THE ROLE OF TEACHING IN THE DEVELOPMENT OF BASIC CONCEPTS IN GEOMETRY: HOW THE CONCEPT OF SIMILARITY AND INTUITIVE KNOWLEDGE AFFECT STUDENT' S PERCEPTION OF SIMILAR SHAPES

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ABSTRACT

In this research we investigate whether students of the Pedagogical Department of Education have the basic geometrical knowledge which is related mainly with the similarity of shapes. We also investigate how they define similarity of shapes and if the intuitive knowledge affects their perception of similar shapes. The results showed that students have developed certain structures in regard to some concepts in geometry based on the teaching that they have received in school. The results showed, as well, that a large percentage of students are not in a position to correctly define the similarity of shapes. Finally, research shown, that intuition affects their responses and their mathematical achievement.

INTRODUCTION

The role of geometry in the development of mathematical idea is very important. The geometrical skills and visual icons are basic instruments and source of inspiration for many mathematicians (Chazan & Yeryshalmy,1998 in Protopapas,2003). The content of geometry is appropriate both for the development of lower level of mathematical thinking, (i.e. the recognition of shape), as well as for higher order thinking, (i.e. the discovery of the properties of shapes), the construction of geometrical models and the solution of mathematical problems (NCTM, 1999). The representation of geometrical objects and the relationships between geometrical objects and their representations constitute important problems in geometry (Mesquita, 1998).

Geometry constitutes a basic part of the National Curriculum for Primary as well as Secondary Education. The concept of similarity between two shapes is taught in the 3rd grade in Secondary School and in the 1st grade in higher Secondary School, with special emphasis on the similarity of triangles. The teaching mainly concerns, understanding of the concept of similar shapes, i.e. that similar shapes are those which their sides are proportional and their angles that are created by the respective angles are equal.

Literature review has shown the concept of similarity is presented and taught through the environment of dynamic geometry and mainly through the use of applets. The concept is taught in coordination to the teaching of symmetry and transformations that can occur in a shape (http://standards.nctm.org/document/eexamples/chap6/6.4).

In addition, the properties of similar shapes are presented and in the proof of Thalis theorem. This theorem has some applications and proofs with the use of the Geometer Sketchpad. Although there are no relationships presented in regard to the results and consequences (proportion of relationships of line segments) of Thalis Theorem and the concept of similarity of shapes (beyond quadrilaterals).

The common teaching environment of geometry is very limited in formal education. For example, the constructions that the children are asked to deal with, the shapes are placed in a horizontal position, i.e. the sides are parallel to the sides of the object on which the construction is done. As a result most students develop an holistic and stereotype view of the geometrical shapes which is very affected by the intuitive rules.

At the university level, the students of the Department of Education are taught geometry through its historic evolution. In order to be able to follow and understand these lectures basic knowledge of geometry is required. This knowledge is mainly provided at the 3rd year of secondary school. Unfortunately, students appear to be lacking knowledge. This may be due to the long interval that has transpired since they dealt with geometry or due to the teaching in higher secondary school where it is mainly expected by the student to memorize relationships instead of understanding and applying them.

It is possible that the level of mathematical thinking may be influenced by some factors which are mathematics specific, such as the specific mathematical terminology which may be in conflict with the meaning we give to these terms in every day life or the conclusions that we reach based on the intuitive view of mathematical knowledge.

The aim of the present study is to investigate whether the students participating in EPA 171 (Basic concepts in mathematics) have the basic geometrical knowledge that is required for this specific course. It aims to investigate students' knowledge in regard to the similarity of shapes and how their intuitive knowledge may affect their perceptions about similar shapes.

THEORETICAL BACKGROUND

Geometry is comprised by three kinds of cognitive procedures which carry out specific epistemological functions (Duval, 1998):

a) Visualization: Is the procedure which is related to the representation of space in order to explain a verbal comment, for the investigation of more complex situations and for a more holistic view of space and subjective confirmation.

b) Construction with the use of apparatus. The construction of shapes can act as a model.

c) Reasoning: Is investigated in relation to verbal procedures and the extension of knowledge for proof and explanation.

