BEHIND STUDENTS' SPREADSHEET COMPETENCIES: THEIR ACHIEVEMENT IN ALGEBRA? A CASE STUDY IN A FRENCH VOCATIONAL SCHOOL

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Research on the use of spreadsheet in mathematics education usually points out its potentialities in the learning of algebra. The link between spreadsheet and algebra is thus often seen in the direction "spreadsheet for algebra". This paper follows the opposite direction, i.e. "algebra for spreadsheet", by questioning the role of algebra in students' spreadsheet competencies. It reports a case study, based on computer tests, in the framework of a French research project studying students' spreadsheet uses and competencies. The results of the test show algebra raising out again, playing a role behind students' achievements and actions with spreadsheets. Algebra, Spreadsheet competences, Computer tests

INTRODUCTION

What role can technology play in mathematics education? Usually, didactic research approaches ICT questions through this direction, i.e. "technology for mathematics". This is the case for many studies on spreadsheets which consider this latter as a good tool to help pupils understanding algebraic concepts.

Here, we take the opposite direction: "what about algebra for spreadsheet?" by questioning the role algebra plays in students' mastery of spreadsheets. This issue comes from the analyses of experimentations in the context of *DidaTab*, a French research project studying students' spreadsheet competencies. To identify the basic competencies students have acquired, the *DidaTab* project realised tests of competencies in several classes. In the analyses of the results, the relation with algebra stands out again, raising issues on the relations between students' achievements and actions with this kind of software and their mastery of algebra.

In the first part, we give a quick description of the *DidaTab* project. The second part focuses on relationships between spreadsheets and mathematics learning. Then, to get a more concrete view of spreadsheet mastery problems, we detail the results of a computer test administrated to 17 y.o. students in a vocational marketing school. The results of this test put in perspective students achievements, actions, and software interaction understanding, with their knowledge (or their lack of) in algebra.

THE DIDATAB PROJECT

According to educational authorities of many countries, ICT has to be used in classrooms. In the case of secondary education, all countries have established detailed recommendations (Eurydice, 2004, p. 24). In general, using ICT to enhance subject knowledge or learning correct use of a word processor or a spreadsheet are part of the objectives at lower secondary level. But, if ICT seems to be included in prescribed curricula, we only have very few data about effective practices in classrooms and ICT competencies of students. Some data from PISA 2003 (Eurydice, 2005) provides interesting results (for example, that less than half of students are familiar with using a spreadsheet to plot a graph) but rely on declarative statements. We don't know whether students under or over estimate their competencies. To get a more comprehensive picture, we considered that it was not fruitful to take into account ICT as a whole, and decided to focus on specific software. Spreadsheets, prescribed in French curricula for ten years now, were a good indicator of ICT mastery. What do students learn about spreadsheets? Which basic competencies do they have acquired at the end of their schooling?

DidaTab (didactics of spreadsheet[1]) was a three year project (2005-2007) founded by the French ministry of research and dedicated to study personal and classroom uses of spreadsheets in French context. The methodology combined questionnaires, interviews (students and teachers), classroom observations, computer tests, content analysis of official curriculum texts, websites and resources, and some comparative studies with other countries (Belgium, Greece, Italy) have been made. As results (Blondel & Bruillard, 2006), we have an almost complete cartography of spreadsheets uses in the French secondary education, including an overview of personal uses, and we began to describe kinds of genealogy of uses, according to subject matters (e.g. mathematics, technology, social sciences, experimental sciences…). But we have not yet built a theoretical framework to explain spreadsheets uses and competencies of students. Some of these competencies relate to knowledge of mathematical nature, especially algebraic one. In a next part, we discuss this particular relation between spreadsheets and mathematics.

SPREADSHEET AND MATHEMATICS COMPLEX RELATIONSHIPS

In the title of this section, we play on the word "mathematics" to relate two points: mathematics as a school subject, this questions the place of spreadsheet within syllabus, or mathematics as knowledge that spreadsheets may bring into play, this questions the place of mathematics within the spreadsheet objects.

Spreadsheets within mathematics syllabus

Spreadsheets have been introduced at many different teaching levels and courses of the French Educational system. Part of the mathematics syllabus since 1997, first in middle school (grade 6 to 9) then in high school (grade 10 to 12), their place varies according to the school streams, as mathematics education appears under different aims. Two main tendencies can be distinguished, each of them promoting a different use of spreadsheets.

