# ICONICITY, OBJECTIFICATION, AND THE MATH BEHIND THE MEASURING TAPE: AN EXAMPLE FROM PIPE-TRADES TRAINING [1]

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*This paper examines an adult student's efforts as he works intensely, with the help of the researcher, to make sense of the fraction patterns on a measuring tape marked in inches. The multi-semiotic analysis of this encounter is framed using Radford's Theory of Knowledge Objectification. From this socio-cultural perspective, mathematics learning involves the social and semiotically mediated process of objectification, i.e. a process in which one becomes progressively aware and conversant, through one's own actions and interpretations, of the cultural logic of mathematical objects. This paper contributes to Radford's notion of iconicity by showing, through fine-grained analysis, relevant aspects of its dynamics as well as by calling attention to a form of iconicity that, to my knowledge, has not been reported elsewhere.*

## INTRODUCTION AND THEORETICAL FRAMEWORK

This paper is based upon a small part of an impromptu tutorial session involving a pre-apprentice in the pipe-trades with the researcher serving as mathematics tutor. It is part of a larger case study that focuses on the manner in which the pre-apprentice attempts to make sense of, and become fluent with, the mathematics embedded in a measuring tape marked in feet and inches–an essential skill for the pre-apprentice's chosen vocation. While Canada has officially adopted the metric system and most students study measurement exclusively using metric units in their mathematics courses in elementary and secondary school, the use of imperial units of linear measure (e.g. feet and inches) remains common practice in the construction trades. Consequently, is it not unusual to find students at the start of workplace training in the construction trades who struggle with the cultural practice of measuring lengths in fractions of an inch using a measuring tape.

The study draws upon Radford's (2002, 2008a, 2008b) socio-cultural theory of knowledge objectification (TO) to examine the manner in which the pre-apprentice begins to notice the mathematics embedded within the inscriptions on a measuring tape. In this theory, learning is conceptualized as the active and creative acquisition of historically constituted forms of thinking. Such an acquisition is thematized as a problem of *objectification*, that is, as a problem of becoming conscious of, and

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critically conversant with, the cultural-historical logic with which mathematical and other objects have been endowed. One of the aspects that makes the idea of objectification distinctive is the close relationship that it bears with the Vygotskian concept of consciousness and the mediated nature of it (Vygotsky, 1979, also Leont'ev, 1978). Consciousness is formed through encounters with other voices and the historical intelligence embodied in artifacts and signs with which we mediate our own actions and reflections. Within this context the efforts that the pipe-trade pre-apprentice undertakes to make sense of the mathematics of a measuring tape are seen as a process of objectification. One of the questions is to investigate how the cultural meaning of the mathematics behind the measuring tape becomes "recognized" by the pre-apprentice. The question is not only the manner in which personal and cultural meanings become tuned, for personal meanings can only arise and evolve against the backdrop of forms of activity. Here the TO departs from other approaches. The problem is precisely the very social formation and evolution of personal meanings as they evolve within goal directed activity and are framed by the cultural meanings conveyed by socio-cultural contexts.

Several contemporary approaches emphasize, for various theoretical reasons, the embodied dimension of thinking (see, e.g. Arzarello, 2006; Lakoff & Núñez, 2000; Nemirovsky & Ferrara, 2008) and the role of artifacts (Bartolini Bussi & Mariotti, 2008). In the TO, the sensuous and artifact mediated nature of thinking leads, methodologically, to paying attention to the semiotic means through which objectification is accomplished. These means are called *semiotic means of objectification*. Much more than being simple aids to thinking, semiotic means of objectification are constitutive and consubstantial parts of thinking and include kinesthetic actions, gestures, artifacts (e.g. rulers, tools), and/or signs, e.g. mathematical symbols, inscriptions, written and spoken language (see Radford, 2008c); they allow one to draw one's own attention and/or the attention of another to particular aspects of cultural objects (Radford, 2003; Radford, L., Miranda, I. & Guzmán, 2008).

In his recent work, Radford has identified two main (and interrelated) processes of objectification, namely *iconicity* and *contraction* (2008a). While contraction refers to the process of making semiotic actions compact, simplified and routine as a result of acquaintance with conceptual traits of the objects under objectification and their stabilization in consciousness, iconicity is a link between past and present action: it refers to the process of noticing and re-enacting significant parts of previous semiotic activity for the purpose of orienting one's actions and deepening one's own objectification (Radford, personal communication, September 29, 2008). One of the goals of this paper is to contribute to this idea of iconicity by showing, through fine-grained analysis, some relevant aspects of its dynamics as well as to call attention to a new form of iconicity that, to my knowledge, has not been reported elsewhere.

