       | 65.3               | 0.55                          | 62.3                     | 137.5              | 0.92                          | 54.6                     | 72.2               | 1.46 |
|         | 4                             | 53.1                     | 131.2              | 1.21                          | 49                       | 127.8              | 1.08                          | 64.2                     | 144.1              | 0.89                          | 51.5                     | 114.2              | 2.18 |
|         | 5                             | 16.2                     | 87.3               | 0.85                          | 54.6                     | 126.2              | 1.1                           | 72.6                     | 147.5              | 0.94                          | 47.2                     | 122.6              | 2.09 |

**Table 2: Distance, time and flight sequence data for each pilot and airplane.**

The activity was uploaded as a *pdf* file on the web-based educational space where all participants of the course could access it. Teachers were organized into groups of three or four members and each of those groups were assigned to a discussion forum where the activity was collectively solved.

### Data collecting and data presentation

As I mentioned before, one of the characteristics of the computer mediated communication is that it can be easily recorded, stored and shared. This feature represents a significant advantage for educational research, because the need of making transcriptions disappears. In my work for instance, I am studying some of the written asynchronous discussions produced in an internet-based educational program. Those discussions are permanently recorded and accessible on the internet-based workspace, ready to be analyzed. These asynchronous discussions may be composed of dozens of utterances. Due to the space available, it will not be possible to present the complete interactions, but only those sections that I consider most significant and illustrative. I will use bracketed ellipsis [...] to denote the omission of certain segments of text; this edition was made for the sake of brevity and to increase the readability of the data. The data that I will present has been translated from Spanish into English; moreover, the original names of the teachers have been replaced to protect their identity.

To start the analysis of an asynchronous discussion, I order all its utterances in a chronological way. From this arrangement, I try to locate those sections in which two or more participants are involved in a discussion of a particular topic. Each of these sections is broken down into individual utterances, trying to 'label' them with some of the communicative characteristics that define the communication IC-Model, according to the content of the utterance and its role within the whole discussion. Let me consider utterance (1) as an example (see 'Results' section below): This is not an *evaluative* or *challenging* act, nor is *getting into contact* with someone else because

Juan is not criticizing, questioning or being referred to the ideas of another person. He is not *reformulating* because this is the first time that he presents these ideas. Juan says “I think the most important is the proximity to the target”, but he did not present any argument to be able to classify the act as an *advocating* one. The utterance could be classified as a *thinking aloud* act, but because Juan is raising different ways of tackling the problem, I have classified it as a *locating* act. A similar analysis was done with every utterance. In some cases it is difficult to carry out the categorization since the differences between some communicative acts of IC-Model are not entirely clear for some utterances.

## RESULTS

### Data analysis – Group A

The working group A was composed of two teachers from Argentina (Juan and Susana) and one mexican teacher (Horacio). The interaction begins with some *thinking aloud* acts where the teachers begin to make public some of their initial ideas on how to address the problem. For instance, Susana suggests that they should find a way to use the three variables contained in Table 2 (distance, length and time). Juan answered to Susana in (1):

1 *Topic: Re: The first message*

*From: Juan*

*Date: Thursday, the 3rd of April 2008, 11:40*

Colleagues. One possible option is to work with some type of weighted mean for the 3 considered variables (length of throw, distance from target and air time). I think the most important is the proximity to the target. Another option is to think on the deviation from the target (because definitely it is a measure of the dispersion) what do you think?

In (1) Juan is *locating*, I mean, he is examining different ways of facing the problem and trying things out. He is doing a specific suggestion on how to relate the three selected variables. He proposes to use a weighted mean where “proximity to the target” is the most important variable.

