SÁMI CULTURE AND ALGEBRA IN THE CURRICULUM

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Abstract: The Sámi culture's richness of patterns and structures give rise to the question whether an implementation of Sámi culture in the teaching of algebra might improve this teaching for the Sámi pupils. The Sámi have their curriculum but Sámi culture does not seem to be implemented in its algebra syllabus. Mathematical archaeology with respect to metonymy upon the Sámi cultural elements duodji and joik indicate possibilities for the teaching of algebra. But a remaining question is the Sámi mathematics teachers' view of the situation and of the suggested possibilities. The paper aims to prepare for empirical studies which focus on the Sámi mathematics teachers' mathematical archaeology upon their own cultural elements, as a basis for the teaching of algebra.

Key words: algebra; curriculum; mathematical archaeology; patterns; Sámi

BACKGROUND AND RESEARCH QUESTIONS

The Sámi are an indigenous people of the arctic who live in the northern part of Norway, Sweden and Finland, and in the Kola Peninsula of Russia (Kuhmunen, 2006). In 1990 Norway ratified the ILO Convention No. 169 concerning indigenous and tribal peoples in independent countries, and after this the Sámi in Norway got their curriculum (KUF, 1997). In the three latest national curricula, the Norwegian Ministry of Education has worked out special Sámi syllabuses for several subjects, but not for mathematics. One quite common interpretation of the curriculum is that the teaching of algebra should be the same for pupils in the Sámi core area in Northern Norway as for any pupil in our capital Oslo in the south. A quite different interpretation is that the Sámi should have their syllabus in mathematics.

This paper constitutes parts of a basis for a project which intends to research the possibilities of a Sámi algebra syllabus. The idea is that one researcher and one group of Sámi mathematics teachers together design and develop a teaching of algebra based upon Sámi cultural expressions. One lower secondary school in the Sámi core area wants to join a meeting where this project is introduced. The aim of this paper is to obtain important basis material for this important meeting. The basis material includes a) an analysis of the present situation regarding the teaching of algebra for Sámi pupils, and b) an analysis of some Sámi cultural expressions with respect to possibilities for a teaching of algebra. This leads to the two research questions of this study: 1: How is Sámi culture implemented in the algebra part of the national mathematics syllabus for lower secondary school? 2: If there are any (algebraic) structures to be found in Sámi cultural expressions, then how may these structures emerge?

THEORETICAL FRAMEWORK

Algebra

According to Lakoff & Núñez' (2000, p. 110), "Algebra is the study of mathematical form or "structure"". According to the latest TIMSS framework (Mullis et. al., 2007) algebra consists of patterns, algebraic expressions, equations/formulas and functions. Barton (1999) describes mathematics as a system of quantities, relations and space. His term "relations" is interpreted to be wider than just algebra. Fyhn (2000) uses the metaphor "pattern" similar to Lakoff & Núñez' (2000) "structure". Lakoff & Núñez (ibid.) focus on the terms "essence" and "structure" in their approach to algebra,

> Algebra is about essence. It makes use of the same metaphor for essence that Plato did – namely, Essence is form. …Algebra is the study of mathematical form or "structure". Since form (as the Greek philosophers assumed) is taken to be abstract, algebra is about abstract structure. (ibid., p. 110)

The analyses in this paper use the term algebra as by Lakoff & Núñez (ibid.).

Aesthetical Expressions as Basis for the Teaching of Algebra

Fyhn (2000) searched for and analysed relations between pupils' participation in different leisure time activities and their score in some TIMSS mathematics tasks from 1995 and 1998. The pupils were categorised according to their participation in different leisure time activities, activities which they performed at least once a week. The results pointed out some common features for the categories "creative-crafts-girls", girls who participate in activities that concern drawing or handicraft, and the "musicians", pupils who play an instrument. The creative-crafts-girls' mean test score was below the mean score, while the musicians scored high above the mean. Geometry was expected to be a domain where the creative-crafts-girls had their highest score, but their score in geometry turned out to be rather low. Actually these girls' highest scores were on tasks which tested the pupils' understanding of patterns. The musicians turned out to have a test score profile that to a large extent was parallel to the creative-crafts-girls' (ibid.). This gave raise to the idea of a teaching of algebra that is based on the pupils' understanding of patterns.

Symmetry is an important part of the two latest Norwegian mathematics syllabuses for primary school (KUF, 1996; KD, 2006b). But the approach to symmetry is limited to be via geometry. Norway give less priority to algebra in school, and algebra is the domain where the Norwegian pupils have their lowest score in the TIMSS (Trends in International Mathematics and Science Studies) (Grønmo, Bergem, Nyléhn & Onstad, 2008). This opens for new ways of teaching of algebra. Due to the Sámi culture's apparently richness of patterns and structures, a good implementation of Sámi culture in the mathematics subject syllabus might lead to an improved teaching of algebra for Sámi pupils. Before any approaches can be done towards the design of new approaches to school algebra, there is a need for investigating how and to what extent structures and patterns from Sámi culture are integrated in the mathematics syllabus.

