THE DRAG-MODE IN THREE DIMENSIONAL DYNAMIC GEOMETRY ENVIRONMENTS – TWO STUDIES

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Dynamic Geometry Environments (DGEs) in 2D are one of the well researched topics in mathematics education. DGEs for 3D-environments (Archimedes Geo3D and Cabri 3D) were designed in Germany and France. In a first study we could show that pre-service teachers with previous knowledge in 2D-systems prefer to work with a real model of a cube instead of the 3D-system to solve certain problems. Furthermore we could find out that previous knowledge in 2D-systems seems to be insufficient to handle the drag-mode in an appropriate way in 3D-environments. In a second study we introduced the students to the special software before the investigation and distinguished different dragging modalities during the solution processes of two tasks.

THEORETICAL FRAMEWORK

During the last three decades, several 2D-Dynamic Geometry Environments (DGEs) have been created to enrich and further the learning process in the mathematics classroom. The most popular DGEs are Cabri-géomètre, GEOLOG, Geometer's Sketchpad, Geometry Inventor, Geometric Supposer and Thales. In Germany, Euklid-DynaGeo, Cinderella, GeoGebra, Geonext and Zirkel-und-Lineal are popular, with Euklid-DynaGeo being the most widespread software in German schools. DGEs are powerful tools, in which the user is able to exactly construct geometrically, discover dependencies, develop or refute conjectures or to get ideas for proofs.

DGEs are characterised by three central properties: the ”drag-mode”, the functionality ”locus of points” and the ability to construct ”macros”. The drag-mode is the most important feature available in these environments, because it allows to introduce movement into static Euclidean Geometry (Sträßer 2002). It is possible to drag basic points (points which are neither intersection points nor points with given coordinates). During this dragging process, the construction is updated, according to the construction commands which were used in the drawing. To the user, it looks as if the drawing is respecting the laws of geometry while the dragging process is in progress.

2D-DGEs are one of the best researched topics in mathematics education and especially within the PME-group (Laborde et al. 2006). For example, we find research on ”DGE and the move from the spatial to the theoretical” (Arzarello et al. 1998, 2002) or ”construction tasks” (Soury-Lavergne 1998). Noss (1994) has shown that beginners have problems to construct drawings, which are resistant to the drag-mode and it is reported that for pupils there exist two separate worlds, the theoretical one and the world of the computer. ”The notion of dependency and functional relationship” (Hoyles 1998 and Jones 1996) is another interesting theme and it has been shown that pupils have heavy problems in understanding the notion of dependency. They have to be encouraged to use the drag-mode to support the understanding of the spatial-graphical and the theoretical level, serving as a tool for externalising the notion of dependency. Several researchers showed that students do not use the drag-mode spontaneously and they have to be encouraged to do it. Most of the students are afraid to destroy the construction by using the drag-mode and they do not like to use the drag-mode on a wide zone (Rolet 1996 and Sinclair 2003). Arzarello and his group elaborated a hierarchy of several dragging modalities, which were linked to ”ascending” and ”descending” processes and reveal students’ cognitive shifts from the perceptual level to the theoretical one (Arzarello 1998, 2002 and Olivero 2002). There is a great variety and number of research reports concerning the use of the drag-mode in proving and justifying processes (for example Jones 2000 and Mariotti 2000). Other fields of study were ”the design of tasks” (Laborde 2001), ”the role of feedback” (Hadas 2000) and ”the use of geometry technology by teachers” (Noss, Hoyles 1996).

THE FIRST STUDY IN 2007

In the following we will give a brief summary of the research design and the results of our first study. For details see Hattermann, 2008. In July 2007, 15 pre-service teachers with previous knowledge in Euklid DynaGeo (2D-DGE) took part in our investigation. Some groups worked with Archimedes Geo 3D and others with Cabri 3D, their actions on the screen and their discussions and interactions were recorded by a screen-recording software called “Camtasia” and a webcam. We used a qualitative approach to get ideas about students’ behaviour in 3D-DGEs. Some important research questions were the following:

- Do the students use spatial constructions like spheres or do they prefer elements from plane geometry? (Task 1)
- What are the preferred tools to work with (paper and pencil, real model, imagination, DGE) to work with? (Task 2)
- Do students use the drag-mode to validate a construction and to find solutions to problems? (Task 1 and 2)
- How do participants behave in 3D-environments and how do they use the drag-mode? (Task 1 and 2)

Task 1 and Results

The first task was: “Construct a cube without using the existing macro!” Five of seven groups constructed the cube. The Cabri groups needed between 20 and 25 minutes to construct the cube, whereas the Archimedes groups needed about 40 minutes. Different groupes tried to utilise transformations as reflections or rotations. While the realisation of a reflection is quite easy in Cabri, rotations seem not to be easy to handle without any instructions. In the Archimedes environment students had problems with every transformation. The majority of the students used the drag-mode to validate their construction only on demand. This result is comparable to the results obtained by Rolet and Sinclair who worked with school children in 2D-environments. Our probands preferred to measure several segments of the cube instead of dragging a basic point. During the construction, elements from plane geometry (circles, segments, straight lines) were preferred. Some groups used spheres to construct intersection points or to construct equidistant segments, but the majority of the groups worked with circles.

Task 2 and Results

The second task was: “A student affirms: The slice plane between a cube and a plane can be:

- an equilateral triangle
- an isosceles triangle
- a right-angled isosceles triangle
- a regular hexagon.

Construct (with the help of the function “cube”) a cube, check the student’s affirmations and justify your results!”

Every group tried to find validations for their conjectures with the help of the real model, the utilisation of the real model prevailed the use of the computer environment. Students preferred ”the old strategy” to examine the cube and to try to imagine the intersection figure. The software was used to validate the conjectures, which were mostly generated outside the software environment. The students defined a plane with the help of three fixed points, so that no dragging was possible. Furthermore, the drag-mode was not understood and it is not sure, if these students did not understand it in the 2D-case or if they could not negotiate it to the 3D-environments. The possibilities of the drag-mode were not understandable to most students. They did not use the drag-mode in an expected manner (to use draggable points on an edge of the cube to define the intersection plane and to drag it to scrutinise different intersection figures). The approach of one group could illustrate this result: The students defined many fixed points on every edge of the cube and defined a plane with the help of three points. After verification, they deleted the plane and constructed another one with the help of other points. Only in exceptional cases the drag-mode was used and more often than not in a manner that a controlled dragging of the plane was impossible, which is the case when students used three arbitrary points in space to define the intersection plane. Students’ statements support the assertion that the “drag-mode” was not understood and previous knowledge in 2D seems to be insufficient to handle 3D-systems!

THE SECOND STUDY IN 2008

Methodology

Our second study took place in February 2008 at the University of Giessen and 15 pre-service teacher students participated in it. The participants had previous knowledge in Euklid DynaGeo (the most widespread 2D-DGE in Germany), but their experiences with DGEs were less than those from students who participated in our first study, because of changes concerning the content of different lectures following new study regulations. There were seven groups (six groups of two students and one group of three students). Three groups worked with Archimedes Geo3D while four groups utilised Cabri 3D to solve the given tasks. Each group worked in a separate room, the actions on the screen were recorded by utilising the screen-recording software "Camtasia". Furthermore, a webcam and a microphone were used to record students' voices and interactions.

In our second study we tried to create an environment in which we could observe different dragging modalities. Due to the results of our first study we opted for an approach with a preparation session in which students were introduced to the special software environment and were encouraged to use the drag-mode. Both groups were taught in:

- dragging basic points in 3D-space in the special software environment with the help of the keyboard
- the distinction between basic points, semi-draggable points and fixed points
- the construction of a midpoint of two points
- the construction of a "perpendicular plane" to a straight line through a given point beyond the straight line
- the construction of a "perpendicular line" in the x-y-plane to a given straight line in the x-y-plane through a given point , beyond the straight line
- in the construction of a circle in an arbitrary plane, devoid of the x-y-plane, with a given centre and through a new point on the plane
- in reflecting the circle on an arbitrary point devoid of the circle's centre
- in constructing a plane which contains a given straight line
- in constructing a plane with the help of three points in such a way that one of these points can be dragged on a straight line

Archimedes-groups were especially introduced to the utilisation of transformations which is quite complicated in this environment. After the first introduction students were urged to solve five task which forced students to use the drag-mode. Here, we followed suggestions from the Centre informatique pédagogique (CIP 1996) for 2D-environments and adapted the ideas to our 3D-environment. There were five files and every file contained a special task. Every task consisted of a body and one or several yellow points which had been constructed by the researchers before. The task was to find hypotheses concerning the construction of the yellow point(s) by dragging a special point which was marked in blue colour. With the help of these preparation tasks, we intended to weaken students' constraints to use the drag-mode and to encourage them. Because of the domination of the real model compared to the software environment in our first study, we decided to forbid paper and pencil and not to provide a real model of the cube.