These different procedures can be carried out separately. Thus the visualization is not based on the construction. There is however access on the shapes and the way that they have been constructed. Even if the construction leads to visualization, the construction is based only on the connections between mathematical properties and technical restriction of the apparatus which are used. Furthermore although the visualization is an intuitive aid, necessary in is some instances for the development of proof, still the justification is solely depended on a group of sentences (definitions, axioms, theorems) which are available. In addition to this visualization is sometimes more deceptive or impossible. Still these three kinds of cognitive procedures are closely linked and their cooperation is necessary for any progress in geometry (Protopapas, 2003).

The concept of similarity:

Similarity constitutes a basic link between algebra and geometry and it also has a close relationship to trigonometry. The theorem which expresses that two similar triangles have their sides proportional and Pythagoras theorem constitute two basic links between geometry and algebra. The connection of geometry and algebra is particularly construction as it allows using the visualization of geometry in algebraic problems and the flexibility of algebraic operations in geometrical problems. Similar triangles and the Pythagoras theorem constitute the cornerstone of Trigonometry. By using similar triangles we can calculate the sides and angles of an object by measuring the lengths of a smaller model.

According to Vollrath (1977) in geometry similarity constitutes a relationship between shapes/figures. A shape F1 is similar to a shape F2 if there is a transformation s such as $s(F_1) = F_2$. i.e. the square is similar to another one only when the ration of their sides is the same. In a didactical situation this constitutes a conclusion. Similar conclusions may be reached in regard to triangles and polygons. The proof is given based on the definition, using the properties of similar transformation. For a spiral approach of geometry it is important to know when it is possible to extract conclusions in regard to the understanding of similarity as it is defined through geometry or based on everyday language before teaching definition. Nevertheless, students do not seem to use the idea of sides' proportion to secure an exact answer about the similarity of shapes in enlargement or deduction in size of a shape (Kospentaris and Spyrou, 2005).

This can form the basis for a general definition of the concept of similarity. For the teaching of similarity at University level it is necessary, the lecturers to know in what extent the link between representation and expression of the concept of similarity can support or inhibit the cognitive procedure for this relationship. Furthermore it is important to know the explanation that the students give to similarity as it is used in everyday life or in a geometrical model (Vollrath, 1977). Kospentaris and Spyrou (2005) confirms in their study that the term similarity in everyday language does not in any way coincide with geometrical similarity, being more close to the meaning of having the same size.

The understanding of the concepts of similarity can be tested with exercises of classifying geometrical objects due to the fact that similarity constitutes a relationship of similarity between shapes/figures. In the teaching of mathematical the exercises of classification direct students in the study of properties and the properties that characterize concept and lead them to the extraction of definitions and they coordinate the understanding of definitions. Due to their importance we use exercises on classification to investigate students' understanding related to similarity irrespective of the mathematical definition. (Vollrath, 1977).

Intuition – and how it affects the teaching in mathematics:

As suggested by Fischbein (1999) intuition constitutes a theme that mostly philosophers are interested in. According to Descartes (1967) and Spinoza (1967) intuition appears to be a genuine source of pure knowledge. Kant (1980) describes intuition as the ability which leads directly to your goals and indirectly to the basic knowledge. Bergson (1954) made a distinction between intelligence and intuition. Intelligence is the way in which one may know the physical world, the world of stability, the extent of the properties of statistical phenomena. Through intuition we have a direct perception of the essence of spiritual life and control of the phenomena, time and motion (Fischbein, 1999).

Some philosophers, such as Hans Hahn (1956) and Burge (1968), have criticized intuition and its effect, in its scientific explanation. They believe that intuition leads to deceptive results and this has to be avoided in the scientific procedure.