In the scientific streams, mathematics is a very theoretical discipline also used to select students. In this "abstract" approach of mathematics, spreadsheets appear as a

[1] In French, spreadsheet is "tableur". See http://www.stef.ens-cachan.fr/didatab/en/index.html for other information and results in English about *DidaTab* project

tool to serve the learning of mathematical concepts. Then spreadsheets' role is to support and enhance learning.

In some other streams, as vocational or literary, mathematics is considered as a more experimental subject oriented towards everyday life problems. This objective favours the use of different kinds of software such as spreadsheets, which allow a more concrete approach of mathematics opened on its everyday applications.

First vision leads to a very small place for working spreadsheet competencies. Moreover, as we will elaborate in the next section, using spreadsheet to enhance mathematical learning is "double-faced" as far as spreadsheet is not neutral on mathematical concepts. Second vision opens a larger place for building some spreadsheet competencies. In these streams, a hypothesis would be that students' difficulties in mathematics could be counterbalanced by some instrumental abilities and some mastery of this software[2]. But the situation is not as simple because of the specific relationships existing between spreadsheet and mathematics: spreadsheet mastery requiring mathematical knowledge.

Mathematics within spreadsheet objects

ICT use in mathematics education is a question among the more general problematic of technology use in human activity, studied in the field of cognitive ergonomics. A theory of instrumentation (Vérillon & Rabardel, 1995); developed in this field, provides a frame to tackle the problematic of the learning in complex technological environments. In this frame, an instrument is not *given* but *built* by the subject (Vérillon and Rabardel, 1995) through a progressive individual *instrumental genesis*. This genesis, is not *neutral*, instruments have impact on the *conceptualisation*. This idea of non neutral «mediation» between subject and tools provides a way to report on the strong imbrications that exist, and have always existed, between mathematics and the instruments of the mathematical work. It led to an instrumental approach in didactics that has been used in several researches on symbolic calculators (CAS) in mathematics education (Artigue 2001, Lagrange 1999, Drijvers 2000, Guin, Ruthven & Trouche 2005). What about spreadsheets?

Some "computer" objects, characteristics of spreadsheets, do not strictly correspond to mathematical knowledge transposed in a computer environment, even not to a computer transposition of school knowledge, but are *however linked with* mathematics. The basic principle of spreadsheet, which consists in connecting cells between themselves by "formula", gives an example of these objects, linking spreadsheet to the domain of algebra. Such a particular relation with mathematics is precisely the reason why many research in didactics from different countries (Ainley (1999); Arzarello et al. (2001); Capponi (2000); Dettori et al. (1995) or Rojano and Sutherland (1997)) give spreadsheets a role in the learning of algebra.at elementary stages, iden-

[2] For instance, in the literary stream, a place is given to concrete aspects of mathematics and this is precisely a stream where spreadsheets take an important part in the mathematics syllabus.

tifying them as tools of arithmetic-algebraic nature. Haspekian (2005a), having adopted an instrumental approach, showed that in spite of an apparent simplicity of use, it is not so evident for teachers to take benefit from these characteristics. The tool generates some complexity: spreadsheets transform the objects of learning and the strategies of resolution by creating new action modalities, new objects, and by modifying the usual ones (as variable, unknown, formula or equation…). Here are some examples.

In a paper and pencil environment, variables in formulae are written by means of symbols (a letter generally for the school levels concerned here). This variable ‘letter’ relates to a set of possible values (numerical here) and exists in reference to this set. In spreadsheet, let us take for example the formula for square numbers. The Fig.1 shows a cell argument A2 and a cell B2 where the formula was edited, referring to this cell argument.

	A	B
1		
2	5	=A2^2

Figure 1 A2 is the cell argument; B2 calculates the square of the value in A2

Here again the variable is written with symbols (those of the spreadsheet language) and exists, as with paper and pencil, in reference to a set of possible values. But this referent set (abstract or materialised by a particular value, e.g. 5 in Fig.1) appears here through an intermediary, the cell argument A2, which is both:

- an abstract, general reference: it represents the variable (indeed, the formula does refer to it, making it play the role of variable);
- a particular concrete reference: here, it is a number (in case nothing is edited, some spreadsheets attribute the value 0);
- a geographic reference (it is a spatial address on the sheet);
- a material reference (as a compartment of the grid, it can be seen as a box)

So, where in paper and pencil environment, we stick a set of values, a cell argument overlaps here, embarking with it, besides the abstract/ general representation, three other representations without any equivalent in paper and pencil (Fig.2).