2 *Topic: Re: The first message*

*From: Susana*

*Date: Thursday, the 3rd of April 2008, 13:05*

Flight partners: I was planning to ask you if you have thought in a linear regression, but I read your proposal of the weighted mean. We just have to decide about the importance assigned to each variable. Since the target is point (0,0) I would give 40% to distance from target, and 30% for the other two, if you agree. [...] Susana

3 *Topic: Re: The first message*

*From: Juan*

*Date: Thursday, the 3rd of April 2008, 19:06*

Fellows. I have been outlining a sketch of the things worked so far and I expressed it on this first draft that I am attaching. [...] Best wishes. Juan

In (2), Susana mentions the possibility of using a linear regression, but this possibility was not further explored because she simply leaves this alternative and without any question she adheres herself to the proposal of the weighted mean. Without a clear argumentation, Susana proposed the weight for each element of the weighted mean. In turn, Juan in (3) contributes to *not locate* Susana's idea of linear regression. In his utterance he completely ignores the timid suggestion of Susana and he only “hear” the proposal of the weights. In a file attached to his utterance (3), Juan *identifies* or clarifies in mathematical terms his perspective on the weighted mean. In this file he defines the concept of “performance” that is used to determine which one is the best airplane. The plane that gets the higher performance will be the winner. This concept is defined as follows:  $Performance = 0.4x + 0.3y + 0.3z$

Where:

$x$  = the arithmetic mean of the distances from target

$y$  = the arithmetic mean of the lengths of throw

$z$  = the arithmetic mean of the air times

Juan never questioned the weights suggested by Susana. He never asks which were the assumptions that Susana considered in order to establish those values, he just includes the values in his own proposal. In general, the interaction between Susana and Juan could be described as uncritical. They experienced a “smooth” interaction where they did not question nor evaluate the proposals from the other. An example of this is in the performance formula. Neither Susana nor Juan noted that this model favoured the airplanes having a landing fare away from the target. On the other hand, Juan’s attitude was not the most appropriate to establish a *dialogue*, apparently Juan was more interested in delivering the solution of the task on time, that in paying attention to the proposals of his colleagues. For example, although the asynchronous discussion forum lasted until the 6<sup>th</sup> of April, Juan showed in (5) a strong rejection attitude towards other proposals to his colleague Horacio (see (4)):

4 *Topic: Re: The first message*

*From: Horacio*

*Date: Friday, the 4th of April 2008, 11:10*

Susana, Juan. I am sorry but my time is very limited. I will try to communicate with you later on. Best regards. Horacio

5 *Topic: Re: The first message*

*From: Juan*

*Date: Friday, the 4th of April 2008, 11:26*

Horacio. We are against the clock, this activity started on tuesday and there is 1 day left...I think you will have to accommodate yourself to the things that Susana and I were working on...there is no time to make any modification... Do you agree? What do you think?

Thus, even though group A was able to successfully solve the mathematical modelling task (i.e. to establish a model to select the best airplane), the interaction inside the team was characterized by a poor exchange of perspectives and ideas on how to address the mathematical task.

### Data analysis – Group C

The group C had three members, but almost all the interaction took place between an Argentinean teacher (Nora) and a Mexican one (Maria). Since the beginning of the interaction, Norma and Maria were *locating* different ways of tackling the problem, but always *maintaining the contact* between them, namely, listening to the proposals of the other, taking them into consideration and evaluating them. At one point, based on Maria's suggestion about excluding the pilots of the analysis, Norma proposed in (6) a new way to find the best paper airplane:

6 *Topic: Some issues*

*From: Norma*

*Date: Saturday, the 5th of April 2008, 06:17*

[...] We could choose the ten shots that are closer to the origin, and then see which of those planes did it in more time and with the biggest length, what you think? [...]

7 *Topic: Re: Some issues*

*From: Maria*

*Date: Saturday, the 5th of April 2008, 21:44*

[...] I propose to choose the other way around, let's say that the best planes are the ones who entered into a circle with center (0.0) and a fixed radio, and then to take the ones who did it in less time [...] you said more time... but are we judging the fastest or the longest stay in the air[?]?... both cases are possible to judge [...] in a model it should be fixed the aspects to take into account and the rest are discarded because it is a model. I think that the idea of the radio is more close to the kind of things that are considered in the accuracy competitions as in archery. Maria

