A TEACHER'S ROLE IN WHOLE CLASS MATHEMATICAL DISCUSSION: FACILITATOR OF PERFORMANCE ETIQUETTE?

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In the improvisation that occurs in a jazz ensemble, a soloist rarely develops a completely new idea but, instead, elaborates and builds on the previous player's input. From an emergent perspective, classroom mathematical practice is akin to such improvisation. How this might happen in a whole-class situation is unclear. In this paper, a description is given of a whole-class discussion that took an unplanned trajectory. The teacher did not impose a particular structure on the lesson but focused pupils' attention on productive mathematical ideas that emerged from the group. In the concluding discussion, it will be shown that the improvisation metaphor, while useful for describing mathematics as a socio-cultural activity, may have a different application in a whole-class situation than in small group settings.

INTRODUCTION

Although plenary sessions are common to mathematics lessons, they are often characterized by traditional approaches that endorse the position of mathematics as a kind of received knowledge and the teacher as sole validator of students' contributions (See, for example, Boaler, 2002; Cobb, Wood, Yackel, & McNeal, 1992) While research shows that whole-class discussion can be fertile ground for higher-order mathematical thinking (Cobb et al., 1992; O'Connor, 2001), the fast pace with which it is usually associated means that there is little scope for students to make comments and build on each others' mathematical ideas (Hodgen, 2007). One consequence of this is that students become disengaged from the subject, perceiving it to be one in which they have little opportunity for participation (Boaler, 2002). However, the orchestration of inquiry-based discussion in mathematics is challenging for teachers. Sherin (2002) alludes to two key tensions whereby teachers, on the one hand, are expected to encourage students to share ideas and, on the other, have to ensure that the lesson is mathematically productive.

In this paper the improvisation metaphor is used to show how a teacher and her pupils co-constructed new mathematical ideas in the context of a whole-class discussion in a primary school. In particular, attention is paid to the way provision can be made for different levels of understanding within the class. In the concluding discussion, reference will be made to limitations of some tools that are used to analyse such research.

THE IMPROVISATION METAPHOR

According to Lakoff and Johnson (1980), metaphors not only help us to understand one kind of thing in terms of another but they can also create a reality and thus act as guides for future action. In relation to the teaching of mathematics the improvisation metaphor is one that serves both of these purposes. Consistent with a view of mathematics as a socially and culturally situated activity, the point of reference in mathematics education is the classroom mathematical practice, a perspective that has been described by Cobb (2000) as *emergent*. Sawyer (2004) maintains that this perspective implies that teaching must be improvisational and 'that the most effective learning results when the classroom proceeds in an open, improvisational fashion, as children are allowed to experiment, interact, and participate in the collaborative construction of their own knowledge' (p.14).

In theatrical improvisation, a group of actors creates a performance without using a script. Because it is characterized by a high level of unpredictability, the performance has associated with it what Sawyer describes as a 'moment-to-moment contingency' (Sawyer, 2006: p.153). As the actors play their parts, several potential possibilities are brought into the frame. What emerges is not decided by any one person but rather is a phenomenon that is produced by the group. In jazz improvisation, each soloist is assigned a number of measures to play before the next soloist takes over. Due to the rapidity of the transition, a player rarely develops a completely new idea but rather responds to and builds on the previous player's input (Berliner, 1994).

Sawyer (2004) maintains that like the improvisation that occurs in theatre or in a jazz ensemble, creative teaching is both emergent and collaborative. It is emergent because the outcome cannot be predicted in advance and it is collaborative because the outcome is determined not by any one individual but by the participants of the group. Martin, Towers and Pirie (2006) used the improvisational lens to analyse collective mathematical understanding. They describe collective mathematical understanding as the kind of learning and understandings that occur when a group of any size work together on a mathematical activity. Central to their analysis is the idea of *co-acting* which they define as

> …a process through which mathematical ideas and actions, initially stemming from an individual learner, become taken up, built on, developed, reworked, and elaborated by others, and thus emerge as shared understandings for and across the group, rather than remaining located within any one individual. (p.156)

They make a distinction between co-actions and interactions. While in interactions there is an emphasis on reciprocity and mutuality, co-actions concern actions that are dependant and contingent upon the actions of other members of the group (Towers & Martin, 2006). Through this co-acting, an understanding emerges that is the property of the group rather than any individual. It is not that all individuals bring the same understandings to the scene but rather that individual contributions will result in something greater than the sum of the parts. Neither does it preclude an individual making his or her own personal advancements.