## METHODOLOGY

### Data collection

The data for this study was collected in a pipe-trades pre-apprenticeship training class being conducted at a trade-union run school in British Columbia, Canada. This program involved pencil and paper work in the classroom as well as practical work in the workshop. It was designed to give the pre-apprentices a head start with important skills that would be addressed subsequently in the early years of their formal apprenticeship training in a number of different pipe-trades.

Throughout this pre-apprenticeship course the researcher served as a math tutor for any pre-apprentices who sought out his help. At other times, the researcher observed pre-apprentices and engaged them in discussion about their mathematics related coursework as they were working on it. The activity of individual and groups of pre-apprentices, working either with the researcher or working on their own, was documented using a video camera. Copies of the course print materials and copies of pre-apprentices' written work were also retained for analysis. The data for this paper was selected from this collection of data.

The individual who is the focus of this analysis, was a secondary school graduate. He had been in the workforce and completed a small number of courses in an electronics-technician training program at a community college during the three and a half years between the time that he finished secondary school and the time he began the pre-apprenticeship program in the pipe-trades. Throughout the pre-apprenticeship course he actively sought out the researcher for help with his mathematics related work.

### Data analysis

A *multi-semiotic analysis* was conducted of the pre-apprentice's and the researcher-as-tutor's joint activity during their one-on-one tutoring session to investigate process of knowledge objectification. This involved the construction of a transcript of the dialogue from the video-recording of the session, along with a detailed account of significant actions, semiotic systems, and artifacts used. This process required, at times, a slow-motion and frame-by-frame analysis of video tape to assess the role and coordination of spoken language with the use of artifacts and gestures during the encounter.

The analysis to be discussed here focuses on an excerpt from the beginning of the tutoring session with the pre-apprentice, who will henceforth be referred to as "C". The researcher will henceforth be referred to as "L". This session took place at a table in the classroom immediately after L discovered that C was having difficulty reading fractions of-an-inch from his measuring tape while working on a pipe-fitting project with his colleagues in the workshop. The focus here is on C's objectification of the difference in the fraction marking patterns on the measuring tape below and above 12 inches, or one foot, where they are marked to thirty-seconds of an inch and sixteenths of an inch respectively. These two marking patterns can be seen in figure one. This is

one of a number of mathematical patterns inscribed on the measuring tape that C comes to notice and coordinate as he becomes proficient with reading the measuring tape over the course of the entire thirty-two minute tutorial.


***Figure 1***. *The marking lines to the left of one foot indicate fractions to thirty-seconds of an inch. On the right side of one foot the markings indicate fractions to sixteenths of an inch. (C has inscribed a line across the measuring tape with his pencil at 11 1/8", partly obscuring the measuring tape inscriptions, and another short line over the marking at 11 5/32".)*

## RESULTS AND DISCUSSION

The shared goal of C and L's work together in the tutoring session is for C to learn how to read fractions on the measuring tape to sixteenths of an inch or, using the language of the TO, to objectify the system of fractions-of-an-inch crystallized within this cultural artifact (the measuring tape). C needs to learn this to be able to complete a pipe-fitting project that he is working on, as well as for his ongoing training, and for his future work as a trades person. L's immediate goal in this particular episode is for C to begin noticing differences and similarities in the marking patterns on the measuring tape.

### Semiotic means of objectification using gestures and signs

The measuring tape from C's tool box is extended on the table top in front of both C and L and the session begins with L asking C what difference he notices between the pattern of spaces on his measuring tape below 12 inches and above 12 inches.

75. L: … What do you notice here between the spaces here, up to twelve [Gesture-uses the index finger of his left hand to sweep up from the zero end of the measuring tape and pauses at 12" just before saying "up to twelve"]
76. C: Yeah its,
77. L: and the spaces after twelve? [G-now pointing with the fourth finger of his left hand to sweep through the exposed interval of the tape measure above 12"]

Here L asks C to explain what he notices while using two distinct sweeping gestures separated by a static pointing gesture at the twelve inch point. This in an attempt to draw C's attention to, and initiate his objectification of, these two intervals as distinct regions of the measuring tape. L emphasizes this distinction by using different pointing fingers to sweep through each of the intervals and a contrasting static pointing gesture at the end of his sweep up to 12 inches to highlight the boundary point between them. As every educator knows, posing a question like this one is an

effective means of drawing a student's attention to, and having him or her engage in a more critical way with, an object at hand. In this short excerpt L's question is framed through the coordinated use of spoken language to describe the two regions of the measuring tape, and the use of a static pointing gesture and two different forms of sweeping gestures. Together, spoken language and gesture serve as semiotic means of objectification for C.

