THE EFFECTS OF MULTIPLE REPRESENTATIONS-BASED INSTRUCTION ON SEVENTH GRADE STUDENTS' ALGEBRA PERFORMANCE

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The purpose of this study was to investigate the effects of multiple representation-based instruction on seventh grade students' algebra performance. The study was conducted on four seventh grade classes from two public schools lasting eight weeks. For assessing algebra performance, three instruments called translations among representations skill test, objective based achievement test, and Chelsea diagnostic algebra test were used. The analyses were conducted by using multivariate covariance statistical model. The results pointed out that multiple representation-based instruction had a significant effect on students' algebra performance compared to the conventional teaching. In addition to this, students from experimental groups found this way of teaching fruitful.

INTRODUCTION

Various meanings can be given to the concept of "representation" in connection with the teaching and learning of mathematics. Seeger, Voight, & Werschescio (1998) summarized some of those definitions in very general terms as follows: "…representation is any kind of mental state with a specific content, a mental reproduction of a former mental state, a picture, symbol, or sign, symbolic tool one has to learn their language, a something "in place of" something else".

Multiple representations can be generally defined as providing the same information in more than one form of external mathematical representation by Goldin and Shteingold (2001). The usage of multiple representations in mathematical learning was investigated in depth by Janvier who defined it "understanding" as a cumulative process mainly based upon the capacity of dealing with an "ever-enriching" set of representations (Janvier, 1987, p. 67). There are two important key terms in a theory of representation that are; "to mean or to signify, as they are used to express the link existing between external representation (signifier) and internal representation (signified)" (Janvier, Girardon, & Morand, 1993, p. 81). External representations were defined as "acts stimuli on the senses or embodiments of ideas and concepts", whereas internal representations are regarded as "cognitive or mental models, schemas, concepts, conceptions, and mental objects" which are illusive and not directly observed (Janvier, et. al., 1993, p. 81).

Another approach to the theory of multiple representations which is called Lesh Multiple Representations Translations Model (LMRTM) has been suggested by Lesh (1979). His theory draws the theoretical framework of this study since he improved a

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model involving translations among representational modes and transformation within one representational mode. According to Lesh, Post and Behr (1987), representations are crucial for understanding mathematical concepts. They defined representation as “external (and therefore observable) embodiments of students’ internal conceptualizations” (Lesh, et al. 1987, p. 34). This model suggests that if a student understands a mathematical idea she or he should have the ability of making translations between and within modes of representations. According to this view, a good problem solver should be able to “sufficiently flexible” in using variety of representational systems. He claimed further, “As a student’s concept of a given idea evolves, the related underlying transformation/translation networks become more complex; and teachers who are successful at teaching these ideas often do so by reversing this evolutionary process; that is, teachers simplify, concretize, particularize, illustrate, and paraphrase these ideas, and imbed them in familiar situations” (p. 36).

A MULTIPLE REPRESENTATIONS TRANSLATION MODEL

After reviewing a number of theories about multiple representations, this study emphasizes investigating particularly students’ ability to use the given representational mode for solving problems, and to make translations among the representational modes. A multiple representational translations model combined from the models belonging to Lesh and Janvier would seem to be perfect modeling for this research study. The five distinct representational modes; namely, manipulatives, real-world situations, written symbols, spoken symbols, and pictures or diagrams in LMRTM were directly included in the model of this study. Some of those representational modes were named differently referring the Janvier Representational Translation Model (JRTM). Instead of “written symbols” from LMRTM, wording of “formulas” from JRTM was included in this study. Besides in lieu of the combination of “situations, pictures, and verbal descriptions”, the researcher decided to use those representational modes separately. Therefore instead of “situations, pictures, and verbal descriptions” in JRTM, “manipulatives,” “pictures or diagrams,” and “spoken symbols” were taken from LMRTM. “Tables” and “graphs” were taken separately from JRTM. Janvier’s Representation Translation Process was revised in light of the Lesh (1979) ideas as appeared in Table 1.

