USE OF WORDS – LANGUAGE-GAMES IN MATHEMATICS EDUCATION

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This article focuses on the introduction of new concepts in mathematics classrooms. A theoretical framework is presented which helps to analyse and to reflect on the processes of teaching and learning mathematical concepts. The framework is based on the theory of Ludwig Wittgenstein. His language-game model and especially its core, the primacy of the use of words, provide insight into the processes of giving meaning to words. The theoretical considerations are exemplified by the interpretation of a scene, in which students are introduced to the concepts of "perpendicular", "parallel" and "right angle".

INTRODUCTION

> "Mathematics education begins and proceeds in language, it advances and stumbles because of language, and its outcomes are often assessed in language." (Durkin and Shire, 1991, p. 3)

A lot of research has been done on communication in the mathematics classroom. Mathematical interactions have been analysed from many different perspectives (cf. Cazden, 1986). This text will focus on the teaching and learning of mathematical concepts in classroom communication. The importance of introducing mathematical concepts is underlined by the multitude of theories used for analysing concepts. In this paper only a few of them can be taken into account: de Saussure (1931), Peirce (CP 2.92) and Steinbring (2005).

By his concept of "language-game" Wittgenstein offers us an alternative view on the introduction of concepts in mathematics classrooms. His perspective has often been used to discuss problems concerning communication in the mathematics classroom (e.g., Bauersfeld, 1995; Schmidt, 1998). Sfard (2008) is using Wittgenstein's theory within her "commogitive model".

Wittgenstein presents considerations we can use to analyse language and especially the meaning of words. His theory of language-games and the construction of meaning will be considered in this paper, which presents first results of scientific research in progress. According to Wittgenstein, the expression of words does not constitute their meaning. Words have another function in the process of constructing knowledge. The main aim of the research is to analyse whether Wittgenstein's theory is useful for reconstructing and thus understanding communication. In spite of the multiple Wittgenstein references, I only know a few examples of using Wittgenstein's theory for analysing communication in the mathematics classroom (cf. the examples of Sfard 2008). More specific aims will be described in the course of this article. The core of the theory, the primacy of the use of words, will be exemplified.

USING WORDS IN LANGUAGE-GAMES

In his later philosophy (cf. the "philosophical investigations" and the "remarks on the foundation of mathematics") Wittgenstein describes a pragmatic theory of language and meaning. He denies every fixed relation between language and objects. Also Wittgenstein is no longer searching for anything, which could be taken as something basically shared by all linguistic acts. Language is not an objective mediator between human beings and objects given. Nevertheless, he considers knowledge – and thus mathematical knowledge – not to be transmitted objectively:

> "Language is a universal medium – thus it is impossible to describe one's own language from outside: We are always and inevitable within our own language […]. Knowledge appears as knowing, and knowing is always performed in language games. Language as languaging or playing a language game is equal to constituting meaning and, thus, constituting objects. There are no objects without meaning, and meaning is constituted by a special use of language within a respective language game" (Schmidt, 1998, p. 390).

For Wittgenstein the construction of knowledge takes place by playing language-games. The term "game" does not imply an option for those who are involved. We cannot choose in the first place whether we want to play the game or not. The problem is that Wittgenstein does not explain in detail what he means when speaking of "language-games". As we will see, this is not because he does not care. Rather it is due to his theory of giving meaning to words.

Words have neither a consistent nor an objective meaning. In different language-games various meanings of a word can occur. Following Wittgenstein there is no direct transformation from a word to its meaning: "[…] experiencing a word, we also speak of 'the meaning' and of 'meaning it.' […] Call it a dream" (Wittgenstein, 1958, p. 216). Moreover, it is the use of a word which determines its meaning:

> "For a large class of cases – though not for all – in which we employ the word 'meaning' it can be defined thus: the meaning of a word is its use in the language" (Wittgenstein, PI, §43).

The term "use" is not limited to the application of words (e.g., in order to solve problems). If we exemplify a word, we also make use of it. One research-guiding problem will be to identify different forms of uses of mathematical words.

A word does not mirror objects and the meaning of a word cannot be observed while looking at its association with a specific object. The meaning of a word is nothing but the role it is playing in the specific language-game and accordingly can be observed only by looking at the use of words. This central thesis might be the reason why Wittgenstein does not define what he means using the term "language-game". He stays consistent: He exemplifies the words he makes use of [1]. Language-games can be different in character. So Wittgenstein (PI, §23) presents the following examples among others:

- "Giving orders and obeying them",

- “Forming and testing a hypothesis” and
- “Solving a problem in practical arithmetic”.

