IMPARTING THE LANGUAGE OF CRITICAL THINKING WHILE TEACHING PROBABILITY

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This paper reports a preliminary study of imparting to students a new kind of language, incorporating elements of ***critical thinking (CT),*** *in the course of a mathematics (probability) lesson. In the paper, we describe and analyse one probability lesson, which is part of an in-depth study that comprises fifteen math lessons of similar constitution. The purpose of this research is to determine whether the teaching methods we developed can improve students' critical thinking. Our approach favors immersion-teaching of CT, i.e. incorporating CT terminology and practice within the framework of a probability lesson, and is based on the specific taxonomy of CT skills proposed by Ennis. We focus specifically on critical thinking while distinguishing it from stochastic thinking, creative thinking and statistical thinking. This study involved 55 subjects. Analysis of interviews conducted with the students and an analysis of their submitted work indicated that students' critical and analytical capabilities greatly improved. These results show that if teachers consistently and methodically encourage CT in their classes, by applying mathematics to real-life problems, encouraging debates, and planning investigative lessons, the students are likely to develop the language of critical thinking as a result. This paper is a description of an initial study, a snapshot that focuses on one lesson and illustrates the orientation of the entire study.*

INTRODUCTION AND THEORETICAL FRAMEWORK

It has already been suggested that teachers should use a language of critical thinking as part of the attempt to change the method of teaching to enable meaningful learning of information (Perkins, 1992). This is an area in which a substantial research literature already exists.

Our focus in this paper is describing our approach and its initial results. In this paper, we are focusing on the language of critical thinking. When defining the term critical thinking (CT), it is important to realize that it is not a new concept; we can find it as early as ancient Greek times: Socrates, as reported by Plato, used to roam the streets of Athens asking people all kinds of philosophical questions about the purpose of life, morality, justice, etc., apparently for the purpose of stimulating a form of critical thinking. These questions and answers were collected and recorded in the Socratic dialogues. In the field of education, it is generally agreed that CT capabilities are crucial to one's success in the modern world, where making rational decisions is becoming an increasingly important part of everyday life. Students must learn to test reliability, raise doubts, and investigate situations and alternatives, both in school and in everyday life. Abundant definitions of critical thinking have been proposed, since

this is a multidisciplinary subject that engaged teachers, educators, sociologists, psychologists and philosophers in all eras, but we would like to focus on Ennis' taxonomy, because for our purposes we needed to employ a hierarchical set of critical thinking skills isolated from other definitions. Ennis (1962) defines CT as “a correct evaluation of statements". Twenty-three years later, Ennis broadened his definition to include a mental element, defining CT as “reasonable reflective thinking focused on deciding what to believe or do” (Ennis, 1985). Our research is based on three key elements: a CT taxonomy that includes CT skills (Ennis, 1987); the learning unit "Probability in Daily Life" (Liberman & Tversky, 2002); and the infusion approach of integrating subject matter with thinking skills (Swartz, 1992).

Ennis’ Taxonomy (Ennis, 1987)

In light of his definition, Ennis developed a CT taxonomy of skills that include intellectual as well as behavioural aspects, e.g. judging the credibility of sources, searching for clarifying questions, defining the variables, searching for alternatives etc. In addition to skills, Ennis's taxonomy (1987) also includes dispositions and abilities. Ennis claims that CT is a reflective and practical activity aiming for a moderate action or belief. There are five key concepts and characteristics defining CT: practical, reflective, moderate, involving? belief and oriented towards? action.

Learning unit "Probability in Daily Life" (Liberman & Tversky 2002)

In this learning unit, which is a part of the formal syllabus of the Ministry of Education, the students are required to analyse problems, raise questions and think critically about data and information. The purpose of the learning unit is to teach the students not to be satisfied with a numerical answer but to examine the data and its validity in order to arrive at a more valid answer and develop their critical thinking. In cases where there is no single numerical answer, the students are required to know what questions to ask and how to analyse the problem qualitatively, not only quantitatively. Along with being provided with statistical instruments, students are redirected to their intuitive mechanisms to help them estimate probabilities in daily life. Simultaneously, students examine the logical premises behind their intuitions, along with possible misjudgments of their application.