Table 1: The combined model of Lesh and Janvier for translations among representation modes

From \ To	Spoken Symbols	Tables	Graphs	Formulas (Equations)	Manipulatives	Real Life Situations	Pictures
Spoken Symbols	_	Measuring	Sketching	Abstracting	Acting out	Acting out	Drawing
Tables	Reading	_	Plotting	Fitting	Modeling	Modeling	Visualiz-ing
Graphs	Interpre-	Reading Off	_	Fitting	Modeling		

	tation						
Formulas (Equations)	Reading	Computing	Sketching	_	Concretizing	Exemplifying	Localizing
Manipulatives	Describing	Exemplifying	Concretizing	Symbolizing	_	Simplifying, Generalizing	Drawing
Real Life Situations	Describing	Exemplifying	Plotting	Modeling, Abstracting	Particularizing	_	Modeling
Pictures or Diagrams	Describing	Describing	Sketching	Abstracting	Constructing	Situationalizing	_

SIGNIFICANCE OF THE STUDY

The issue of what instructional approaches should be used in algebra classes does not have a single and clear answer. No matter which instructional approach is used, the primary goal of mathematics instruction should be to help students in forming conceptual understanding. Janvier (1987) mentioned that if teachers enrich their algebra classrooms by placing multiple representations, the students can more efficiently make connections between the meaning of algebraic concepts and the way of representing them, therefore they simply "go for the meaning, beware of the syntax" which results in conceptual understanding.

The improvement of mathematical understanding and representational thinking of students require flexible use of multiple representations and the interaction of external and internal representations (Pape & Tchoshanov, 2001). Since making meaningful translations in representational modes plays a crucial role in acquisition of mathematical concepts and there are still unanswered questions about the instructional outcomes of using multiple representations, we believe that it would be worth to investigate the multiple representations in this respect.

Since this study focuses on the effects of multiple representation-based environments in mathematics classroom, its results should help mathematics educators who seek alternative pedagogical instructions in classroom settings. Furthermore, if a teacher is aware of his/her students' understanding of the multiple representations and what kind of learning is supported by multiple representation-based environments, s/he can better choose and utilize appropriate type of methods, manipulatives, or activities to meet the needs of students. Moreover, providing students with a multiple representation-based algebra instruction would promote a conceptual shift to thinking algebraically. Therefore, receiving such kind of instruction makes students more competent in the area of algebra.

RESEARCH QUESTION

The purpose of this study is to examine the effects of a treatment based on multiple representations on seventh grade students' performance in algebra, and this study attempted to answer the following research question;

"What are the effects of the multiple representations-based instruction compared to conventional teaching method on seventh grade students' algebra performance when students' gender, mathematics grade of previous semester (MGPS), age, prior algebra level are controlled?"

METHOD

The research question was examined through a quasi-experimental research design since this study did not include the use of random assignment of participants to both experimental and control groups. The target population of this study consists of all seventh grade students from public schools in Çankaya district in Turkey. There were 103 public schools in this region. However, two schools from this district were determined as the accessible population. There were 2 seventh grade classes in School A, and 7 in School B. One experimental and one control group were selected from both schools. There were 15 girls and 13 boys in experimental group and 16 girls and 13 boys in control group taken from School A. On the other hand, the experimental group from School B consists of 17 girls and 21 boys and in the control group the number of girls and boys were equal, that is 18. The participants in this study ranged in age from 11 years to 14 years old.

INSTRUMENTS

To assess algebra performance, three distinct instruments namely Algebra Achievement Test (AAT), Translations among Representations Skill Test (TRST), and Chelsea Diagnostic Algebra Test (CDAT) were used. The rational of using combined instruments is to perceive algebraic learning in a multi dimensional way. It includes procedural, conceptual, and translational knowledge and skills in its nature (Lesh, Landau & Hamilton, 1983). By utilizing three instrument it was aimed to assess algebraic learning within its all dimensions and each instrument was tried to assess different aspect of algebra learnings. It can be claimed that when a student gets higher scores from three instruments s/he can be called as successful in algebra since getting high score means that s/he can use procedural algebra knowledge in problem solving, understand algebra conceptually, and also make simple translations among representations.