The investigation of intuitive knowledge appears mainly in the work of people that are interested in scientific and mathematical understanding of students (for example Clement et al., 1989; DiSessa, 1988; Gelman and Gallistel, 1978; McCloskey et al., 1983; Resnick, 1987; Stavy and Tirosh, 1996; Tirosh, 1991 in Sierpinska, 2000). There is not an accepted definition of intuitive knowledge. The term: "intuition" is used mainly as a mathematical basic term such as the point or line (Sierpinska, 2000).

The importance of definition is probably respected just like the elements that are based on intuition. The basic common properties of these are based on individual proofs which are in conflict to logical and analytic attempts.

The problem of intuitive knowledge has earned an important place in scientific attempts. On one hand scientists need intuition in their attempt to discover new strategies, new theoretical and empirical models and on the other hand they need to be acquainted with what does not constitutes intuitionv– according to Descartes and Spinoza – basic guarantee, fundamental basis for objective truth.

The interest in regard to intuition also stems from the teaching of science and mathematics. When you need to teach a chapter in science or mathematics you often discover that what was already a fact for you – after university level studies – comes in conflict with basic cognitive obstacles that the students exhibit in their understanding. As a teacher you often believe that students are ready to memorize what they have been taught, actually they understand and memories relative knowledge. Intuitive perception of phenomena is often different that to their scientific explanation.

In mathematics, the belief that a square is a parallelogram is intuitively very strange for many children. The belief that by multiplying two numbers we may get a result that is smaller than one or both the numbers which we have multiplied is also difficult to be accepted. Intuition affects many of our perceptions. The educator discovers that the knowledge which s/he is supposed to transfer to the students is in conflict, very often, with the beliefs and explanations which are direct and solid and at the same time in conflict with the scientifically accepted perceptions.

THE STUDY

Aim:

The aim of the study is to investigate whether the students participating in EPA171 (Basic concepts in mathematics) have the basic geometrical knowledge that is related mainly with the similarity of shapes. How do they perceive the concept of similarity of shapes and how their intuitive knowledge may affect their understanding of similarity of shapes?

The three hypothesis of the study were:

1. The students have specific difficulties in basic concepts in geometry.
2. The students define similarity of shapes based on similar triangles or intuitive knowledge.
3. Intuitive knowledge affects their perception of similar shapes.

Subjects:

The participants in this study were 85 students of the Pedagogical Department of Education. 42 had mathematics as a major subject in higher secondary school, 39 had mathematics as a core subject and 4 did not specify.

Design of the study:

In order to examine the hypothesis of this study a test was administered to all the students that took part in the study. The students had 40 minutes available to respond to the test. The tasks of the tests were related with basic geometrical concepts (definition and construction of obtuse angle, application of properties of parallel lines and of the Pythagoras theorem in the solution of relevant exercises), definition of similarity of shapes, recognition of similar shapes as well as tasks which were used to examine whether the students had the necessary knowledge which is required to teach the lesson.

For the analysis of the results descriptive statistic as well as the implicative analysis have been used. More specifically for the data analysis the following elements of implicative analysis have been utilized: (a) The similarity tree-diagram which shows the variables according to the similarity they show (b) the hierarchical tree-diagram which presents the implicative relationships according to the order of significance.

Results:

The first hypothesis is confirmed in that basic knowledge of geometry where no special attention is given in school, such as the ability to give the definition of concepts. For the examination of this hypothesis which concerns basic geometrical concepts four questions were posed.

The first two questions were related mainly to the mathematical terminology which the students use. Students were asked to give a definition and construct an acute angle and it's supplementary. The analysis of the results shows that 83% can draw an obtuse angle but they only refer to the fact that it has to be bigger than 90^o but they do not specify that it has to be smaller than 180^o. 14% of the students who are mostly the ones that had mathematics as a major subject in higher secondary give a complete answer, whereas 3% of the students can not answer this basic question at all. In regard to the question related to the supplementary angles 95% give a complete answer since only one condition is requested (sum 180^o) and only 5% does not answer or gives a wrong answer.