The infusion approach (Swartz, 1992)

There are two main approaches to fostering CT: *the general skills approach* which is characterized by designing special courses for instructing CT skills, and the *infusion approach,* according to Swartz (1992), is characterized by providing these skills through teaching the set learning material. According to this approach, there is a need to reprocess the set material in order to combine it with thinking skills. In this report, we will show, on the example of one lesson, how we combined the mathematical content of "probability in daily life” with CT skills from Ennis' taxonomy, and evaluated the subjects' CT skills.

METHODOLOGY

The main paradigmatic aspects of methodology in mathematics education research have been broadly established (Scherer & Steinbring, 2006). Our methodological challenge was to investigate the development of the "language of critical thinking" through critical thinking skills incorporated into a structured mathematics lesson, such as a probability lesson. In this regard, the methodological approach is closest to the "Design Experiment" (as discussed by Cobb, Confrey, diSessa, Lehrer and Schauble, 2003). Through careful instructional design, a lesson sequence was constructed with the goal of consistently and methodologically encouraging and promoting crtitical thinking by applying mathematics to real-life problems, encouraging debates and using investigative lessons, in order to develop the "language of critical thinking". The research process examined student classroom products (primarily student submitted work) and post-lesson interviews with students to document changes in students' analytical capabilities. These changing capabilities could then be related to classroom activities, which were documented by video.

Setting, Population, and Data

Fifty-five children between the ages of fifteen and sixteen participated in an extra curriculum program aimed at enhancing the critical thinking skills of students from different cultural backgrounds and socio-economical levels. An instructional experiment was conducted in which probability lessons were combined with CT skills. The study consisted of fifteen 90 minute lessons, spread out over the course of an academic year, in which the teacher was also one of the researchers.

Data sources were: Students' products, Pre and post questionnaires, Personal interviews and Class transcriptions.

The students' products (papers, homework, exams etc.) were collected. Five randomly selected students were interviewed at the end of each lesson and one week after. The personal interviews were conducted in order to identify any change in the students' attitudes throughout the academic year. Not only was the general attitude examined, attention was paid to the development of critical thinking language (e.g., by asking the student to define critical thinking and to explain how they viewed critical thinking in the scope of the lesson; furthermore, they were also asked to assess whether they considered themselves to be critical thinkers, and it was the answer to this question that was used to establish the nature and frequency of critical thinking among them). All lessons were video-recorded and transcribed. In addition, the teacher kept a journal (log) on every lesson. Data was processed by means of qualitative methods intended to follow the students' patterns of thinking and interpretation with regards to the material taught in different contexts. Following Ennis' taxonomy (Ennis, 1987), data was analysed by employing three principles: (1) As the student is asked to articulate the question dealt with in a particular lesson, the level of critical thinking was deciphered (as will be discussed later on); (2) students' reactions to the teacher's attempt to induce critical thinking were examined through their responses as well as

from the interviews; (3) proposition of alternatives was employed as an interview technique, in an attempt to identify critical thinking abilities.

The Intervention- Unit Description

As already mentioned, the probability unit combines CT skills with the mathematical content of "probability in daily life". This new probability unit included questions taken from daily life situations, newspapers and surveys, and combined CT skills. Each of the fifteen lessons that comprised the probability unit had a fixed structure: a generic (general) question written on the blackboard; the student's reference to the question and a discussion of the question using probability and statistical instruments; and, an open discussion of the question that included practicing the CT skills. The mathematical topics taught during the fifteen lessons were: Introduction to set theory, probability rules, building a 3D table, conditional probability and Bayes theorem, statistical connection and causal connection, Simpson's paradox, and judgment by representativeness. The following CT skills were incorporated in all fifteen lessons: A clear search for an hypothesis or question, the evaluation of reliable sources, identifying variables, “thinking out of the box,” and a search for alternatives (Aizikovitsh & Amit, 2008). Each lesson followed the same four part structure.

1. Given Text

At the beginning of the lesson the teacher presented a short article or text.

2. Open Class Discussion in Small Groups

Discussion in small groups about the article and the question.

•Initial suggestions for the resolution of the question

•No intervention by the teacher

3. Further Discussion Directed by the Teacher

Open class discussion. During the discussion the teacher asked the students different questions to foster the students’ thinking skills and curiosity and to encourage them to ask their own questions.

• Various suggestions from students in class.

• Interaction between groups of students.

• Reaching a consensus across the whole class (or just across the group).