Among three instruments Algebra Achievement Test (AAT) was administered to analyze students' computation skills in algebra intensively. 10 essay type questions were used in this instrument which combines traditional school algebra test items including symbolic manipulations and computations in algebra. The items which are related with the procedural skills in school algebra are criterion-referenced tasks addressing key learning goals specified in the Mathematics Curriculum for Elementary Schools, published by Turkish Ministry of National Education (MEB, 2002). The required time for this instrument was 30 minutes. The internal reliability value of Cronbach alpha was calculated as .90. To score the students' responses to each question in AAT four-point rubric was used. The highest point of 4 indicated a complete understanding of underlying mathematical concepts and procedures while the lowest point of 0 was given for irrelevant or no responses. The minimum and maximum possible scores from the test items are 0 and 40 points, respectively. Students who got scores above mean score of the group was accounted as high achievers.

Another instrument for assesing students' algebra performance was the Translations among Representations Skill Test (TRST). The purpose of this test was to obtain data about students' abilities in making translations among different representational modes. TRST contains 15 open-ended items which were designed to measure skills of translation among representations, use of certain representations, and creating new representations. The items in TRST required a translation from one representation type to another, such as from tabular representation to graphical one. In the last two items all type of representations were required to solve the problem. Duration of the test was 40 minutes. It was scored by using a three-point holistic scoring rubric. The highest point of 3 was awarded for responses showing that the problem was solved correctly and that the appropriate translations among representations were used. The lowest point of 0 indicated if the response is completely wrong or immaterial to the. The possible minimum score was 0, and the possible maximum score was 36. The internal reliability estimate of TRST was found to be .79 by calculating the Cronbach alpha coefficient.

The last instrument was Chelsea Diagnostic Algebra Test (CDAT) which was developed by the Concepts in Secondary Mathematics and Science Team (Hart, Brown, Kerslake, Küchemann, & Ruddock, 1985) to determine 13-15 years old children's algebraic thinking levels. This test was designed to measure the conceptual knowledge of elementary algebra. In CDAT there are six different categories of interpreting and using the *"letter"*. Apart from these six categories, four levels of algebra understanding were developed with respect to the children's responses and the items themselves. In Level 4, children can deal with the items that require specific unknowns and which have a complex structure (Hart, et al., 1989) and they can be accounted as successful in algebra. The students answered the items in this test approximately in 60 minutes. The discrimination power of the items ranged from 0.20 to 0.60. Reliability measure as based on KR-20 coefficient was found to be 0.93. There were 53 items in the adapted version of CDAT. The possible minimum and maximum scores were 0 and 53 respectively. Besides, CDAT was used as a pretest to find out experimental and control group students' conceptual algebraic knowledge before the intervention. It was considered that seventh grade students' algebra knowledge coming from their previous mathematics background might affect the experiment therefore CDAT as pretest was also taken to MANCOVA statistical model as a profounding variable.

TREATMENT BASED ON MULTIPLE REPRESENTATIONS

For this study, the instructional design for experimental groups consists of daily lesson plans in which several activities took place. There were 21 activities which were involved in the lesson plans of the instructional unit in order to aid in teaching of a unit of algebra. All 21 lesson plans which had distinct contexts and problem situations were developed in order to reflect the procedure of translations among representations, transformations within a specified representation, usage of any representational mode in dealing with algebraic situation. In particular, students were required to learn constructing the multiple representations of algebraic situations, including expressing them in tables, graphs, and symbols. Instead of teaching these representation skills in isolation, it was anchored within meaningful thematic situations. Instead of direct instruction in how to construct and use mathematical representations in algebra, students were only guided in the activities to explore different representations and to develop their understanding of each one. In experimental groups students were frequently given tasks that require them to make translations among different representations. This approach was used to present and develop concepts from verbal, algebraic, graphical, and tabular standpoints. To illustrate, for instance, a concept first introduced a numerically intuitive approach in which tables were used to collect and work on data. Then a verbal representation was used to verbally complement the relationship among numbers in the tabular representation. Finally, a transition was made to the algebraic representation. The usage of multiple representations varied for each activity presented in this treatment. For instance, for the topic of equations, first the tabular representation then the verbal representation were constructed; however, for conceptualizing the concept of graph, first, the algebraic representation, and then the other representations were used.