The third question of the test concerns the use of basic relationship between angles and is based on parallel lines and the solution of a problem. These relationships are used quite extensively in secondary education something that leads students to a direct recognition and use of the relationships. This is illustrated by the results in the test since the majority (90%) that dealt with the task in question 3 managed to give correct answers.

The forth question of the test require a direct application of Pythagoras theorem twice. The application of Pythagoras's theorem without its proof constitutes a basic chapter in the teaching of geometry in secondary school. Thus 82,5% of the students were able to solve the exercise, 4,5% were able to solve only half of the task and 13% either gave a wrong answer or did not provide a response.

The second hypothesis was not fully confirmed. More than a third of the students could give a complete answer and a significant percentage of students referred to the similarity of the appearance of the shapes or the similarity of triangles. In order to examine this hypothesis the questions 5a and 5b were given.

In the question 5a, which asked students to answer "what are similar shapes?" only 36,5% of the students were able to give a complete answer (5iv). 21% referred to the similarity in the appearance of the shapes (5iii) and 14% referred to the similarity of triangles (5ii) which plays a significant role in the teaching of similarity in secondary education. A significant percentage of the students 12% referred to equality (5i), whereas 16% of the students either did not provide any answer or gave a wrong response (5i).

In order to examine whether the students have the ability to use the definition of similarity of shapes in an exercise regarding similar triangles, the second part (5b) of exercise 5 was asking students to find the relationship of similarity between given triangles. Differently to their responses in the 1st part of the exercise where 53% could give a complete answer, only 30% were able to reach a mid way to the solution. 17% could not solve the problem or did not give any response.

For the application of the theory regarding the relationships of similarity and also for the examination of the third hypothesis, exercise 8 was presented where students were asked to find which polygons are similar. In contrast to exercise 5b where they had to write some relationships algebraically in order to prove the similarity of the shapes, in this exercise, they needed mental representations of the relationships so that the right choices could be made. Just like in question 5, some students confuse similarity with the relationships regarding the appearance of the shape. That is probably why 87% responded that the parallelograms that have equal angles one side proportional and one side equal are similar (8i). It is very likely that they have reached this answer because both of them are parallelograms. 13% of the students believe that the rectangles are similar to the square (8iv) in the shape. This may be due to the fact that all three of them are parallelograms (appearance of the shape). Similarly 6% believe that the right angle triangle is similar to the scalene triangle (8v), most probably because both of the triangles have the same appearance. 80% recognize the similarity of the rectangles that are presented (8iii) and of the right

Question 5: What are similar shapes?
5i: referred to equality or no answer or wrong response.
5ii: referred to the similarity of triangles
5iii: referred to the similarity in the appearance of the shapes
5iv: complete answer

Exercise 8: students were asked to find which polygons are similar.
8i: responded that the parallelograms that have equal angles one side proportional and one side equal are similar.
8ii: recognize the similarity of the right angle triangles
8iii: recognize the similarity of the rectangles
8iv: believe that the rectangles are similar to the square in the shape.
8v: believe that the right angle triangle is similar to the scalene triangle