These examples may indicate that language-games are little “passages” or specific situations in our daily communication, but Wittgenstein also presents a larger field:

> “I shall also call the whole, consisting of language and the actions into which it is woven, the ‘language-game’.” (Wittgenstein, PI, §7)

Language is constituted by a “multiplicity” (Wittgenstein, PI, §23) of language-games. And all these language-games bear a temporal dynamic:

> “And this multiplicity is not something fixed, given once for all; but new types of language, new language-games, as we may say, come into existence, and others become obsolete and get forgotten. (We can get a *rough picture* of this from the changes in mathematics.)” (Wittgenstein, PI, §23)

The temporal dynamic indicates once more that there is no specific meaning for words fixed forever. Changing the meaning of a word is accompanied by a change of the language-game. Learning also means to realize changing meanings of words. Learning includes learning how to play different language-games. Thus, learning implies partaking in changing and new language-games.

USING WITTGENSTEIN

In mathematics education there has been a lot of research to consider and to analyse concepts and how students get used to them. Some work (e.g., Duval, 2006) is based on de Saussure’s (1931) relation between signifier and signified (fig. 1). The theory of de Saussure provides a subject-object dualism and thus implies some problems:

> “If there would be a correspondence between language and reality, then, surely, one could arrive at true verbal statements about the world. Descriptions (and teaching), then, would become a case only of an adequate selecting and of providing for sufficient precision of the verbal means (denotations), as well as an adequate fit of these means with the object” (Bauersfeld, 1995, p. 277).

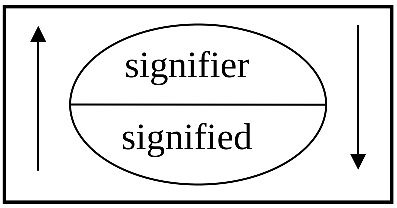

Figure 1: De Saussure’s (1931) relation between signifier and signified

Peirce (CP 2.92) offered a more detailed framework. His triadic relation between the sign, its object and its interpretant (fig. 2) has been used to analyse and to describe verbal or non-verbal interaction (e.g., Hoffmann & Roth, 2004; Presmeg, 2001; Sáenz-Ludow, 2006; Schreiber, 2004). The reconstruction of classroom interaction based on this framework has to deal with the difficulty that it is problematic to determine the object to which the sign is related. Contrarily, Wittgenstein's theory is a more pragmatic one. He does not regard any ontology of a sign. According to his theory words only get their meaning by their use and do not transport any given meaning. There is no fixed relation between words and objects.

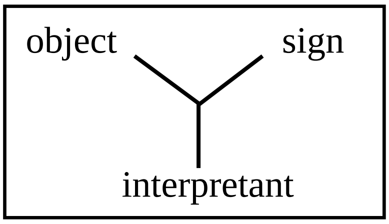

Figure 2: Peirce's triad

By his epistemological triangle (fig. 3) Steinbring (2006) provides a way to analyse static moments in the process of giving meaning to words. He presents a triadic relation between "sign/symbol", "object/reference context" and "concept":

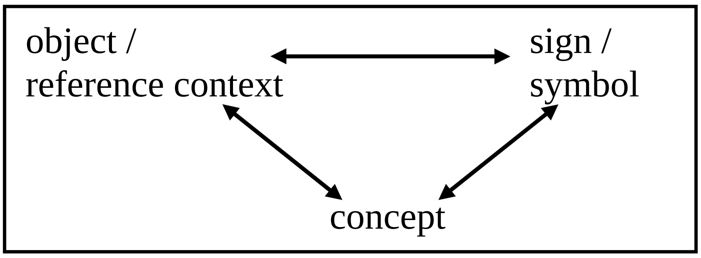

Figure 3: Steinbring's epistemological triangle (2006, p. 135)

The importance of the context can also be observed in Wittgenstein's writings, as he is considering the use of a word in the specific language-game. And language-games are depending on the situation:

> "Here the term 'language-game' is meant to bring out into prominence the fact that the speaking of language is part of an activity, or of a form of life." (PI, §23)

Wittgenstein points out that there is no direct transport of meaning from the teacher to the student, nor a direct understanding. We only can analyse the meaning of a word by looking at the use of that word in a specific language-game, which is at the same time influenced by other language-games. If we take a look at the language-game "mathematics education", we are also confronted with influences of every-day language-games of the students (and the teacher) and, all the more, of the rather mathematical language-games the teacher is able to participate in with mathematics experts outside of the classroom.

Words can be used in more than one language-game and thus each word can exhibit different meanings. If the teacher is going to introduce a concept in mathematics education, the children might immediately associate some meaning to it – due to the use of that word in another language-game the student took part in. This might be an every-day language-game or a language-game of mathematics education of a previous era (e.g., subtraction means to remove things, which does not work for negative numbers).