4. Critical Thinking Skills and Mathematical Knowledge (Teaching)

The teacher referred to the questions raised by the students and encouraged CT, while instilling new mathematical knowledge: the identification of and finding a causal connection by a third factor and finding a statistical connection between C, and A and B, Simpson's paradox and Bayes Theorem.

Case study- The Aspirin Case

Below, I have provided a detailed description of one lesson called the Aspirin Case. Following the description, I outline the analysis of the lesson using the following techniques: referring to information sources, raising questions, identifying variables, and suggesting alternatives and inferences. The lesson topic was conditional probability. The CT skills practiced in the lesson were evaluating source reliability, identifying variables, and suggesting alternatives and inference.

1. A Given Text

Your brother woke up in the middle of the night, crying and complaining he has a stomachache. Your parents are not at home and you don't know what to do. You gave your brother aspirin, but an hour later he woke up again, suffering from bad nausea and vomiting. The doctor that takes care of your brother regularly is out of town and you consider whether to take your brother to the hospital, which is far from your home. You read from a book about children's diseases and find out that there are children that suffer from a deficiency in a certain type of enzyme and as a result, 25% of them develop a bad reaction to aspirin, which could lead to paralysis or even death. Thus, giving aspirin to these children is forbidden. On the other hand, the general percentage of cases in which bad reactions such as these occur after taking aspirin is 75%. 3% of children lack this enzyme.

(Taken from "probability thinking" p. 30+slight changes made by researcher)

2. Open Class Discussion in Small Groups

Discussion in small groups about the generic question:
Should you take your brother to the emergency room? What should you do?
Can aspirin consumption be lethal?

3. Further Discussion Directed by the Teacher

The generic question on the blackboard was:
Should you take your brother to the emergency room? What should you do?

21 Teacher: What do you think?
22 Student 1: Where is the information taken from? Can we see the article for ourselves?
23 S2: Is the source reliable? How can we check it?
24 S3: Where is the article taken from? What is its source?
25 S1: Should I answer the identification of the sources question?
26 T: Not yet. We are focusing on searching for questions. Please think of other questions.
27 S3: What connection does the article discuss?
28 S2: first we need to identify the variables!!!
29 T: Right. First, we ask what the variables are.
30 S4: You can infer it from the title that suggests that a connection exists between aspirin and death.
31 T: According to the data from the article, Can we find a statistical connection? (the student already know this subject)
32 S2: I know! We can ask: suggest at least 2 other factors that might

cause the described effect.
33 S5: The question is what causes what?
34 S6: Can aspirin consumption be lethal?
35 T: What do you think?
36 T: How can you be sure?
37 S6: Umm…
38 S3: Are there other factors, such as genetics!?
39 T: Very good. What did student 3 just do?
40 S1: He suggested an alternative!!
41 T: How can we check it? Do you have any suggestions? Can you make a connection between this problem and the material we have learned in the past few lessons? Can you offer an experiment that would solve the problem?
42 S3: Of course. An observational experiment.

In paragraph 21 we encounter skills such as "searching for the question"- a fundamental skill. First there is a need to clarify the starting point for the interaction with the student. We also need to clarify to ourselves what is the thesis and what is the main question before we approach decision making. The paragraph also demonstrates relevance to daily life. In paragraph 26 the students are taking a step back, we refer to "identifying information source and evaluating the source's reliability" skill. This step is crucial, as it helps us to assess the quality and the validity of the article discussed. This skill was practiced in past lessons. See paragraph that summarizes the article. In paragraph 26 we encounter "searching for the question" skill again. We will continue searching for the main question through practicing the "variables identification" skill. Raising the search for alternatives. Posing questions enables the practice of this skill. Paragraph 30 deals with identifying the variables and understanding them by a 2D table and a conditional probability formula. In paragraph 36 the teacher builds the students' self esteem by encouraging them to express their ideas and opinions (even if they are not always correct or relevant). She prevents any intolerance of other students. The method of instruction that aims at fostering the confidence and the trust of the students in their CT abilities and skills is, according to Ennis "referring to other peoples points of view" and "being sensitive towards other peoples' feelings". In paragraph 23 the student is referring to other sets and finding the connection between them. Paragraph 31 depicts the skill of "Searching for alternatives". Paragraph 42 refers to a controlled experiment or an observational experiment. An additional grouping and finding the connection between the variables by Bayes theorem or a 2 dimensional table.