Gestures dominate C's response to L's question. This is clear by considering his spoken words alone, which provide only a vague and partial response. It is only through C's use of spoken language, interspersed with an elaborate and coordinated sequence of ten gestures, each positioned in a precise way relative to the measuring tape that it becomes clear that he is, indeed, becoming consciously aware of the way in which the marking patterns on the measuring tape are different from one another.

(Transcript note: The spoken words in the transcript below are printed in bold to assist the reader to differentiate these from the descriptions of the accompanying actions.)

78 C: **There's,** [G(Video frame 1, 26:52)–sweeps up through the first few inches of the tape measure with the fourth finger of his left hand in a manner similar to the gesture just enacted by L]

**there's more.** [G(Video frame 2, 26:53)–makes two chopping motions aligned with the markings on the tape measure with his left hand, the first significantly larger than the second just before he says "there's more" in reference to the markings inscribed on the measuring tape.

G(Video frame 3, 26:54)–points to the 12" mark with the fourth finger of this left hand before withdrawing it from the measuring tape].

**Video frame 1** (26:52). C sweeps up through the first few inches of the measuring tape.

**Video frame 2** (26:53). C makes two chopping motions aligned with the markings on the measuring tape.

**Video frame 3** (26:54). C points to the 12" mark.

In line 78, C begins his description of the difference between the two marking patterns on the measuring tape. He starts by sweeping the fourth finger of his left hand upwards through the first few inches of the measuring tape (Video frame 1). This is the same type of one finger indexical sweeping gesture that L had just used

(albeit using a different finger) to draw attention to this region of the measuring tape. C embellishes L's original gesture sequence by including a chopping gesture midway up this interval. This chopping gesture is aligned with the series of parallel markings inscribed on the measuring tape and reflects the familiar action of physically dividing or chopping up the interval on the measuring tape in the same way as is indicated by the inscribed measuring tape markings (Video frame 2, 26:53). Immediately following this gesture C says "there's more" (line 78), a confirmation that he is, indeed, referring to the closely packed markings inscribed on this region of the tape measure. C resumes and finishes his sweep through this region of the tape measure by pointing with the same finger of his left hand to the 12 inch point, the endpoint of this interval (Video frame 3, 26:54), before taking this hand away from the measuring tape. This use of a static single-finger pointing gesture at the 12 inch point separating the two regions of the measuring tape is the same type of gesture that L used a few seconds earlier to separate his sweeping gestures at the 12 inch point as well.

(line 78 continues) **It's like it's more spread out** (in reference to the markings on the tape measure after the 12 inch point.) [G(Video frame 4, 26:55a)–points briefly to the 12" mark on the tape measure now with the first finger of his right hand, replacing the previous pointing gesture expressed by the fourth finger of his left hand.

G(Video frame 5, 26:55b and Video frame 6, 26:56a)–starting with his thumb positioned at the 12 inch point, sweeps his right hand up the measuring tape a short distance while holding an approximately 2.5" wide interval between the thumb and first finger.]

**Video frame 4** (26:55a). C points again to the 12" mark on the measuring tape.

**Video frame 5** (26:55b). C begins to sweep an approximately 2.5" wide interval up the measuring tape starting with his right thumb at 12".

**Video frame 6** (26:56a). C continues his wide-interval sweep up the measuring tape.

(line 78 continues) **when** [G(Video frame 7, 25:56b)–grasps the tape measure with his right thumb and first finger on opposite edges at the 12" point and G(Video frame 8, 26:57)–sweeps his hand in this configuration upwards a short distance from 12"] **you pass one**,

79 L: **Yeah,**

80 C: **one foot** [G(not shown)–while maintaining the same grasping position, repeats this sweep upwards for a second time]

**Video frame 7** (26:56b) C grasps the measuring tape at the 12” point.

**Video frame 8** (26:57) C sweeps his hand in this configuration upwards a short distance from 12” and then repeats this motion a second time.

