SPEAKING OF MATHEMATICS – MATHEMATICS, EVERY-DAY LIFE AND EDUCATIONAL MATHEMATICS DISCOURSE

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The aim of this paper is to describe and analyze how discourse as a theoretical and didactical concept can help in advancing knowledge about the teaching of mathematics in school. The collection of empirical data was made up of video and audio tape recordings of the interaction of teachers and pupils in mathematics classrooms when they deal with problem-solving tasks. Discourse analysis was used as a tool to shed light upon how pupils learn and develop understanding of mathematics. The results underline that a specific and precise dialogue can contribute towards teachers' and pupils' conscious participation in the learning process. Teachers and pupils can construct a meta-language leading to new knowledge and new learning in mathematics.

INTRODUCTION AND AIM OF THE STUDY

This research deals with teachers and pupils discussing with each other in different situations within and about mathematics in school. The theoretical point of departure is first and foremost an in-depth study of the meaning of and relationships between concepts, words and signs in order to demonstrate how mathematical discussions can be understood. The concepts of context, mediation and artefacts are central to the socio-cultural perspective chosen and thus play an important role in this research, (Vygotsky, 1978, 1934/1986, 2004). The concept of context can be described as being the environment where our actions take place and thus create and re-create the context as such. Mediation implies that human beings interact with external tools in their perception of the world around them. Linguistic as well as physical artefacts are created by mankind to perform actions and solve problems. They are cultural resources which contribute towards maintaining and developing knowledge and abilities in society (Vygotsky, 1978, 1986). Using semiotic tools one can demonstrate how a linguistic element is connected to its meaning, (Ogden and Richards, 1923; Melin-Olsen, 1984; Johnsen-Hoines, 2002). We can picture a semiotic triangle made up of concept, expression and reference. If we look upon language as a medium for communication based on conventional signs it is by applying language that the reference to the world at large is created.

The relationship between thought and symbol is, like the one between thought and reference causal and direct in a semiotic triangle. The relationship between symbol and reference, on the other hand, is indirect and attributed. Concepts within a socio-cultural perspective which may be applied to the semiotic triangle are expression, content and reference. These three functions of a sign can only be understood when they are applied simultaneously. Thus we can see signs such as words, numbers, symbols, diagrams, equations and letters. The sign expresses something separate from the sign itself. Signs, objects are related to the meaning or conception of them. Mathematical knowledge must be actively constructed in relationship to signs, words and symbols.

I have chosen to describe mathematical discussions out of a discourse perspective. The concept of discourse can be understood in different ways. It can be interpreted as a set of conventional rules for discussing, understanding and conceiving the world and its different phenomena (Winther-Jörgensen & Phillips, 2000; Sfard, 2002). A discourse can be understood as a linguistic system which delineates issues of exclusion and inclusion, borders on what is excluded and inner standardization (Gee, 2005; Börjesson & Palmblad, 2007).

Foucault (1972/2002) wants to clarify how we are caught up in and blinded by lines of reasoning without really being conscious of what we say. We can refer to this as an invisible discourse. In the discourse on teaching mathematics there is an invisible element which is difficult to affect unless we make ourselves aware of its existence.

From a socio-cultural perspective discourse is defined as the language which gives and is attributed meaning in various contexts and which excludes and includes things to be understood (Säljö, 1999, 2000). In this work I have chosen to metaphorically regard discourse as a network where signs, concepts and references make up the nodes. Nets can be chosen or created in such a way that meaning is constructed in situated action as well as socio-cultural practices which transgress defined situations. Thus, a discourse can also be a set of rules for talking, writing and thinking about a specific content. Many discourses are mixed in school which both teachers and pupils must learn to become involved in, understand and master. This includes knowing when borders between different discourses are crossed. Mathematical instruction means that teachers and pupils are placed in different discourses, ranging from those applied to every-day life to purely mathematical ones. This means that they move over borders and between registers all the time. An example of this occurs when pupils work with concrete materials and are to express themselves using numbers and symbols. In doing so, they will move over different borders. When working in school we must learn to understand when we are situated in a specific discourse.

A mathematics lesson contains a number of words and expressions from every-day life. The language applied is rich and we talk departing from many different perspectives and towards many different aims. To be able to conduct conversations in a context as specific as school mathematics we have to develop a meta-language which makes it possible to put what we want to express into perspective. In every-day life we build models in order to understand reality and we use every-day methods for solving problems in order to describe connections to mathematics. We seek the history of mathematics to be able to see how every-day application developed into pure mathematics. This paper mirrors how teachers and pupils apply different types of discussions to deal with problem-solving tasks in and about mathematics. In these discussions we develop our thinking and our methods for learning and it is in the same discussions that we shed light on the transitions required in order to move from concrete to abstract activities. A knowledge rendered in linguistic terms is required. This is something that I aim to disclose in my empirical studies. In the discussions in and about school mathematics an oscillating movement between reality and mathematical concepts and expressions is to be seen.