In our preparation session, we tried to provide students with competencies to solve the tasks which were given in our study without giving them exact hints. So we broached the issue of constructing a perpendicular line to a straight line through a given point on a special plane without mentioning that this construction could be useful to construct a cube. For another example, students had to construct a plane in such a way that one point of this plane could be dragged on a straight line. The idea behind was to show students how to construct a "draggable plane" without telling them that it could be an appropriate way to scrutinise different intersection figures of a plane and another body by using three defining points of the plane on appropriate segments of the body, which seems to be a reasonable way to solve our second task in the study.

Research questions

First of all we are interested in the general behaviour of our students in a 3D-environment; especially we looked for differences in students' behaviour during the solution process of different tasks compared to the first group in July 2007 which had no preparation session. Are there important differences among the two DGEs? Because of the importance of the drag-mode in DGEs, we want to know more about the utilisation of it, especially we are interested in different dragging modalities in 3D-environments. Do students use the drag-mode to validate their construction in task one (construction of a cube)? A validation of the construction with the help of the drag-mode assumed, how do they use it? Are they more "courageous" than their predecessors in July 2007 and do they use the drag-mode on a "wider zone"? What are the preferred tools to construct a cube? Is one preparation session enough to get students familiar with a 3D-DGE in such a way that elements like spheres or 3D-reflections will be used to construct a cube or do constructions like circles (elements from planar geometry) prevail the construction?

Do students use the drag-mode to discover different intersection figures of a cube and a plane or do they try to avoid the utilisation of the drag-mode in task two? Is it possible to identify different "ways of dragging"? What solving strategies are preferred by students who do not possess neither a real model of a cube nor a paper and pencil environment?

Task one and Results

We used the same task as in our first study in July 2007:"Construct a cube without using the existing macro!"

Every group constructed the cube. The Cabri-groups needed 17, 19, 26 and 41 minutes for the construction, whereas the Archimedes-groups needed 34, 37 and 45 minutes. Furthermore every group utilised the drag-mode to validate their construction and two Cabri-groups did it in a "courageous way" so to say, they used it on a wider zone. One Archimedes-Group was very careful by dragging basic points. Every group was very happy by observing the invariance of the constructed cube under dragging and jubilation and pleasure were recognisable in nearly every group. This fact shows that dragging can motivate and emotionally affect students which underlines the importance of this feature.

By comparing the periods of construction it seems as if Cabri-Groups work faster. In our first study we came to the same statement and argued that one reason for this could be the "base plane (x-y-plane)" which exists in Cabri. In Archimedes this plane has to be constructed first. We can't support this hypothesis with our actual data, because during the preparation session the construction of the x-y-plane in Archimedes was mentioned and every Archimedes-group had no problems to construct it in a short time not exceeding 3 minutes.

No group tried to construct the cube with the help of spheres, only circles, planes and perpendicular lines were used to construct cube vertexes. An explanation for this result lies in the preparation session, in which circles, but no spheres were explicitly mentioned.

One Archimedes-group utilised reflections on a plane and reflections on a straight to construct cube vertexes. One Cabri-group utilised the function of a parallel plane to a given plane but furthermore no reflections were used by students. In our first study no Archimedes-group used reflections to construct the cube. Due to the fact that "transformations" are not easy to handle without instructions, this fact was not surprising to us. After an introduction in defining and utilising transformations in Archimedes, one of three groups used "reflections", but the size of the sample seems to be too small to interpret this fact in more detail.

Besides we observed students who had problems with "parent-child-relations" (see also Talmon 2004). Several situations occurred, which prove that dependencies of construction objects are not understood completely. Some groups did not understand that objects disappear by deleting an object on which they depend on.