Parts of this investigation will take place in cooperation with the teachers; the rest will take place in this paper. In addition the apparently richness of structures and patterns in Sámi cultural expressions need to be confirmed and described before they can be treated as a basis for the teaching of algebra.

Mathematical Archaeology

Mathematics can be integrated into an activity to such a degree that it disappears for both the pupils and the teachers. According to Skovsmose (1994, p. 94) "Mathematics has to be recognised and named, that is the task of a mathematical archaeology." It makes a difference whether the teaching is built upon situations that contain possibilities for application of mathematics or just for descriptive purposes. Many sorts of descriptive uses of mathematics can be possible as well as appropriate through mathematical archaeology; mathematics can be treated as an emerging subject (ibid., p. 90). It is important to a project, which contains mathematics as an implicit element, to spend some time on mathematical archaeology. The reason is: "If it is important to draw attention to the fact that mathematics is part of our daily life, then it also becomes important to provide children with a means for identifying and expressing this phenomenon" (ibid., p. 95). If there exists any algebra in the Sámi culture, it has to be implicit and hidden. A result of a mathematical archaeology may be that such algebra is recognised, named and described. A description of such algebraic structures may lead to an increased consciousness about possibilities for the teaching of algebra.

METHOD

The first research question will be answered by a) a survey of the development of the Sámi Curriculum in general and analyses of the treatment of algebra in it, b) a survey of the mathematics textbooks for Sámi pupils and analyses of their treatment of algebra, and c) analyses of the treatment of algebra in the national tests for Sámi pupils in mathematics and Sámi language. The second research question concerns the emergence of mathematics from elements in the Sámi culture. The research question will be enlightened by performing mathematical archaeology (Skovsmose, 1994) upon duodji and joik. Duodji is the name of Sámi craft, handicraft and art (KD, 2006a), while joik is the old Sámi folk music (Graff, 2001). The emergence of mathematics is categorised into three different levels; 1: recognition, 2: naming and 3: description.

ANALYSES

The Sámi Curriculum

The Sámi's right to take care of and develop their language and culture has not always been accepted in Norway. The *norwegianisation* (assimilation) of the Sámi has been extensive and long-lasting (Minde, 2005). The norwegianisation also has led to a disparagement of Sámi culture, and this gives reasons to believe that there are few tracks of Sámi culture to be found in the Norwegian curriculum. In 1989 the Ministry of Education published the Sámi syllabuses (KUD, 1989) as a special supplementary

booklet to the national curriculum for the compulsory school. The intention was to adapt the traditional syllabuses to the Sámi culture and the Sámi surroundings. Some subjects got their own syllabus, but mathematics did not. The 1997 national curriculum (KUF, 1996a; KUF, 1996b) included a special Sámi curriculum (KUF, 1997). The mathematics syllabus was identical with the Norwegian one except for the illustrations.

The national curriculum of 2007 (KD, 2006a) includes a special Sámi syllabus for seven subjects, but not for mathematics. Reasons for a particular Sámi mathematics syllabus are that the Sámi and the Norwegian numerals are structured differently (Nickel, 1994), and that the traditional Sámi measuring units are based on body measures and not on the SI-system (Jannok-Nutti, 2007). For the pupils who learn Sámi as their first language, Sámi units of measurement and mathematical methods are treated as Basic Skills in mathematics, as an integrated part of the subject Sámi language (KD, 2006c). And "skills in mathematics require understanding of form, system and composition" (ibid., p. 3). According to Lakoff & Núñez (2000), this is algebra. But according to the curriculum, this is part of the syllabus in Sámi language. For the pupils who learn Sámi as their second or third language, basic skills in mathematics mean general concept development, reasoning and problem solving as well as the understanding of quantities, amounts, calculations and measurements (KD, 2006a). For these pupils the syllabus has no aims regarding their understanding of form, system and composition.

In the Norwegian national curriculum (KD, 2006b), the subject area "numbers and algebra" for the lower secondary school is presented this way

> The main subject area numbers and algebra focuses on developing an understanding of numbers and insight into how numbers and processing numbers are part of systems and patterns… Algebra in school generalises calculation with numbers by representing numbers with letters or other symbols. This makes it possible to describe and analyse patterns and relationships. Algebra is also used in connection with the main subject areas geometry and functions. (ibid., p. 2)

As for the pupils who learn Sámi as their first language at school, the understanding of form, system and composition may be integrated with descriptions and analyses of patterns and relationships in the mathematics lessons. But this message is only implicit in the curriculum. Thus an interesting question is whether the Sámi culture is integrated in the teaching of algebra for the Sámi pupils.