Communication in a mathematics classroom can be described in terms of learning a mathematical register, (Duval, 2006). It can also be looked upon as a situation where there are two parties involved – two individuals who speak, think, write, read and listen. It is therefore highly interesting to study what learners and teachers have to say in and about mathematical practices.

The over-riding aim here is to raise this issue: “How can discourse as a theoretical and didactical concept contribute towards further developing mathematical teaching?”

Method

I have for many years been interested in communication and interaction within and about mathematical teaching. In my studies I have chosen to monitor how teachers and pupils have generated knowledge in discussions on mathematical concepts, problem-solving and formal mathematics. I did so in order to be able to establish what happens in interaction between teachers and pupils and between pupils.

In these studies I have made use of video and audio recordings. Video recordings were applied in order to make sure that it became clearly visible what went on in the interaction within a classroom. It also proved to be fruitful in that the activities on both teachers’ and pupils’ part became evident. The audio recordings were used as a means of analyzing the discussions as interactive situations. Group interviews are a well-chosen strategy for trying to capture discourse as regards what they include and exclude. The table below describes the environment used to acquire data in the respective studies.

Design of the Empirical Studies

Study I	Study II	Study III	Study IV
7 teachers	14 groups	26 groups	68 groups
Teacher-pupil interaction Classroom	Pupil interaction Classroom	Group talk Three pupils	Group talk Three pupils
Video	Video	Audio tapes	Audio tapes
Mathematical content			
The Area of a Triangle	The Area of a Triangle	Problem-solving	Rational Numbers

Table 1. Data acquisition in the empirical studies I-IV

Seven teachers took part in my first study. They were assigned to plan and carry out an introductory lesson on the area of the triangle in year 5 in compulsory school. Choosing mathematical content was a regular concept to the teachers who took part. Focus for these video recordings lay on documenting the public and the teacher-led interaction in the classrooms involved. Each recording lasted between forty and sixty minutes. Twenty-five occasions were recorded and focused on interaction between teachers and pupils. The study further describes how teachers cross discourse borders in teaching on the area of the triangle and in what ways they carried out their lessons as regards interaction between teachers and pupils, as well as what types of questions they used in their talks with pupils.

The introductory lesson on the area of the triangle is carried on into this second study but here focus is on pupils' interaction in a laboratory situation, where the teacher gives explicit directives to the groups of pupils. Varying directives from the teacher in the classroom lead to different trains of action and linguistic concepts on the pupils' part. In total the interaction of fourteen groups has been recorded and analyzed in the classes involved. The groups were made up of five to six pupils. The laboratory situations are described as regards activity and linguistic interaction. The pupils are active in that they draw, cut and fold pieces of paper. Every-day language is used to a great extent and retains its every-day character.

The point of departure for the third study was to monitor 26 groups of pupils when they set about a written mathematical task. The task is of an open variety and contains different pieces of information that the pupils are to decide on. One of the concepts which stay in focus for the pupils is the word fairness. Pupils seek, talk, make guesses, test and calculate an answer. There is, however, no evident way to go about solving the task. On the one hand the pupils end up in an every-day discourse and on the other hand in a mathematical discourse. They have difficulties making judgments as they reason with each other. Each group has been recorded on audio tape which has then been transcribed and analyzed. The pupils were put into groups on the basis of their mathematical skills as deemed by their teachers. The recordings took place in a small room next to the classroom.

For the fourth study one of the assignments from the National Test of mathematics for year five was used. The assignment deals with rational numbers. Five different partial studies were carried out. Sixty-eight groups of three pupils each and 120 individual pupils took part in the different studies. The first partial study was carried out with 30 pupils in year five who solved the assignment on their own and were asked to provide a written explanation. The second study took place in three classes of 30 pupils each. For the third partial study I used five schools from different parts of a large municipality. Thirty-one group interviews with pupils in year five were carried out, each group consisting of three pupils. When the pupils solve their assignment they rely on an every-day discourse. The next study involved 31 new groups of pupils. They were allowed to use a pocket calculator and they engaged in a solely mathematical discourse. The last part of this study was carried out with six groups of three pupils each and it deals with the issue of reasoning with the help of a numerical line. The results show that, depending on what tools are applied and what situation the pupils are in, the outcome turns out differently in different discourses.