Furthermore we could identify several dragging modalities in 3D-environments. Students used the drag-mode in our first task to

- validate the construction at the end of the construction process.

- see that there are only two draggable points (the points that define the first edge of the cube) and to see that the other points are fixed.
- find out the function of a semi-draggable point on the edge of the cube that had been constructed before. (Students forgot for what reason they had it constructed)
- adapt the length of a segment to the measure of the first edge. (students did not really construct a cube in this attempt, they created a cube which was not invariant under dragging)
- find out more about the degrees of freedom of draggable points, for instance to scrutinise if points are draggable on a plane or only on a straight line.
- find an error in the construction. (Actually the construction was correct, only one point was wrong and this fact was discovered by dragging)

Task two and Results

The second task was changed compared to the version used in July 2007. Task two was the following: "Construct with the help of the function "cube" a cube and try to find by experiment all Polygons (n = 3, 4... n = number of vertexes) which exist as intersection figures between the cube and a plane." The second task was changed slightly in comparison to the first study, because we intended to further the need for the utilisation of the drag-mode. In the first study we gave four intersection figures and asked students to confirm or refute our statements, whereas the assignment is more open in our second study. We hoped that trying to discover new intersection figures would motivate students and moreover we tried to create an environment in which dragging could help students to find solutions. Finally we intended to observe and distinguish different "ways of dragging" during the solution process.

Except of one group, everybody found the equilateral triangle and the isosceles triangle as an intersection figure. Approximately the half of the participants mentioned an arbitrary triangle as intersection figure, whereas only one group could find a parallelogram. The rectangle and the square were the easiest figures which were found by every group. Half of the groups found the trapezoid as intersection figure, whereas the other participants found it was well, but did not identify this quadrilateral as a trapezoid. Nobody looked for an isosceles trapezoid. Three groups found a pentagon, four groups found a hexagon and four groups found the regular hexagon. There were groups that found the hexagon and not the regular hexagon and vice versa.

During the solution process we observed different dragging modalities. Students used the drag-mode by

- defining the intersection plane by one point on an edge of the cube and two vertexes.

- choosing two points in a Cabri-environment to define the plane (now a plane appears) and to observe the behaviour of this plane by moving the cursor on the screen. (a special type of dragging only available in Cabri-environments)
- defining three points on different edges of the cube to define the plane.
- using three arbitrary points in space to define the intersection plane.
- defining one draggable point on a straight line that is defined by two vertexes of the cube and to use two other points in space to define the plane.

Students used the drag-mode to:

- find out the function of a special point which had been constructed before. (a point was used to define a plane for example)
- vary the volume of the cube so that the intersection points between the cube and the plane become visible (which is not always the case).
- identify new intersection figures.
- get an idea how to construct the intersection figure afterwards with the help of fixed points to define the plane.
- identify more special figures/more general intersection figures from an existent figure. (find an equilateral triangle from an arbitrary triangle or vice versa)
- scrutinise if there are intersection figures with more than 4 vertexes. (with the special type of dragging in Cabri)
- move the cube, instead of varying the plane, to scrutinise different intersection figures.
- identify draggable and non draggable points.

It is really worth mentioning that we could observe happiness in every group by realising different intersection figures with the help of the drag-mode. “Wow” or “that’s really great” are only two short examples that underline our affirmation.

Conclusion

We succeeded in our second study to get the probands more familiar with the special DGE and to observe different dragging modalities in task one and two. There are still situations in which students utilised the drag-mode very careful and not on a wider zone, but the majority of our participants utilised the drag-mode to validate and to discover in a “courageous” manner without hesitation. So we claim that it is possible to prepare students in an appropriate time to use the drag-mode in 3D-systems and to encourage them.

For a classification of different dragging modalities it will be interesting to categorise them theoretically and to analyse the “instrumental genesis” of the drag-mode according to Rabardel’s theory (Rabardel 1995). It will be an exciting task for further research to observe the progress of the utilisation of the drag-mode. It should be possible to define different theoretical stages in the utilisation of the drag-mode from a "beginner's stage" which will be characterised by nearly no dragging or careful dragging up to an "expert's stage".

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