4. Critical Thinking Skills and Mathematical Knowledge (Teaching)

This phase of the lesson focused on encouraging critical thinking and instilling new mathematical knowledge (Bayes formula) statistical connections by referring to students' questions and further discussion.

A teacher-led discussion focused on methods of analysis using such Critical Thinking skills as: Source identification: Medicine book; Source reliability: High; Variable identification: A – enzyme deficiency, D – adverse reaction to aspirin; Mathematical Knowledge: Data: P(D/A)=0.25 P(D)=0.75 P(A)=0.03, To prove: P(A/D)=?
Using Bayes formula (or a two dimensional matrix) the result is:
Lesson Conclusion is that only 1% of the children without the enzyme develop an adverse reaction to aspirin, thus there is no need to go to the hospital.
Even so, is it worth taking the risk? What do you think? (question to the class).

DISCUSSION

Research analysis according to critical thinking skills in this case study
Through the infusion approach, students practice their CT while acquiring technical probability skills. In this lesson, the following five skills are exercised: raising questions – asking question about the article and probing on the main question about the connection between aspirin and death; referring to information sources and evaluating the source's reliability - the text took from Medicine book; the students skepticism and identification of variables – students identified the enzyme deficiency and adverse reaction to aspirin. Following these skills, another skill, searching for alternatives (paragraph 38), was presented. In class the teacher and the students spoke about suggesting alternatives, not taking things for granted, but examining what had been said and suggesting other explanations. Hence, the skills that were practiced in the described lesson were: raising questions, evaluating the source's reliability, identifying variables, and suggesting alternatives and inference. In order to understand and monitor the students' attitudes toward CT as manifested by the skills specified above, interviews were conducted with five students after the aforementioned lesson. In these interviews, the students acknowledged the importance of CT. Moreover, students were aware of the infusion of instructional strategies that advance CT skills. Examples from two of the interviews follow.
Student 4 was interviewed and was asked to define CT. His answer was:
"I think CT is important when you study Mathematics, when you study other topics and when you read the paper, but it is most important when you deal with real life situations, and you need the right instruments in order to do so (deal with these situations)."
When Student 2 was asked about important components during the last few classes and the present class, she answered: *"first we should check the information source's reliability and despite all the numerical data, I don't accept the researcher's conclusion."*
Additional data, consistent with these two examples suggest that infusion of CT into the formal curriculum in mathematics can equip students with CT skills that are applicable to wider disciplines.

RESEARCH LIMITATIONS

This case study presents one lesson which was designed in a fixed pattern – a generic question, a discussion of the question, the practice of statistical connection, introduction to causal connection and experiencing the use of CT skills such as: raising questions, evaluating the source's reliability, identifying variables, and suggesting alternatives and inferences. On the basis of the interviews conducted and questionnaires that were qualitatively analyzed, it is not established, at this stage, the extent to which these skills have been acquired. Skill acquisition will be evaluated in much greater detail at a later phase in this study, using quantitative measures – the Cornell Critical Thinking Scale and the CCTDI (Facion, 1992) scale. At this stage we have provided only an introductory picture of our approach and an indication of the form of our analysis and results. However, this case study provides encouraging evidence of the effectiveness of this approach and further investigation in this direction is needed.

CLOSING REMARKS

The small scale research described here constitutes a small step in the direction of developing additional learning units within the traditional curriculum. Current research is exploring additional means of CT evaluation, including: the Cornell CT scale (Ennis, 1987), questionnaires employing various approaches, and a comprehensive test composed for future research.

The general educational implications of this research suggest that we can and should lever the intellectual development of the student beyond the technical content of the course, by creating learning environments that foster CT, and which will, in turn, encourage the student to investigate the issue at hand, evaluate the information and react to it as a critical thinker. It is important to note that, in addition to the skills mentioned above, in the course of this lesson it appears that the students also gained intellectual skills such as conceptual thinking and developed a class culture (climate) that fostered CT. Students practiced critical thinking by studying probability. In this lesson, the following skills were demonstrably practiced: referring to information sources (paragraph 22), encouraging open-mindedness and mental flexibility (all questions), a change in attitude and searching for alternatives. A very important intellectual skill is the fostering of cognitive determination – to be able to express one's attitude and present an opinion that is supported by facts. In this lesson, students could be seen to be searching for the truth, they were open-minded and self-confident. In other words, they practiced critical thinking skills. A new language was being created: the language of critical thinking.

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