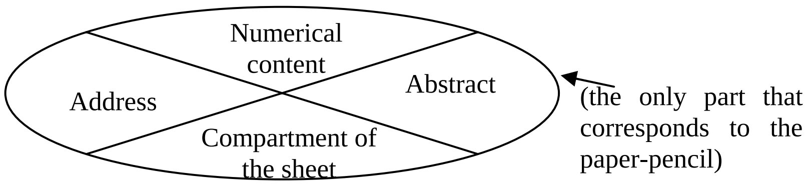

Figure 2: The “cell variable”

Other examples of the changes due to spreadsheets are given in Haspekian 2005a.

From an institutional point of view, these changes have different impact following the different way chosen to introduce algebra. As the recent ICMI study showed (Stacey et al., 2004), different aspects of algebra can be focused on: as a tool of generalisation, a tool of modelling, or a tool to solve arithmetical, geometrical or everyday life problems through what is called since Descartes, the « analytical method ». Following the case, different mathematics is brought forward: variables, formulae and functions on one hand, unknowns, equations and inequations on the other hand. In the French school culture, it is traditionally the analytic way that is chosen, the resolution, though equation solving, of various problems appears as emblematic of pupils introduction to algebra. Table 3 gives a quick insight of the distance between the algebraic culture in the French secondary education and the algebraic world carried out by spreadsheets.

"Values" of algebra	In paper pencil environment	In spreadsheet environment
Objects	unknowns, equations	variable, formulae
Pragmatic potential	tool of resolution of problems (sometimes tool of proof)	tool of generalization
Process of resolution	"algorithmic" process, application of algebraic rules	arithmetical process of trial and refinement
Nature of solutions	exact solutions	exact or approached solutions

Table 3: distance between different “algebraic worlds”

More generally, the mathematical culture sustained by spreadsheets is an « experimental » one: approximations, conjectures, graphical and numerical resolutions, implementing everyday life/ concrete problems, statistics… Thus, this vision fits with the aim of mathematics in particular streams of the French Education, especially where students not very good at mathematics are supposed to use spreadsheet with stronger objectives. It is thus interesting to investigate with students of non scientific streams and test them at the last year of their schooling (grade 12).

As we will see next, the computer test confirms the complex relation between spreadsheet and mathematics. Algebraic aspects; especially the use of cell-variables in formulae, stand out again as one of students’ main difficulties with the tool.

STUDENTS’ SPREADSHEET COMPETENCES: A CASE STUDY

We report here the example of a one hour computer test administrated in 2005 in a class of 13 students of vocational school[3] (17 year old) preparing a marketing diploma. After presenting the objectives and a brief description of the test, we first give an overview of the general results and then an analysis on the algebraic aspects that these results lead to focus on.

Objectives and description of the test

For this part of the *DidaTab* project, the objective was to assess students’ spreadsheet competences in a computer test. In order to design such tests, a first step consisted in the identification of basic spreadsheet skills, that have been actually organised in five categories (see below), then the definition of some general and simple tasks corre-

[3] This school is identified as rather difficult in the sense that students have behavioural difficulties and social problems.

sponding to each ability, and finally the construction of a database of skills, questions and tasks (for more details on this step of the project, and especially on the design of the tests, see Tort and Blondel, 2007).

From the database we selected 24 exercises relevant to the school year of the students and covering all categories of skills. Then, the students' mathematics teacher chose 11 exercises from this list according to the competences she assumed that her students have. With regard to the classification, the 11 exercises are divided in the following way:

1. "Cells and Sheets Editing" (3 were selected)
2. "Writing of formulae" (4 were selected)
3. "Translating data into graphs" (1 was selected)
4. "Managing data tables" (2 were selected)
5. "Modelling" (1 was selected)

The tasks were proposed in the computer test with increasing order of difficulty, in a spreadsheet file. Students had to answer directly within the tool and record their work at the end of the test. The collected data are constituted by these file records, observation and the complete recording of the actions for one of the students.