8 *Topic: Re: Some issues*

*From: Maria*

*Date: Saturday, the 5th of April 2008, 22:32*

Colleagues: I am writing you because I think that a good size for the radio could be 20 because it is one fourth of the distance from the point of departure to the target point. With this we only have six throws with three planes, I mean, the fourth plane does not participate, it does not surpass the first filter, then we can evaluate the next point.... and if we estimate the maximum speed [...] It would be like the thing that I am sending you ...What do you say? [...] I will wait for your criticism

In (7) Maria is *challenging* Norma's proposal by suggesting replacing the ten shots criterion with the radio criterion. I think this intervention is particularly valuable because explicitly brings into the discussion the need to establish the criteria, assumptions or variables to consider for building a mathematical model. Her next

sentence sums up this point: “[I]n a model it should be fixed the aspects to take into account and the rest are discarded because it is a model”. This is the kind of meta-reflection that I was looking to produce through the activity.

Maria’s utterance (8) includes a spreadsheet file that illustrates with more detail the ideas presented in (8) and (8). She concludes that the winner is the plane number 3. As a reaction, Norma in (9) *evaluates* the proposal of Maria, and qualifies as arbitrary the choice of a radio with longitude 20. Norma agrees with Maria about using the proximity to the target as a first filter for selecting the best plane, but she suggested to use the mean of the distances from target instead of the radio proposed in (7) and (8).

9 *Topic: Re: Some issues*

*From: Norma*

*Date: Sunday, the 6th of April 2008, 12:19*

Girls, Maria: The radio that you mention is a bit arbitrary, why do not we take advantage of the fact that we already have the mean of the distances from target, and then to select the planes that were above that mean??? [...]

10 *Topic: Re: Some issues*

*From: Norma*

*Date: Sunday, the 6th of April 2008, 13:03*

Well, here you have what I made according to the previous observation about the radio. But I would also mention that I love your conclusions, Maria.

If you agree, let’s vote; choose one of the three options, or choose all of them because for me all of them are ok. I mean, they are all equally valuable and correct. There are as many answers as aspects and ways of evaluating we have agreed previously.

In (10) Norma attached a file showing her new calculations, in which the winner is the plane number 4. Despite she is *advocating* a different model and getting a different winner, Norma recognizes the validity of the model suggested by her colleague Maria, in fact I think that this recognition is the basis for issuing the comment made by Norma in (10), a comment linked to another reflection implicitly sought for the modeling activity: the recognition that there may be different valid answers or mathematical models to answer the same question. It may be noted that the discussion has reached an interesting point: the participants in the discussion have been able to *locate* different ways (or models) that can serve as a mean to answer the original question which one is the best airplane? Moreover, apparently they have recognized as valid each of those models, then ... what model to choose?

This discussion continued even addressing issues of *responsibility* (see Alrø & Skovsmose, 2002, p. 217). At one point Maria asked, “[I]f the owner of the plane 3 shows up, with what criteria would we justify that we do not chose the early drafts in which he would win and instead we took the other one[?]”. No doubt, this was a rich interaction in terms of the reflections achieved by the teachers.

## CONCLUSIONS

The analysis of the interactions through the IC-Model shows that there are some differences in the communicative characteristics that are present in the interactions of groups A and C. For example, the interaction within the A team can be described as uncritical because there is a lack of communicative acts such as *challenging* or *evaluating*; additionally they did not seize the opportunities to find additional ways to address the problem (see for example the utterances sequences (2)-(3) and (4)-(5)).

In the team C, participants were able to *locate* several ways to tackle the problem. There was a general interest in hearing (or *keep the contact*) and evaluate the proposals of the other, and they were able to recognize the existence of multiple perspectives to solve the problem.

I argue that members of team C team were able to establish a *dialogue* that fostered the emergence and recognition of multiple perspectives to solve the problem. I think that the existence of this dialogue encouraged the emergence of meta-reflections about the modeling process.

It is necessary to continue working in a more explicit characterization of the concept of reflection. It is also necessary to discuss how the characteristics that are specific to the internet-based communication affect the emergence of reflections. Methodologically speaking it is necessary to find appropriate tools to detect or to point out when a reflection takes place in an online setting, but particularly in an asynchronous interaction.

## NOTES

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