Textbooks

The Sámi mathematics textbooks are Norwegian textbooks translated into Sámi language. The Norwegian Directorate for Education and Training (Udir, 2004) presents two mathematics textbooks in Sámi language for lower secondary school; one of them is approved for the curriculum of 1997, and the other one is Finnish. For economic reasons Norway offer the lower secondary school pupils no Sámi mathematics

textbooks which are approved for our latest curriculum. However, these pupils have their right to get appropriate books: The United Nations' Declaration on the Rights of Indigenous Peoples Article 14 claims that "Indigenous peoples have the right to establish and control their educational systems and institutions providing education in their own languages, in a manner appropriate to their cultural methods of teaching and learning" (UN, 2007, p. 6). The Sámi parliament, the Sámediggi, is an elected representative assembly for the Sámi in Norway (Kuhmunen, 2006). The Sámediggi's Youth Committee underlines the importance of getting Sámi textbooks. They sent an open letter to the Norwegian Minister of Education where they demand that the Ministry carry out necessary actions in order to improve the school-days for Sámi children (Nystø Ráhka, 2008). The textbook situation for Sámi pupils is far from satisfactory. Thus it is not any surprise that no attention is paid towards including Sámi culture in the algebra paragraphs in the existing textbooks.

National Tests

From 2003 Norwegian pupils have taken part in national tests as part of a national system for quality assessment (KD, 2003). From 2007 the mathematics tests were replaced by tests in mathematics as a basic skill in every subject. One result of this is that algebra is no longer part of the tests. The tests are translated from Norwegian to Sámi language; the Sámi pupils are offered no special tasks. The Ministry of Education and Research have decided that pupils who have Sámi as their first or second language will be tested in mathematics as a basic skill in this subject (KD, 2007c; KD, 2008). The Norwegian Directorate for Education and Training will carry out the translations of the mathematics tests into the three Sámi languages (ibid.). The national tests in mathematics as a basic skill do not reflect the pupils' achievement of any goals which are particular for the Sámi curriculum, and the algebra goals for the Norwegian pupils are neither reflected in these tests.

Duodji and Joik

The ornamentations in Sámi handicraft, *duodji*, and the Sámi folk music, *joik*, are both rich on structures and patterns. This claim is based upon the doctoral dissertations of Dunfjeld (2001) and Graff (2001). According to Dunfjeld (2001) the Sámi people's understanding of their own ornamentation differs from the pure formal understanding of ornamentation that we find in Western Europe. Thus she introduced the term "Tjaalehtjimmie" which has a meaning beyond pure decoration; "it is composed by signs, ornamentals and symbols which together may give meaning "(ibid., p. 102, my translation). For example may the meaning of the triangular engraving be decided from its localisation and orientation related to other symbols in a composition like in figure 1.

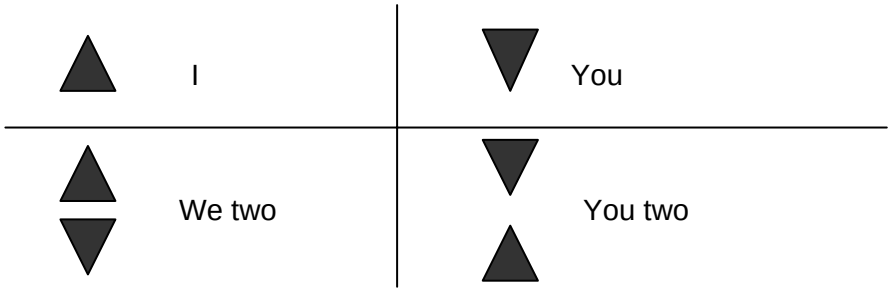

Figure 1. How the triangular engraving may symbolise personal pronouns in first and second person, and in singular and dual (Parts of figure from Dunfjeld, 2001, p. 109)

In duodji there are several more or less advanced plaited patterns. Fyhn (2006) describes hair plaiting by first splitting the hair into three equal parts. Plaiting can be further described by numerous repetitions of "take the right part and cross it over the mid-part. Then take the left part and cross it over the mid part". The right part, whichever it is, can refer to all of the three parts of the hair, and so is for the mid part and the left part as well. This is what we understand with conceptual metonymy (Lakoff & Núñez, 2000), and this exists outside mathematics.