I have used a discourse analysis to analyse the group discussions and the discussions in the classroom, (Wertsch, 1985, 1998; Kozulin, 1998; Fairclough, 1992, 1995, Gee, 2005). A discourse analysis is based on details in what is written and said in a particular situation. In the restricted discourse language can be seen as “language-in-action” which is always an active process in constructing knowledge. My study focuses on the interaction between individuals and in what ways knowledge, language and mathematical skills develop.

Results

Discourse analysis can be used as a tool with help of which descriptions of how pupils learn and develop their understanding of mathematics can be made clear. Looking at my empirical material I have come to discern the discourse in school mathematics which can provide the bridge upon which teachers and pupils can meet and become mutually involved.

In school mathematics teachers and pupils talk using every-day concepts and mathematical concepts, signs and words. This intercourse demands that a mutual understanding takes place. The analysis of what is said in the different groups shows that the discussions are situated somewhere on a scale between two extremes – on the one hand every-day concepts, on the other hand purely mathematical concepts. Words such as “put on” and “put together” are based in every-day practice whereas words such as “add/addition” and complex numbers are situated in a purely mathematical discourse. Any individual is to be found somewhere in this continuum depending on how far this individual has come in the process of developing an understanding of abstract reasoning. If we consider signs and expressions the same thing can be said for them.

In my empirical data where teachers talk to pupils in whole-class discussions and in group talks, teachers utilize different signs and change registers in their teaching. They go from geometrical into arithmetical/algebraic discourse and back. Analysis of these talks clearly reveals how pupils talk about and understand the concepts. Most pupils use every-day language and it demonstrates that teachers are situated in one discourse and pupils in another. The same thing can be seen when pupils work with concrete materials, performing acts but not acquiring the mathematical concepts which the teacher had planned. Pupils find themselves in a distanced discourse rather than an inclusive one as the teacher had intended. In one of my excerpts the pupils are engage in a group discussion of how to move from a rectangle made of red paper to a triangle. The teacher has told the pupils to prove that the triangle’s square is half of the rectangle. Here we can follow their discussion:

Måns: Mine is so smeary. Nobody can think about that it is so smeary.

Kalle: We can fix this so it will be the half.

Beatrice: It’ll be a square.

Stina: Do you know how to fold all pieces of papers. I can’t fold anything.

Måns: You can learn how to fold if you know how to fold.

Kalle: The fundamental form to fold frogs, but I can’t, they don’t jump like this.

Stina: I can fold aeroplanes.

Here you can see pupils being in an every-day and distanced discourse. They try to follow the teacher’s goal to prove but they got into another discourse.

In another assignment of a problem-solving character about decimals the pupils first had to work with an every-day picture as a point of departure and their talks are thus carried out in an every-day discourse. Some pupils do not arrive at the mathematical terms and an understanding of them. Other groups are given a formal assignment to be solved using a pocket calculator and they remain there, locked up in the system of signs and decimals. Yet another group of pupils draw lines together in order to understand the decimals and can accommodate the mathematical signs and words, which makes them involved in the discussion and solving of the assignment. They start to speak, think and write “Mathematish”.

I: Now I want you to explain why you think that this is right.

H: Nine is a whole number, it’s one smaller, only a whole number. 9,12 is nine whole and one tenth and two hundredths, I think, 9,2 then there is nine whole number and two tenths.

E: Nine is such a whole one. 9,12 there is a tenth smaller than two tenths so then 9,2 will be bigger than 9,12.

N: Nine is a whole number the second number in 9,12 is a hundredth and 9,2 the second is a tenth.

The connections are created between every-day references and mathematical concepts and expressions and it becomes easier for pupils to leave the idea of “doing”. Meaning has been attributed to mathematical concepts and signs and these have been created for defined ends. But the meaning can only be understood by those who are able to take part in a mathematical discourse.

By analyzing how teachers and pupils talk about mathematical phenomena in different situations I can use the concept of discourse to establish that connections are often not created between every-day concepts and their mathematical counterparts. If pupils cannot interact and thus form networks of concepts which assist them on their path to conscious mathematical thinking this becomes a major problem for them. Consequently teachers and pupils must develop their mathematical language in concord with every-day language.

Discourse analysis can thus be used as a tool where descriptions of pupils’ learning processes and understanding of mathematics can be made clear. I have displayed the results of my documented discussions and will place discourse in focus and further develop it as a means of establishing a direction.