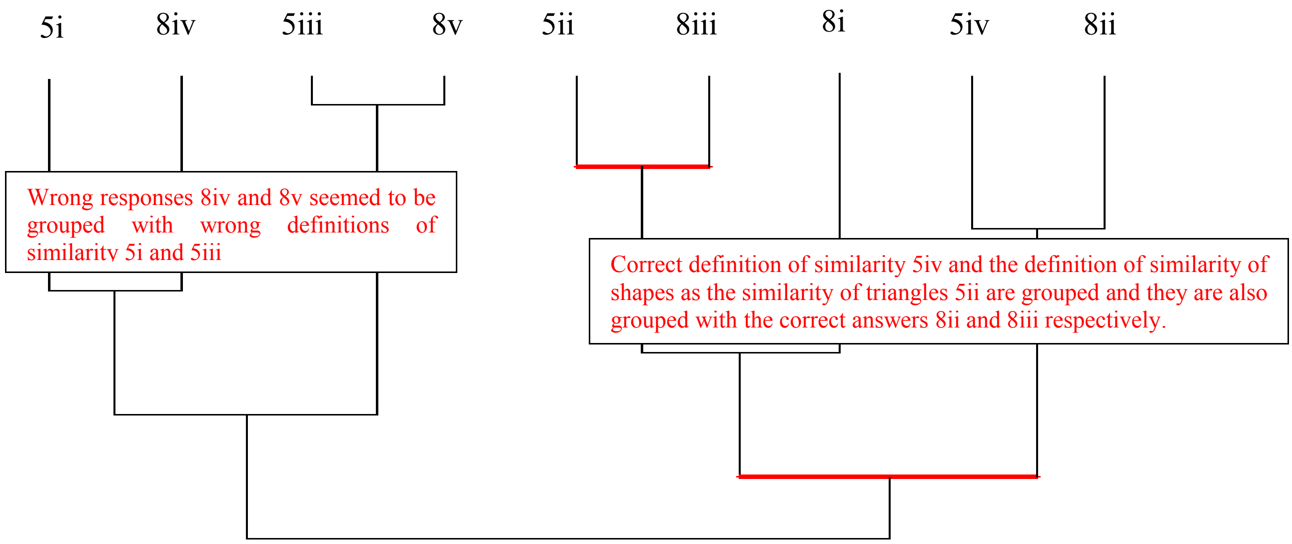

Figure 1: similarity tree diagram

In order to examine whether the definition that students give for the similarity of shapes affects their answer in exercise 8 where they are asked to recognize similar shapes we have used the similarity tree diagram (Figure 1). In the tree diagram the wrong responses in exercise 8 seemed to be grouped with the variables 8iv and 8v (similar shapes: square-rectangle, variable 8iv and right angle triangle and scalene triangle 8v) with the variables 5i and 5iii respectively of exercise 5 which refer to wrong definitions of similarity (5i: equality of shapes or wrong answer and 5iii: similarity in the appearance of the shape). In addition to this, the correct definition of similarity (variable 5iv) and the definition of similarity of shapes as the similarity of triangles (variable 5ii) are grouped and they are also grouped with the correct answers in exercise 8, and the variables 8ii and 8iii respectively. The variable 8i which is the wrong answer in 8 since it presents the similarity of two parallelograms that their sides are not proportional appear to be grouped with the correct definitions (mainly with the definition of similar triangles and the correct answer in regard to rectangles) and the correct answers. This may be due to the fact that most students perceive as the correct answer, something that indicates that students are depending on the perception of shapes and not the definitions and the properties of the shapes.