In a more fine-grained analysis of the improvisational metaphor, Martin and Towers (2007) have introduced the notion of performance etiquette. In jazz terms this refers to a situation where players drop their own ideas in deference to a better (in the view of the collective) idea if that works. It means that due attention and equal status have to be given to all players' ideas and intuitions. According to Martin and Towers, '(in) mathematics, 'better' is likely to be defined as a mathematical idea, meriting the attention of the group, which appears to advance them towards the solution to the problem' (p.202). Although much of the work done by Martin et al. concerns small groups there is evidence that the metaphor is also applicable to whole class discussion (See, for example, Dooley, 2007). King (2001) contends that in lessons where students and teachers co-create classroom discourse, 'one can view students as other participants in [the] improvisation, following the direction of the lead improviser, the teacher'(p.11). She proposes that the teacher is rather like the soloist who must modulate her performance to her instrumentalists and audience. There is some danger that this analogy leads to the teacher's role being perceived as centre of (as opposed to central to) the learning process. Sherin (2002) suggests that, in order to achieve a satisfactory balance between process and content, the teacher engages in *filtering* by which is meant a narrowing of ideas generated by students so that so that there is a focus on mathematical content. An implication for whole class discussion is that the teacher is more facilitator of group etiquette than lead improviser. This idea is pursued further in the account below.

BACKGROUND

The aim of my research is to investigate the factors that contribute to the development of mathematical insight by primary school pupils. The methodology is that of 'teaching experiment' which was developed by Cobb (2000) in the context of the emergent perspective and in which students' mathematical development is analysed in the social context of the classroom. For a period of six months, I taught mathematics to a class of thirty-one pupils (seven girls and twenty-four boys) aged 9 - 10 years. The school is situated in Ireland in an area of middle socio-economic status. Although I taught the lessons, the class teacher played an active role as co-researcher, advising on the suitability of lesson content, clarifying any confusion that arose in whole class discussions, working with pupils during group work and making observations in post lesson discussions. Many lessons took place over two or three consecutive days, each period lasting forty to fifty minutes. I visited the class on a total of twenty-seven occasions. All phases of the lesson were audiotaped. When children were working in pairs, audio tape recorders were distributed around the room. Each pupil maintained a reflective diary. Follow-up interviews were held with students who had shown some evidence of reaching new understandings over the course of a lesson.

Forman and Ansell (2001) contend that analysis based on isolation and coding of individual turns is too limited to bridge the individual and social. Therefore, I conducted ethnographic microanalysis, which according to Erickson (1992) is especially appropriate when the character of events unfolds moment by moment. The approach adopted was top-down starting with the molar units (lessons) and moving to progressively smaller fragments. I transcribed all lessons and isolated those in which pupils showed evidence of constructing new mathematical insight. Thereafter I identified constituent parts of the lesson, starting with major events and moving progressively to the actions of individuals. A comparative analysis of lessons was also undertaken.

The lesson described here took place on a third consecutive visit to the class during a week of the Spring term. On the previous two days, the pupils had been working on a lesson entitled ‘Chess’, the object of which had to find the minimum number of games that could be played by participants in a competition where each competitor had to play all other players. At the conclusion of this lesson some pupils had found the answer for one hundred players (i.e., the sum of 1 - 99) by using a calculator while others had latched onto the discovery made by one pupil, David[1] that the solution could be found ‘by multiplying by the number less than it and halving it’ ((100 x 99) ÷2). It was my intention on the third day to begin a new lesson but first told the story of Gauss (the mathematician who, as a boy, had amazed his teacher by his rapid calculation of the sum of integers from 1 to 100) in order to see if the pupils would make any connections between it and the chess problem. I expected that talk on this problem would last no longer than five or ten minutes. However, a rich discussion followed in which I truly had to improvise. Although this lesson is not being promoted as exemplary, I learnt from it something about the power of ‘letting go’ and ways in which group etiquette might be facilitated.