The actualization of treatment can be illustrated in one activity namely; *"Inequalities"*. In this activity students were responsible to find out the main characteristics of inequalities using the tabular representation. At the beginning the activity sheets were given to the students, and then they examined the activity. They filled the given table by required numbers, and then the translation from one representation to another came. For this, the daily life situations and the algebraic representational modes were selected. Students were required to give one daily life example to the inequality of "$x–3<7$". Students' examples were like;

"There are x number of teachers in one school, then 3 of them are appointed to another school, and the number of the remaining teachers was less than 7".

"Let us say that the number of the desks in our class was x, we get rid of 3 of them, then there are less than 7 desks in our class".

After getting students translations among representations, all of them were discussed in class. It is compulsory for the students to keep the activity sheets in the folder that the researcher gave them, since they did all the works on those papers. They were also responsible to bring their folder to the class every mathematics lesson.

In the treatment, particularly the translations among representational modes were stressed and valued by the researcher. In conventional algebra teaching, however, translation among representations might occur only when the students are required to draw a graph. In this case, instead of constructing a table to represent the given equation, they only identified two points where the line passess through. Then, by the help of this information, a graph could be drawn. However, the multiple representations-based instruction emphasizes the translations from variety of representational modes to the other modes. Therefore, students could have the opportunity to notice that one mathematical concept can be represented in several ways and these ways can be complementary to understand this concept. The same task of drawing the graph of a linear equation is taken in a way that, students analyze the equation through daily life situations, plain language, tables, and graphs. In that sense, drawing the graph of an equation is not an end but it is a means of interpreting the existing mathematical situation. The treatment lasted eight-weeks. Each week experimental groups received four lesson hours, with each session lasting 40 minutes.

RESULTS

To test the null hypothesis related to the research question, the statistical technique of Multiple Analysis of Covariance (MANCOVA) was used for comparing the mean scores of control and experimental groups separately on the AAT, TRST, and CDAT. MANCOVA was carried out by putting experimental groups together as a one experimental group and control groups as one control group as well (Cohen & Cohen, 1983).

Initial descriptive analysis revealed that the experimental groups had the higher scores on all the instruments compared to the control groups. Before conducting MANCOVA the assumptions called normality, multicollinearity, homogeneity of regression, equality of variances, and independency of observations were verified (Green, Salkind, & Akey, 2003).

The MANCOVA results revealed that, there was a significant effect of two methods of teaching on the population means of the collective dependent variables of seventh grade students' scores on the AAT, TRST, and CDAT after controlling their age, the MGPS, and PRECDAT scores. 37% of the total variance of MANCOVA model for the collective dependent variables of the AAT, TRST, and CDAT was explained by group membership of the participants. Using the Wilks' Lambda test, significant main effects were detected between the groups experimental group and control group (λ = .63, p = .000). Therefore, the results of this study were of practical significance. The significant finding of a group effect from MANCOVA, allowed further statistical analysis to be done in order to determine the exact nature of significant differences found in main effect. Therefore univariate analyses of covariance (ANCOVA) were carried out on each dependent variable in order to test the effect of the group membership. From the analyses, it can be stated that, multiple representation based instruction has a significant effect on the dependent variable scores of CDAT [$F(1,125) = 38.005$, $p = .000$)], TRST [$F(1,125) = 25.942$, $p = .000$], and AAT [$F(1,125) = 18.271$, $p = .000$]. Furthermore, for the observed treatment effects, it was obvious that the values of eta squared for the scores of the CDAT, TRST, and AAT were .233, .172, and .128 respectively which are equal to the medium effect size. This explains 23% of the variance in CDAT, 17% of the variance in TRST, and 13% of the variance in AAT related with the treatment. Power for the scores of the CDAT, TRST, and AAT were found as 1.00, .92, and .78 respectively. Step-down analysis was carried out as significant MANCOVA follow up analysis. By the help of this analysis, the unique importance of dependent variables which were found as significant in the MANCOVA analysis was investigated. Since there are three significant dependent variables namely, CDAT, TRST, and AAT, three step-down analyses were conducted. By doing so, any possible variance overlap among the dependent variables was planned to be detected. According to these results, the effect of multiple representation-based instruction had still significant effect on each dependent variable.