Words could be used in more than just one way. Accordingly, they can convey different meanings or meanings, which cannot be grasped only by knowing one form of their use. Thus, the use of a word in a specific situation must not lead to the whole range of possible meanings. Also, some concepts are restricted or expanded in the course of mathematics education (e.g., the concept of numbers). Therefore, this study is going to focus on the introduction of new concepts in the mathematics classroom and their development during following lessons. Some research-guiding questions are: How do students make use of words? What might be the meaning of a word for them? How do teachers influence the play of another language-game?

METHODOLOGY

The empirical data emerged from classroom observations in different grades (1-10) in Germany. Classroom communication has been videotaped by teacher students acting as researchers. The videographed units comprised 4-8 lessons of 45 minutes each. The teacher students were observers; they were told to exert no influence on the classroom communication and on the teachers' way to introduce the concepts. Altogether eight classes were visited.

The qualitative interpretation of the classroom communication is founded on an ethnomethodological and interactionist point of view (cf. Voigt, 1984; Meyer, 2007). Symbolic interactionism and ethnomethodology build the theoretical framework which is going to be combined with the concepts of "language-game" and "use".

According to Wittgenstein we should not ask: What is the meaning of a word? Rather we should analyse what kind of meaning a word gets in the classroom. Therefore, we have to analyse social processes. Thus, we have to follow the ethnomethodological premise: The explication of meaning is the constitution of meaning.

Analysing students' languaging for mathematical concepts, the development and the alteration of meaning by the use of the according words, we are able to reconstruct the social learning in the mathematics classroom [2].

The main aim of this study is to get a deeper insight into the processes of giving meaning to words in the mathematics classroom. Therefore, alternative ways of introducing concepts are going to be considered. Comparing possible and real language-games can help to understand the special characteristics of the actual played language-game.

THE USE OF WORDS IN CLASSROOM COMMUNICATION

The following scene emerged from a 4th grade classroom in Germany (students aged from 9 to 10 years). It is the first time that these students get in contact with the concepts of "parallel", "perpendicular" and "right angle" in this mathematics class.

The teacher starts the lessons by writing the words on the blackboard and asking the students to associate anything coming to their mind about these words. Afterwards a painting by Mondrian (cf. fig. 4) is presented on the blackboard [3].

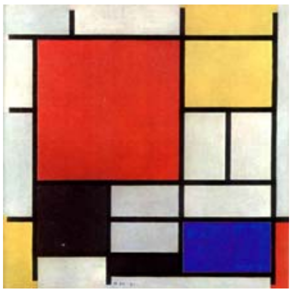

Figure 4: Painting by Mondrian on the blackboard

Teacher: Why do I fix such a picture on the blackboard? And why are these concepts written down on the blackboard? I have a reason to do so. Jonathan, it is your turn.

Jonathan: Because the painter has done everything in parallel, perpendicular and in right angles.

Teacher: You are right. You seem to know what parallel, perpendicular and right angle means. Maybe you can show it to us on the picture.

Jonathan: Perpendicular is this here (points first at a vertical, afterwards at a horizontal line). Parallel is this here (points at two vertical lines). A right angle is this (pursues two lines he former would have called perpendicular).

By pointing to different things on the blackboard, Jonathan makes use of the words “perpendicular”, “parallel” and “right angle”. He must have been in contact with practices of using them and thus with meanings of these words in a language-game outside this classroom. In this situation the words get a meaning by him pointing at something. This use can be described as an *exemplaric use*: An example is used to show the meaning of a word.

The use Jonathan makes of the words need not imply that those words could also be used in different ways, but *this* use and respectively *this* meaning get established in this classroom communication.

The teacher does not have any further questions. The teacher accepts the use of the words Jonathan must have known from another language-game. Thus, it seems that the exemplaric use is an acceptable one and that the meaning of the words is “taken-to-be-shared” in the classroom (cf. Voigt, 1998, pp. 203).

Certainly, in another language-game the meaning of the words “perpendicular”, “parallel” and “right angle” can be different. They can be defined by using other concepts. A right angle can be defined as an angle of 90 degrees. Also the word “right angle” can be used in coherence with Pythagoras’ theorem or in relation to the shortest distance of parallel lines. Perpendicular can be described by using the concept of “right angle”. All of these uses describe other language-games and not all of them can be played in a 4th grade classroom. Altogether, the words can have different uses and, thus, different meanings. In this classroom the words are used in order to represent things (cf. de Saussure’s model).

In the next few minutes the students had to create a mindmap, which should contain “something which can fit to the picture”. Then, afterwards “perpendicular” gets exemplified on the picture again. Now the classroom communication goes on with “parallel” and “right angle”:

Teacher:	Now we just have two problems: parallel and right angle.
Sebastian:	Right angle is easy (holds the set square at the blackboard).
Teacher:	Can you show it here (points at two lines on the painting by Mondrian which have been used to show “perpendicular”). (After five seconds) Doris just say it. Wait! Before you go ahead, let –
Doris:	You can make out four right angles out of it.
Teacher:	This is the sign for the right angle (draws ⊾ on the blackboard). Maybe you can just draw it into the picture? (After three seconds) You can also choose another one.
Doris:	John
Teacher:	John and Tim come here. Doris said you would be able to find four right angles.
John:	You two, me two (speaks to Tim while pointing at two lines).
Teacher:	That is not right. No. Doris, show him were they are.
John:	There is a right angle.
Teacher:	Ah, yes!