The focus of this paper is on the discussion that took place after I first related the story of Gauss. Although space does not allow the full transcript to be presented, an effort is made to give as full as possible a sense of the lesson trajectory (a problem described by O'Connor (2001: p.144) as ‘the competing requirements of data reduction and interpretive explicitness’). The following transcript conventions are used: T.D.: the researcher/teacher (myself); Ch: a child whose name I was unable to identify in recordings;…: a hesitation or short pause; […]: a pause longer than three seconds; ( ): inaudible speech; [   ]: lines omitted from transcript because they are extraneous to the substantive content of the lesson.

THE IMPROVISATIONAL CREATION

On telling the story, some pupils suggested that Gauss may have found his solution by adding fifty and fifty or five twenties, considering addends of rather than the sum to a hundred. When I focused their attention on the problem conditions, Barry had this idea:

18	Barry:	Eh, you add up all the numbers that are in ten like one, two, three, four, five, six, seven, eight, nine, ten…

[1] Pseudonyms are used throughout the paper.

19	T.D.:	Hmm.
20	Barry:	and then multiply by ten.
21	T.D.:	Ok, so you would add up as far as ten and then multiply the answer by ten?
22	Barry:	Or nine, I'm not really sure.
23	T.D.:	Ok, why do you think it might be nine?
24	Barry:	Eh, because you have already counted up to ten and it's ten tens in a hundred.

Here he was making an assumption that the sum of numbers between 1 and 10 would be the same for all decades. Brenda then asked if she could check the answer on the calculator which was interesting given that she had thus correctly established the solution for forty players in the Chess activity.

Anne and Fiona then built on the idea proposed by Barry:

48	Anne:	I think it's thirty multiplied by ten.
49	T.D.:	Sorry?
50	Anne:	Thirty multiplied by ten.
51	T.D.:	Thirty multiplied by ten, why would you say it's thirty? [ ]
54	Anne:	Because if you add from one up to ten it's thirty.
55	T.D.:	How do you know if you add one up to ten it's thirty?
56	Anne:	If you add one to five, that's fifteen…
57	T.D.:	Hm, hm
58	Anne:	and then fifteen and fifteen is thirty so then if you multiply that by ten.
59	T.D.:	Ok, possibly that would get it for you. Fiona?
60	Fiona:	Well, could you em, oh, em, do, eh, you could do one plus two and up to fifty and then double it...

I chose not to correct misconceptions at this point but wrote the suggestions on the blackboard. This proved a good judgement in this instance because a short while later two pupils commented on Anne's input:

66	Alan:	Em, well, I don't think Anne's one is right.
67	T.D.:	Why?
68	Alan:	Cos ninety-nine plus ninety-eight plus ninety-seven plus ninety-six to ninety would be around over five hundred and when…
69	Ch:	Oh!
70	T.D.:	Ok, [ ] you are thinking ninety plus ninety one plus ninety two plus ninety three would give you approximately how much?
71	Alan:	Em, I don't know.
72	T.D.:	But it's…
73	Alan:	But it would probably be over five hundred.
74	T.D.:	It would be over five hundred, so in that section, if you are thinking about all those numbers there that would give you about, even just adding ninety to a hundred so you are thinking that would give you about five hundred. [ ]. Barry?

75	Barry:	Eh, well, I disagree with Anne as well because, eh, I counted, I counted up all the numbers up to ten and I got fifty-five.