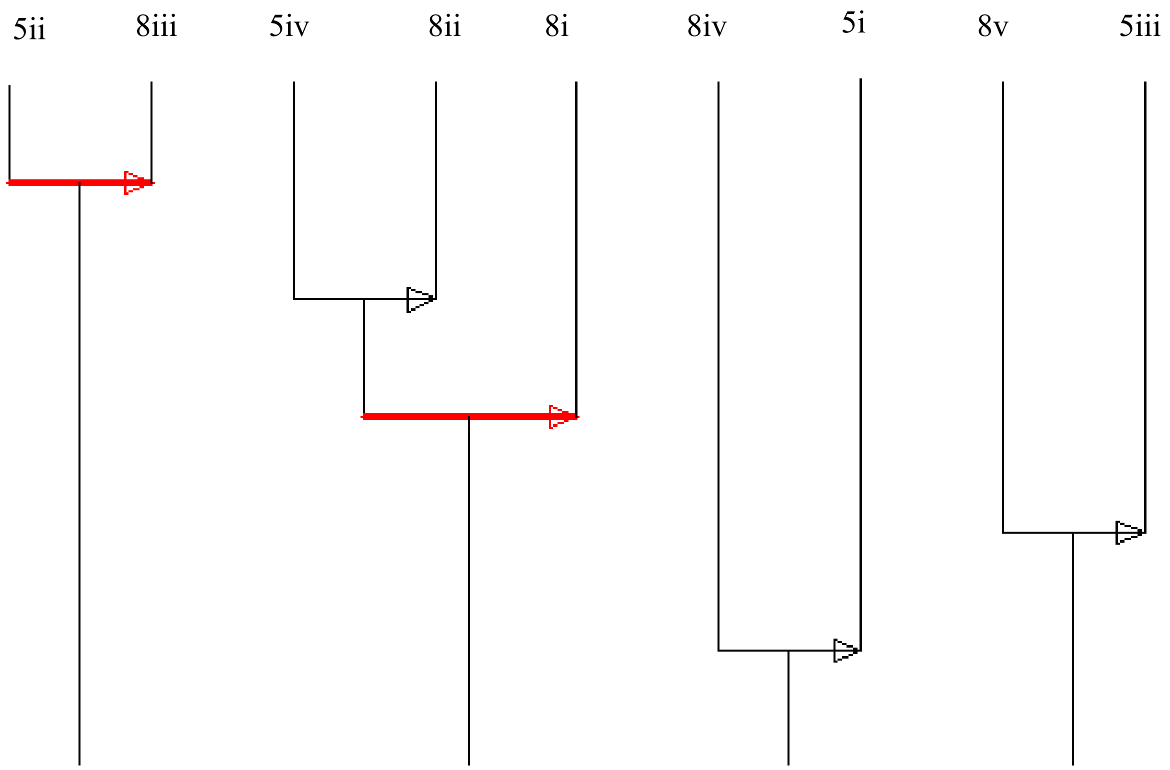

Figure 2: hierarchical diagram

The hierarchical diagram (Figure 2) shows that success in the definition constitutes success in the tasks in exercise 8, whereas in the wrong responses higher in line are the tasks in exercise 8, something that results to difficulty in giving a correct definition for the similarity concept.

CONCLUSIONS

The data of the study suggest that students have developed certain structures in regard to some concepts in geometry based on the teaching that they have received in school. The fact that in secondary education more emphasis is placed on the practical application of theory and less on the understanding of concept, leads to students΄ difficulty in giving complete definitions that require conditions, which in the practical application are implied without being presented (for example, the representation of an obtuse angle is never presented opposite to angles bigger than 180^o and that is why students never refer to the condition that an obtuse angle needs to be smaller than 180^o).

Based on this it appears that students are in a position to carry out operations by using certain formulas (Pythagoras's theorem) or recognize relationships in shapes which they were taught in school and they are expected to apply these in exercises similar to exercises 3 and 4 of this test.

For a spiral approach and development of geometry, it is important to know when it is possible to extract conclusions in regard to the concept of similarity as it is defined in geometry. As it appears from the data, a large percentage of students are not in a position to correctly define the similarity of shapes. However they are able to apply the relationships of similarity in triangles since teaching in secondary education is related to the similarity of triangles

In the search for similarity relationships in exercise 8 students influenced by their intuition found relationships that were based on the similarity of the appearance of the shape but they were not mathematically similar. This indicates that intuition affects their responses and their mathematical achievement since a number of these students have not received adequate mathematical training in order to base their answers on definitions, properties of the shapes and not on the perceptual appearance of the shape.

The data suggest that the wrong similarity definition leads to wrong responses in the practical applications, whereas the wrong representations of concepts create students' erroneous structures and definitions of the specific concepts.

In conclusion, in regard to the teaching of geometry at University level it is important to give more attention in the teaching of basic geometrical concepts and skills. As it was shown by the results in this study the teaching that many students receive in secondary school is inadequate, something that affects their perception and achievement in geometry. The lack or limited knowledge that students have, lead, to the use and translation of mathematical definitions based on wrong mental representations which are affected by intuitive knowledge and not by the correct mathematical definitions and correct representations.

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