When line 78 continues, C replaces, briefly, his left hand pointing gesture at the 12 inch point with the first finger of his right hand (Video frame 4). This reflects, in part, L’s earlier set of indexical gestures, i.e. using different pointing fingers to distinguish between the two different regions of the measuring tape. C then forms a wide-interval gesture using his right thumb and first finger and without hesitation sweeps this up the measuring tape with his right thumb starting from the 12 inch point (Video frame 5 to Video frame 6). As he does this he says “it’s more spread out” (line 78). This reflects the wider interval spacings between adjacent fraction markings inscribed here. C then grasps the measuring tape at 12 inches with his right thumb and first finger in a position that looks like he is grasping or pinching it (Video frame 7), and then sweeps his hand up the measuring tape from 12 inches and Video frame 8) and then repeats this a second time (not shown). This series of three sweeps up the measuring tape from the 12 inch point (one wide-interval sweep and two grasping sweeps) serves to sustain both his own and L’s attention on this region of the measuring tape.

(line 80 continues) **and when you’re before one foot its more um,** [G(Video frame 9, 27:01)–makes a very brief and narrow-interval gesture with the thumb and first finger of his right hand with this hand now positioned above the region of the tape measure between 0” and 12”.]

**Video frame 9** (27:01) C makes a very brief narrow-interval gesture with the thumb and first finger of his right hand with this hand now positioned above the region on the tape measure between 0” and 12”.

81 L: **Okay.**

82 C: [silence]

C’s explanation comes to an end as he says “below one foot its more um” (line 80) while making a very brief but distinct narrow-interval gesture with the thumb and first finger of his right hand (Video frame 9). This gesture is positioned above the region of the measuring tape between 0 and 12 inches and reflects the narrower

intervals between adjacent markings on this region of the measuring tape in comparison to the intervals above 12 inches that C had described using a wide-interval gesture seconds earlier.

By responding to L's question in lines 78 and 80, C enacts a coordinated series of semiotic actions that serve to draw his own awareness to the marking patterns on the tape measure and thus mediate his thinking and deepen his consciousness of these patterns. This was, after all, the outcome L was aiming for by posing his initial question in lines 75 and 77. C's use of gestures and spoken language in this excerpt are examples of semiotic means of objectification for oneself.

**Forms of iconicity and mathematics as reflexive praxis**

Radford describes iconicity as the process of noticing and re-enacting significant parts of previous semiotic activity for the purpose of orienting one's actions and deepening one's own objectification. We can find three forms of iconicity within this brief and intense exchange between L and C.

The first form of iconicity involves C noticing and re-enacting all of the hand gestures and corresponding hand positions that L had used while posing the question to him at the start of their exchange. These included his use of different fingers for pointing at the different regions of the measuring tape in line 79–Video frames 1 and 4, the sweeping gesture for identifying the region of the measuring tape below 12 inches in line 78–Video frame 1, and the static one-finger pointing gesture directed at the 12 inch point in line 78–Video frame 3.

The second form of iconicity involves C noticing the different inscription patterns on his measuring tape below and above 12 inches and re-enacting these using different forms of semiotic actions, in this case using hand gestures. The examples here include C's chopping gesture to describe the closely packed pattern of marking lines below 12 inches in line 78–Video frame 2, his wide-interval gesture to describe the relatively wide intervals between markings above 12 inches also in line 78–Video frame 6, and his narrow-interval gesture to describe the relatively narrow intervals between the markings below 12 inches in line 80–Video frame 9.

The third form of iconicity to be found coincides with the second form of iconicity just described in this set of data. It involves C noticing a form of gesture that he has enacted himself and then re-enacting this within a different context. I refer here to C's use of a narrow-interval gesture using this thumb and first finger to describe the marking pattern below 12 inches on the measuring tape in line 80–Video clip 9. This occurs after he has enacted a similar wide-interval gesture using his thumb and first finger in reference to the marking pattern above 12 inches in line 78–Video frame 6.

We can infer that C became consciously aware of the possibility and/or usefulness of utilizing this form of interval gesture as a result of using it to describe the intervals above 12 inches because he then backtracked to elaborate on his previous description of the region of the measuring tape below 12 inches using this same form of gesture. The finding of this third form of iconicity–noticing and re-enacting parts of one's

own semiotic activity in a new context–is a new contribution to the theory of knowledge objectification.

## CONCLUDING REMARKS

The brief excerpt that is the focus of this paper is taken from the beginning of a tutoring session involving a pre-apprentice in the pipe-trades learning to read the mathematical meaning embedded within a measuring tape marked in inches with the researcher serving in the role of tutor. This analysis illustrates the sensuous and artifact mediated nature of mathematical thinking and knowledge objectification. Particular features of the theory of knowledge objectification were evident including: examples of semiotic means of objectification–for another as well as for oneself–and three forms of iconicity: re-enactment using matching semiotic actions, re-enactment using different forms of semiotic action, and a newly reported form of iconicity, re-enactment of one's own previous form of semiotic actions in a different context.

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