Discussion

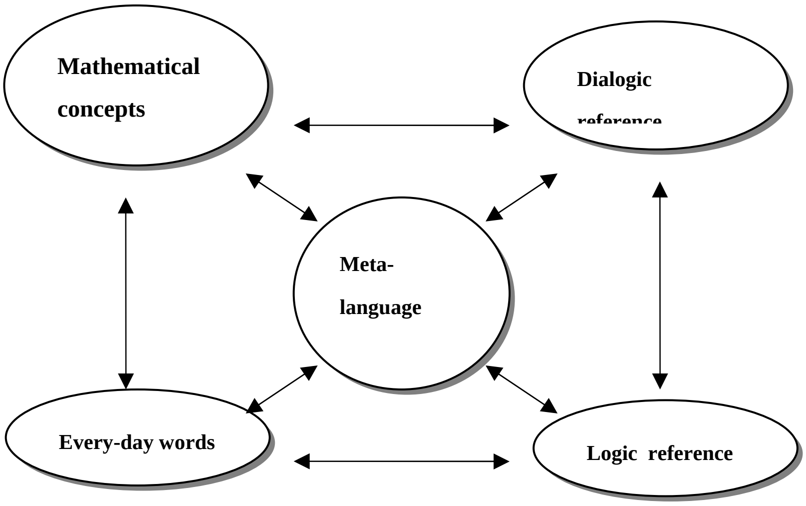
If the discourse is viewed as a distinct means of establishing the direction for teaching mathematics, it becomes the teacher’s task to bring to a conscious level the different ways pupils use for passing borders between different discourses, so that pupils become aware of the nature of mathematical concepts. A discourse is made up of artefacts and products created by mankind for specific ends and the language used can be understood only if the discourse itself is understood (Säljö, 2005). Teaching should invite pupils to become participants in a mathematical discourse.

The words *speak*, *think* and *write* can be viewed as parts of a discourse and when teachers and pupils apply them in the teaching and learning process, it can reinforce consciousness and participation in mathematical thinking. This could constitute the formative discourse. Furthermore, teachers and pupils must learn to realize what is changed when going from one discourse to another in mathematics. To be able to discern whether the discussion is carried out in an every-day or a mathematical discourse, to be able to recognize whether one is situated in a geometrical or an algebraic discourse and how the movement between registers manifests itself in mathematics is important knowledge for teachers, student teachers and pupils. When an individual speaks the way language is applied can develop qualitatively by the process of learning to value, scrutinize and put forth arguments in both every-day and mathematical discourses. In these, thinking is developed and by using linguistic and concrete artefacts in interplay thinking is further prompted. We can thus create a connection between every-day life and mathematics. Since mathematics started in a culture which used conventional signs and written language it has also developed texts and thus reading is a part of mathematics. The concepts of listening and reading should also be entered into the discourse, leading onto the concept of interpreting. In this perspective pupils will actively form and interpret their knowledge.

Discourse can be defined as a "way of speech" but I would prefer to widen the definition in so much that I view discourse as a network where teachers and pupils acquire knowledge by moving between and utilizing mathematical and every-day concepts, expressions and situations by talking, thinking, writing, listening and reading.

It has been my ambition to put the concept of discourse into perspective in the following manner. By adopting a discourse perspective we can direct attention to linguistic dimensions of mathematics teaching. It would also assist us in letting individual, silent calculation interact with a communicative aspect. By formulating and interpreting their mathematical knowledge pupils can acquire new knowledge. We will create a recognizing nearness through experience and distancing, fostered in a development and a familiarity with the system of mathematical signs. Through quality in the discussions which arise in a learning process we can develop the language concerned and thus improve understanding. In this context quality means that teachers and pupils use words, signs, concepts and situations in awareness of the specific discourse. We should also keep in mind that a mathematical discourse is something that develops over time.

Current research presents many images of the existent situation – "this is what it is like". My discourse perspective, however, focuses possible changes. I want to present a discourse theory which recognizes qualities in language and knowledge from both the every-day world and the mathematical sphere and in doing so clarifies both every-day and mathematical concepts. In this context quality means that we communicate around a concept, a sign, a reference and a situation by looking critically at it, putting forth arguments for and against, and eventually arriving at understanding what I take with me from this learning process. It is absolutely clear that the further our acquisition of new knowledge develops into an issue of learning to apply abstract and complex intellectual and practical tools, the more essential it becomes to engage in communicative practices. Thus we can learn how to apply and co-ordinate these tools, both linguistic and physical, with an outside world to reach new mathematical knowledge. Models and symbolic representations can be tested critically as regards their connections to the every-day world and other concepts as well as their logical consequence and explanatory value. The table below reinforces discourse as a theoretical and didactical concept.



Model describing the passing of borders between discourses.

By placing focus in learning processes on the concept of discourse our teachers and pupils can grow to master a meta-language for school mathematics. This will then constitute a specific and precise language in and about mathematics. Language is constructed in our actions and how we express ourselves using the appropriate signs. By putting forth arguments and making interpretations in a dialogical environment we can acquire knowledge as regards knowing when borders between discourses are passed, as well as regarding the interplay between thought and experience in mathematics.

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