An overview of the results

Among the five categories of skills, clear differences between basic skills linked to superficial manipulations (not requiring knowledge of the contents) and abilities requiring deeper knowledge appear.

The best rates of success for the 13 students, concern cell formatting: italic (10), bold (11), date format (9). The results decrease then as the tasks require more understanding of spreadsheets objects. Some tasks requiring deeper knowledge of spreadsheet functionalities have been moderately achieved: recopying a format (6), sorting out data (6), or representing data with a graph by choosing the best type of representation (4). Finally, more specific knowledge as the conditional format (0) or specific displays either of numerical data (fractional format: 1) either of graphics (displaying labels on the X axis: 2) seem rather unknown from these students.

All exercises of the formulae category are part of the competences that have been failed in. Actually, the success rates for the four tasks of this category are the lowest of the test, varying from 0 to 2 good answers for each item: Writing a formula to calculate the AVERAGE of a line of data in adjacent cells (2), Writing a formula calculating a subtraction (0), a product (0), a division (0), Writing and copying down a formula using relative and absolute references (0, only 1 student answered: he gave a number…), Writing a conditional formula (using the IF function) (0).

Three main issues can be raised from these observations:

1) The inadequacies between the skills we thought students have and their actual level of competence. Students' abilities were clearly lower than expected.
2) The teacher tended as well to overestimate the skills of her students. The exercises she has chosen were globally too difficult.

3) The very bad results concerning the formulae category raise the question of spreadsheet's relation to algebra. Obviously, the formulae, the copying of formulae, the use of relative/absolute references as variables in formulae and the conditional formulae appear in students' results, as the less achieved competences.
In the next section we analyse this last point in more details.

Algebraic aspects in students' achievements

Competences just mentioned are all linked with algebraic knowledge of students, their understanding of the concepts of variable and formula. These results join other research in didactics of mathematics (Capponi, 2000, or Dettori & al. 2001). For Capponi, benefiting from spreadsheet potentialities requires from the user the understanding of some algebraic knowledge such as the notion of formula, and students' difficulties with spreadsheets show their needs in this domain: the work remains at the numeric level (data tables, numbers, operations) without reaching the level of an algebraic treatment (dynamic sheet of calculations, formulae).

About formulae

Looking further the tasks of the formulae category, we note that sometimes, not only the correct formula had not been found, but not even wrong formulae have been tried. Some students edit, instead of formulae, the corresponding arithmetic operations, some others edit directly the results they calculate by hand, but most of them do not answer anything. Another surprising point concerns the calculus of the average: we did not find any formula such as "(A5+B5 +… +N5)/ 14" or equivalents and only 2 students achieved the calculus of this average.

Observation during the test brings out some more elements. One of the students who succeeded in the average used the AVERAGE functionality (and seemed yet surprised to have directly the response). This can seem paradoxical, but to calculate the average of the given numbers, he directly used the function "AVERAGE" provided by spreadsheet; the references to the adequate cells are then automatically made. The student has to calculate an average, he has an "average" function (as a key of calculator), and he uses it without controlling more what this feature produces. The use of "AVERAGE" can thus mask its lack of understanding of what is really a formula in spreadsheet and the way it can be used. We have the same observation for the other student who used the average function. Finally, in the whole test, we did not find any other formula at all except these automatic formulae as average or sum. And the very surprising result that is coming to light with these analyses is that no student used a single relative reference in the entire test! According to us, this is precisely linked to the problem of the cell variable. Very few students used formulae which send back automatically the cell references[4] (such as SUM or AVERAGE) and not even a sin-

[4] The spreadsheet used in this experiment is Microsoft Excel. The interface provides buttons that you can directly activate and obtain the writing of a formula including cell references

gle student was able to write a formula which requires finding and entering the cell variable.

About the cell variable

The use of cells as variable in a formula seems more difficult than the use of formulae itself. In tasks which require a formula which does not automatically send back the cells references, either students do not find any formula or they use again an “automatic” formulae (AVERAGE or SUM) even when these functions have nothing to do with the task! For example, a correct answer to a task was a formula with a multiplication and one student has written the following formula: “=SUM (C12*10)”. SUM is here totally useless and used in a non standard way. The student invoked the function and turned the usual argument automatically written by the software (“=SUM (C12)” in this case) into a multiplication. By using this automatic formula, he did not enter himself the cell reference in the formula.