DISCUSSION

This research study has documented that, compared to conventional instruction, multiple representations-based instruction did make a significant influence on the algebra performance of seventh grade students. There might be various reasons to this result. Visualization of algebraic objects, connections among algebraic ideas, and the improvement of translational abilities in algebra problem solving (Lesh, Post, & Behr, 1987) can be counted as what multiple representations-based instruction provide for students. By the help of this instruction, students avoid memorization in algebra learning, and understand concepts meaningfully. As suggested in Swafford and Langrall`s (2000) study; multiple representations-based instruction promotes conceptual understanding of algebra and makes students conceptualize algebraic objects. The results of this study are supported in the literature by numerous studies. One of them is Brenner`s (1995) and her colleagues study. They conducted only 20 days multiple representations unit including variables and algebraic problem solving. After treatment they implemented four instruments related to algebra learning to the seventh and eight graders. Significant difference was found between experimental and control group of students in favor of the students in experimental groups. The findings of this study are also consistent with the findings of previous studies (Ozgun-Koca, 2001; Pitts, 2003) that provided evidence for the effectiveness of multiple representations-based instruction in engaging students in meaningful algebra learning. Additionally, in Herman`s (2002) study similar results were found. It was stated that after multiple representation based instruction in college algebra course, students were better able to establish connections between varieties of representational modes.

This study confirmed the need for considering other kinds of representations, such as; representations used in graphic calculator and computer programs or representations that students create and unique for them. As it was suggested by Ozgun-Koca (2001), computer-based applications can be used to provide linked and semi-linked representations, and graphical form of representations. These applications can make students to abstract mathematical concepts from virtual world. Besides, allowing students to create their own representations for solving algebra problems makes them more creative and flexible in mathematics (Piez & Voxman, 1997). In this study it was observed that, students were mainly restricted by four types of representations which are tabular, graphical, algebraic, and verbal. This can be due to the activities or researcher's emphasize on those representation types. However, students should be given an opportunity that they can use representations that they invent or create. Moreover, it can be suggested that future research can focus on teachers and teaching strategies in algebra classrooms. All of the data for this study was collected from students. Future research could combine data from students and their teachers, because teachers have also impact on shaping students' representation preferences. What teaching strategies and representation types are used within algebra classrooms by teachers and how those representations are conceptualized by the students seems to be worthwhile to investigate.

According to the researcher, mathematics educators ought to recognize making establishment between concepts for the mathematics instruction for all students. Nowadays, many attempts can be observed to improve mathematics instruction. Multiple representation-based instruction for conceptual algebra understanding is just the one that the researcher implemented and appreciated the benefits of using this method. Giving opportunity to new instructional methods like multiple representation based instruction in mathematics classrooms enables students better mathematics learner. As Klein (2003) implied; *`Learning to create and interpret representations using specific media such as texts, graphics, and even videotapes are themselves curricular goals for many teachers and students` (p. 49).* As a three-year experienced mathematics teacher before, the researcher could say that in traditional mathematics classroom, there is a need to encourage students to think more deeply on mathematical concepts, to intrinsically motivate for learning, to make students appreciate the nature of mathematics by getting rid of rote memorization, and to avoid overemphasizing mathematical rules and algorithms. In fact, new instructional methodologies like multiple representation-based instruction can address this need.

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