The class is going to consider the last two “problems” (parallel and right angle), which have not been exemplified a second time. Doris identifies four right angles on those lines, which had been used before in order to show the meaning of the word “perpendicular”. John shows an example for a right angle. Again we can speak of an *exemplaric use*. The meaning of the word “right angle” is connected to the examples on the blackboard. Now and again, it seems that the meaning of “right angle” is “taken-to-be-shared”, but the students do not yet express characteristics of right angles, they only have examples.

Now the scene is going on:

Tim:	Ah, this corner which is coming from the right side (marks the angle with the teachers’ sign)
Teacher:	Correct! Just make it a little bit thicker, so that the other ones can see it.

Tim: This is a left angle. (points at the opposite side of the vertical line)

Teacher: No!

Lisa: That is always a right angle.

Tim recognizes the examples as examples for the use of the word “right angle”. He explains why John’s example can be called “a right angle”. Thus, he abstracts from the concrete example and presents a use of the word “right angle” by a kind of definition: The word “right angle” can be used, if a line for the angle comes from the right side. Tim tries to give an *explicit-definitional use* (cf. Winter, 1983) of the word: The student describes a general characteristic when and how the word “right angle” has to be used. He relates the word “right angle“ to other words. Contrarily to the former use of the word “right angle“, Tim uses another ethnomethod to constitute meaning.

The concept of the word “left angle” is used by an implicit reference. It is implicit, because the pair of concepts “left-right” indicates that an orientation in space is considered – a relation between observer and object. Thus, the word “left angle” gets an *implicit-definitional use*. The exemplaric use Tim makes of the word “left angle” can be seen as a test of his proposal. It is a probable consequence of his first definition. In other words: It is a hypothetic-deductive approach of verification (cf. Meyer, 2008).

Tim’s use of the word “right angle” can be explained only because there is use of the word “right” in common practice. Here the word “right” can be used to show a certain relation between observer and object. So Tim was able to combine the two uses of the words “right” and “angle” to establish a constructive meaning of the conglomerated word “right angle”. The comment of the teacher harshly shows that the new language-game is not acceptable.

Tim’s use shows that the former meaning of the word “right-angle“ only seemed(!) to be “taken-as-shared”. It has not been shared. Tim has been trying to give a theoretical fixation of the concept. The language-game he initiated is not an acceptable one. Lisa does not take part in the new language-game. She seems to play the former game and to explicate a routine: We need to have more examples to grasp the meaning of the word “right angle”.

FINAL REMARKS

The episode shows that de Saussure’s model is not sufficient to analyse classroom communication. Mathematical concepts are in need of a fixation by other concepts (a theoretical fixation). An empirical way can be used to introduce words, but the language-game has to change afterwards. In this scene a student initiates another language-game, which is condensing in (not acceptable) theorems.

The use of Wittgenstein’s theory shows that concepts can be observed by looking at the way teacher and students make use of the words at hand in the specific language-game. In this scene we have seen an *exemplaric*, an *explicit-definitional* and an *implicit-definitional* use. The exemplaric use consists of pointing at examples to illustrate the words. The explicit-definitional use consists in giving an explanation for the word in relation to other concepts. Thus, it provides a deeper insight in mathematical coherences: Characteristics of the underlying concept get expressed. The concept gets a general character, not being linked to special examples any more. An explicit-definitional use is also in need of a deeper mathematical insight, as it has to be known what counts as a definition. The implicit-definitional use in this scene requires a common pair of concepts ("left-right") and an explicit-definitional use of the other word.

Wittgenstein's theory itself is not a theory of interpretation. Rather he presents a theoretical framework, which can be used on top of a theory of interpretation. Symbolic interactionism and ethnomethodology fit to Wittgenstein's considerations of social processes in languaging. Future analyses have to show the fruitfulness of this framework.

NOTES

1. " 'The meaning of a word is what is explained by the explanation of the meaning.' I.e.: if you want to understand the use of the word 'meaning', look for what are called 'explanations of meaning'." (Wittgenstein, PI, §560).

2. As proposed by Bauersfeld (1995) I will speak of "languaging" to accentuate the connotation of language use.

3. Many thanks to Johannes Doroschewski and Philipp Heidgen for the video. The translation has been done and simplified by the author of this article. The original transcript will be sent on demand.

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