We have exactly the same phenomenon in another task: 2 students used SUM in both columns although the answer has nothing to do with a sum. One of them, after using the function SUM transformed the separating sign ":" in the syntax of this function into a subtraction (and the result is then correct)! All the others had not answered or had put either an operation or directly the numerical result instead of a formula. Once again, the use of cells as variables in a formula seems to be problematic, the type of functions as SUM or AVERAGE being apparently the only type of formula those students manage.

The problem of the cell-variable is also revealed by the use of the recopy. Here again, a deeper analyse of the answers of the whole class shows that it is not so much the fill handle that raises problem than copying downwards formulae. The recopy becomes problematic when it puts at stake some cells references which have to be incremented. This principle of the spreadsheet functioning, which is one of its most basic interesting feature, but which has an algebraic nature (the recopied cell playing the role of variable in the formula and the spreadsheet keeping the structure of the formula during the recopy), seems not to be understood by students. Results concerning recopy are quite different whether the recopy does concern cell-variables (copying a formulae with references: 0) or not (as copying down a date: 6).

In conclusion, it seems clear that these students do not master the ability of self editing a cell-variable in a formula or the way the recopy affects the cell references.

DISCUSSION AND PERSPECTIVES

The computer test reveals difficulties of grade 12 students, not so much in surface manipulation skills, but in their lack of understanding of algebraic concepts. Using a formula in a spreadsheet requires having understood the concept of "variable" in the spreadsheet (the cell argument in the formula). Using a recopy of a formula requires seeing the increment of the references produced by the recopy as a means for the spreadsheet to preserve the algebraic structure of the formula along the copy. The syntactic writing varies in every line but the algebraic structure is preserved. These types of knowledge were analyzed as algebraic competences which constitute a difficulty for students at the pre-algebraic level (Capponi, 2000, Haspekian 2005b).

In an exploratory study with younger students (grade 7), which consisted of a first approach to algebra through the use of spreadsheet, Haspekian (2005) found similar results. The students were asked to write, interpret or transform formulae. The observations have shown that the technique of using a formula and copying it down was the competence the longest to acquire and created most difficulties to the students. The difficulties were the following ones:

- comprehension of formulae (some remained in a use of arithmetical level of the spreadsheet);
- use of the fill handle, in particular at the beginning. But even afterward, when they experienced it several times, they had difficulty in appropriating it and its use was not systematic.

The experiment in vocational high school shows that students of grade 12 have the same difficulties as regard to these algebraic concepts embarked in the tool. It would be interesting to make paper-pencil test on their level in algebra to validate this hypothesis.

Another interesting point is the question of the modalities of spreadsheet learning. In the experiment of Haspekian (2005b), half of the students had followed a training course about spreadsheet (hands-on work) some months before the experiment. In particular they had seen formulae and recopy of formulae, and the teacher of this course had asserted that these students would have no difficulty with the tasks of the experiment. The results showed that they had the same difficulties and took the same time to answer the exercises that the other half of the students, those who had never used spreadsheet previously. Our computer test points out the same difficulties.

It is also interesting to compare with students of other professional fields or students of general fields. In *DidaTab*, another computer test has been administrated in a class of literary stream. Results show that students have less difficulty with recopy and formulae but have much more difficulties with manipulation skills. Yet in France, this stream is the general stream where spreadsheets use is the most strongly prescribed by curriculum... Certainly, as mentioned in part II, spreadsheets change too much the traditional mathematics that live in the general streams, teachers do not seriously enough take into account spreadsheet learning (not enough time devoted to spreadsheet learning, lack of structured training sessions, etc.) in these general streams, and many students are not able to manage important spreadsheet features. This result is confirmed by many interviews of students in the *DidaTab* project. Thus, our small experiment with 12th graders gives a rather different picture from general discourses about students great competencies. It seems that intrinsic difficulties of spreadsheet concepts are not sufficiently taken into account in mathematics education, even in the school streams where mathematics objectives and views are connected to every day life. In conclusion, students of professional fields who are mostly supposed to use spreadsheet due to their school profile are unfortunately those who are precisely blocked by their difficulties in algebra, and students of general streams with a better level in mathematics are those who will not “meet” spreadsheets enough because of the specificity of their stream…

To go further, it would be interesting to deepen the research with more computer tests in different levels and settings, and try to define thus kinds of students trajectories of uses.

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