Enda then said that multiplying five by twenty or adding fifty plus fifty (both ideas were written on the blackboard) didn't 'actually have much to do with this'. Anne now corrected her earlier idea:

91	Anne:	I don't think…my answer wouldn't work.
92	T.D.:	What were you thinking your answer was?
93	Anne:	I thought it would be thirty multiplied by a hundred.
94	T.D.:	Why would it not work?
95	Anne:	Em, because you would have to, cos I did eh one plus two plus three plus four plus five and then em I got fifteen and then I added fifteen and fifteen equals thirty but then it would be more because you would have to add six, seven and that.

Anne seemed to have reached a new understanding about the addition of a series of numbers. It is possible that she began to reflect on her thinking because Barry and Alan disagreed with it. Colin then arrived at a new approach to the problem:

97	Colin:	It could like eh add the, say you could have ninety-nine, add the closest and the furthest and then the second closest and the second furthest.
98	T.D.:	So give me an idea what you are talking about now. Tell me, elaborate a bit on that. [ ]
101	Colin:	Eh if it was ninety-nine, you add one, if it was ninety-eight you add two, if it was…
102	T.D.:	Ok, so you are thinking - very interesting because that's - you could have ninety-nine plus one, go on!
103	Colin:	Ninety-eight plus two, ninety-seven plus three, ninety-six plus four, eh, ninety-five plus five, ninety-six or ninety-four plus six (*teacher records on blackboard*)…
104	T.D.:	Ok, so what's that giving you, why are you putting those numbers together?
105	Colin:	They all go up to a hundred.
106	T.D.:	So what's that telling you then, what do you think it might be, have you any idea what the answer might be?
107	Colin:	Eh, no.
108	T.D.:	Do you see what Colin is doing there? He is matching up numbers, he is taking the numbers at the very beginning and he is matching them up with the numbers at the end.

I was quite excited when I heard this input as this was the method used by Gauss as a young boy, hence my remark, on line 102, 'very interesting because..'. I wrote his suggestion on the blackboard but also 'revoiced' his input (line 108), a teacher strategy that serves to repeat or expand a student's explanation for the rest of the class (Forman & Ansell, 2001; O'Connor, 2001). Enda then proposed a different way of grouping the numbers. However, I did not grasp his idea:

113 Enda: Eh, well, I think one possible way it would probably would be just as hard, it would be harder than one plus two plus three, it's probably not going to help us, what I was going to say is eh adding…when adding ninety plus ninety-one plus ninety-two and all that sort of stuff…

114 T.D.: Hm, hm.

115 Enda: It's the same every time, you would just, all you would probably, eh, you would probably need to go backwards and just take way ten from the answer above every time. That would ( ) if you took away ten from the answer every time.

116 T.D.: Hm, hm

117 Enda: So add up the numbers going from a hundred backwards. [ ]

120 T.D.: If you went a hundred plus ninety-nine plus ninety-eight plus ninety seven…

121 Enda: Yeah

122 T.D.: all the way back as far as one, would you still get the same answer?

123 Enda: The same answer, even though it would just be easier to do it backwards with that way em you just need to take ten away from it every time. If you were on ninety, if you got a hundred back to ninety and you were on eighty, just take ten away from the answer above.

Enda had found an interesting solution method, that is, adding from 100 to 91 and then finding the solution for the sum from 90 to 81 by subtracting ten. In fact this is a very viable method (if one hundred is subtracted each time). I had assumed he was talking about commencing the addition from a hundred rather than one. It is very possible that I did not comprehend his approach because it was one I had never considered. I did, however, ask him to pursue his idea in his diary.

Liam then made another observation about Colin's list:

135 Liam: I don't think like if you go back to Colin's way…if you go back, you wouldn't be able to do it, if you go back to one then you might double it, the whole thing.

136 T.D.: Sorry?

137 Liam: If you go all the way to one, then you double the whole thing.

Neal then suggested that the list should terminate at 50 + 50 and I urged pupils to think about the number of 'hundreds' there might be. Anne then proposed that the answer would be a thousand and this led to an interesting contribution by Brenda:

166 Anne: I think the answer would be a thousand.

167 T.D.: You think it's going to be a thousand. Do you agree with Anne that it's about a thousand? Brenda?

168 Brenda: Eh, no cos when I em added up forty for it and, em, I got more than a thousand.

This is the first time in the lesson that a direct reference has been made to the chess activity. Fiona confirmed that the answer for 40 children (i.e., the sum from 1 to 39 although this was not as yet clear) was 780. Anne picked up on this idea:

183 Anne: Well, in the one we did yesterday, when the number of children was a hundred, then the number of games was four thousand, nine hundred and fifty so that there would be the answer.