> This everyday conceptual metonymy …plays a major role in mathematical thinking: It allows us to go from concrete (case by case) arithmetic to general algebraic thinking… This everyday cognitive mechanism allows us to state general laws like "x + y = y + x", which says that adding a number y to another number x yields the same result as adding x to y. It is this metonymic mechanism that makes the discipline of algebra possible, by allowing us to reason about numbers or other entities without knowing which particular entities we are talking about. (ibid, p. 74-75)

According to the curriculum the Sámi ornamentations are geometry (KD, 2006a). Dunfjeld (2001) denotes these structures as geometry, too, and she refers definite to figures as triangles, rhomboids, squares and rectangles. Her mathematical archaeology is at level two; naming. When she refers to the organisation of the geometrical figures and the patterns they shape, she does not denote it as mathematics anymore. Fyhn's (2006) description of ornamentation as metonymy is mathematical archaeology at level three, description.

Graff (2001) claims that researchers have focused on joik from different perspectives: as text, as melodies and rhythms, and as communication. To "joik" a person means to perform a particular joik which is dedicated to this person; the joik is an expression with a meaning (ibid.). The pitch constitutes an analogy for conceptual metonymy in music, when two or more people sing together. The structure of the song is given on beforehand; independent of what particular pitch to use. Graff (ibid.) points out, among other things, that the melodic motive in joik is based upon melodic patterns which in turn might have different shapes. The structuring of the joiks which he investigated, show that a rhythmic motive might be repeated throughout the complete joik (ibid.). Algebra is the study of mathematical form or "structure" (Lakoff & Núñez, 2000), and joik is just a way of expression that like other music is built up by

given structures. According to the curriculum, the understanding of how different patterns and structures influence artistic and musical expressions is part of mathematical skills in the subject music (KD, 2006a). Graff's (2001) term "rhythmic motive" is the name of a structure and could be denoted as mathematical archaeology at level two. He gives thorough descriptions of the structures as well, and he uses words like "ascending –descending melody line (inverted U-form)" (ibid., p. 210, my translation) and "transposition" (ibid., p. 214, my translation). But there is no mathematics connected to the names and the descriptions of these structures. The structures that constitute a basis for duodji ornamentations and for joiks may be identified and described by mathematical terms. The process in which algebra is emerging from these aesthetic expressions can be carried out as mathematical archaeology (Skovsmose, 1994) at three levels. But because joik as well as duodji express more than just aesthetics and structure, the meaning aspect need to be focused and enlightened.

CONCLUSION

The Sámi curriculum (KD, 2006a) offers a special Sámi syllabus for several subjects, but not for mathematics. "The understanding of form, system and composition" is part of the syllabus for Sámi as first language. Together with "descriptions and analyses of patterns and relationships" from the algebra syllabus, this opens for an integration of elements from the Sámi culture in the mathematics lessons. But that depends on whether the Sámi mathematics teachers are aware of and agree to these possibilities, and how the Sámi language teachers approach "form, system and composition" in their lessons. Due to the norwegianisation (Minde, 2005) there are reasons to believe that the teachers are not aware of the possibilities of integrating elements from their culture in the teaching of algebra.

The United Nations' Declaration on the Rights of Indigenous Peoples (UN, 2007), states that the Sámi lower secondary school pupils have their right to get appropriate mathematics textbooks in their own language. There are Sámi versions of Norwegian textbooks for primary school and for some of the grades in lower secondary school, but many of these books are based on a lapsed curriculum. And no special attention is paid towards including Sámi culture in the algebra parts of these textbooks. The lack of Sámi mathematics textbooks results in extra work for the teachers. Sámi pupils are offered translated versions of the Norwegian national tests in mathematics as a basic skill in every subject. The fact that these tests do not concern any algebra is an example of how Norway gives less priority to algebra in school. The national tests neither reflect the pupils' achievement of any goal in the Sámi curriculum. Aesthetic expressions may become a resource in the teaching of algebra: According to the Sámi curriculum (KD, 2006a) the relations between aesthetics and geometry are elements in the work with duodji decorations, while the music syllabus focus on the understanding of different patterns and structures. No connection between aesthetics and algebra is found in the Sámi curriculum.

One question for the further research is whether and to what extent the Sámi mathematics teachers find the project relevant and worthwhile taking part in. One more question is how metonymies might function in bridging the gap between Sámi cultural discourses and the algebra teaching discourse. The term "discourse", is here used as by Foucault (2004, p. 53), "…discursive practice is a place in which… objects is formed and deformed." These questions are closely interwoven and the further development of the project depends on the meeting between the researcher and the teachers. Maybe the teachers really want to join the project. But one other outcome is that the teachers dislike the ideas of creating an algebra teaching based upon Sámi cultural expressions. Another outcome might be that the teachers give priority to other parts of mathematics than algebra at the moment. A third possible outcome is that the teachers want to take part in the project, but that metonymies turn out to be less useful than they seem at the moment.

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