I wrote 4950 on the blackboard as one other possibility. Hugh however noticed the error:

197 Hugh: I think it would be, em, five thousand, nine hundred and fifty.

198 T.D.: Where are you getting that from?

199 Hugh: Em, because eh yesterday we didn't add on the hundred.

200 T.D.: Ok […] so

201 Hugh: So then it would be …five thousand…and fifty.

Liam now saw that 50 + 50 should not be included in the list:

209 Liam: Well on the last one in Colin's one you have to do a triple sum kind of ( ) because it would be forty nine plus fifty one and then add fifty on to it.

David confirmed that the solution was 5050 and explained his reasoning as follows:

213 David: Em, well if you do Colin's way and then, em, you get, em fifty ( ) and then when you get to forty nine plus fifty one and you have to add the fifty on and that gives you about five thousand and fifty.

At this point in the discussion the class teacher indicated that a small group of pupils had taken out their diaries and were working on solution methods in them. In particular, Declan seemed to be very keen to complete the listing suggested by Colin. The pupils embarked on paired/individual work during which the class teacher sat with Declan. In the plenary session that was held at the conclusion of the lesson, Fiona and Clare discussed possible answers for the sum of numbers up to 200 (they proposed 5050 x 2). Some pupils spoke about the solution they found on the calculator. Declan described how he solved the problem using Colin's method. Miles began to consider that the answer might be obtained by multiplying a hundred by a hundred and then halving it 'to take way the pluses that you add on to get one hundred'. David, however, did not use the formula he had found for the chess problem to add the numbers from 1 to 100.

DISCUSSION

There is evidence that co-acting took place in this lesson. For example, in the early part of the lesson, Fiona and Anne picked up on Barry's idea of adding a section of numbers and applying proportional reasoning (albeit incorrectly). Later Anne reconsidered her reasoning on the basis of input by Alan and Barry. Colin's idea may well have emerged because of the discussion around addition of numbers between 1 - 10 and 90 - 100 (see lines 68 and 75). Enda's method could be an elaboration of that proposed by Colin. Brenda made the explicit connection with the previous day's lesson which prompted solutions by Anne and Hugh. However, the co-acting is not as linear as might be the case in small group discussion. Rather there is a weaving in and out of ideas. Lines 135 and 209, where Liam broke the flow of conversation to transform Colin's listing, are instances of this. It also seemed that some students who made no contribution to the dialogue reported above were nonetheless actively engaged. For example, Declan, a student who is not confident about his mathematical ability, pursued Colin's idea with great zeal. An implication of this is that tools used to analyse whole class discussion must extend to include those who are silent but participating in the enquiry.

O'Connor (2001) ponders the difficulties of looking objectively at transcriptions and attempting to discern the motives of the teacher in taking certain actions. As the researcher/teacher on this lesson, I am in a position to say, at least to some extent, why I took certain courses of action. A primary concern was keeping things, to continue with the jazz metaphor 'in the groove', for the group while at the same time respecting the input of individuals. Enda's idea (lines 115 and 123) did not become part of the collective because I did not understand it. Recourse to a diary allowed him to pursue his own investigation, however. My position in this lesson was not that of lead improviser because the lesson took an unexpected trajectory, but I feel that I facilitated group etiquette by drawing attention to ideas that would lead to solution to the problem.

With regard to the future direction of this research, the ways in which whole class discussion can impede or facilitate pupils' mathematical insight will be further analysed. In particular attention will be paid to the ways in which the making public of ideas by writing them on the blackboard and the revoicing of pupils' input stimulates the filtering process.

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