GREEK STUDENTS' ABILITY IN PROBABILITY PROBLEM SOLVING

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This study aims to contribute to the understanding of the approaches students develop and use in solving probabilistic tasks and to examine which approach is more correlated with students' ability in probability problem solving. Participants were students from the 12th grade. Implicative statistical analysis was performed to evaluate the relation between students' approach and their ability to solve problems. Results provided support for students' intention to use the algebraic approach and avoid Venn's diagrams. Students who were able to use the coordinated approach by using multiple representations had better results in problem solving. In addition the results suggest the flexibility in multiple representations is a trivial predictor of probabilistic problem solving.

Keywords: Probability, problem solving, 12th grade students, representations.

INTRODUCTION

There is an increasing recognition that statistical and probabilistic concepts are among the most important unifying ideas in mathematics. Statistical concepts form the single most important idea in all mathematics, in terms of understanding the subject as well as for using it for exploring other topics. The reasons to include probability and statistics teaching refer to the usefulness of statistics and probability for daily life, its instrumental role in other disciplines, the need for a basic stochastic knowledge in many professions and its role in developing a critical reasoning (Gal, 2002).

The understanding of probabilistic and statistical concept does not appear to be easy, given the diversity of representations associated with this concept, and the difficulties presented in the processes of articulating the appropriate systems of representation involved in probabilistic and statistical problem solving (SPS) (Anastasiadou, 2007).

Probability is difficult to teach for various reasons, including disparity between intuition and conceptual development even as regards apparently elementary concepts (Chadjipadelis and Gastaris, 1995). Since an education that only focuses on technical skills is unlikely to help teachers overcome their erroneous beliefs, it is important to find new ways to teach probability to them, while at the same time bridging their content knowledge and their pedagogical content knowledge (Batanero et al, 2005).

There is general consensus in the mathematics education community that teachers need a deep and meaningful understanding of any mathematical content they teach (Chadjipadelis, 2003). Biehler (1990) suggests that teachers require meta-knowledge about probabilities and statistics, including a historical, philosophical, cultural and epistemological perspective on statistics and its relations to other domains of science.

In primary and secondary school levels, probability and statistics is part of the mathematics curriculum and primary school teachers and mathematics teachers frequently lack specific preparation in statistics education (Anastasiadou and Gagatsis, 2007; Chadjipadelis, 2003). According to Batanero et al., (Batanero et al, 2005) probability is increasingly taking part in the school mathematics curriculum; yet most teachers have little experience with probability and share with their students a variety of probabilistic misconceptions. The understanding of probabilistic concepts has been a main concern of statistics education that is an important focus of interest for the International Statistical Institute and of the International Association for Statistical Education.

In the field of statistics learning and instruction, representations play an important role as an aid for supporting reflection and as a means in communicating statistical ideas. Furthermore the NCTM's Principles and Standards for School Mathematics (2000) document include a new process standard that addresses representations and stress the importance of the use of multiple representations in statistical learning. In addition, an important educational objective in statistics is for pupils to identify and use efficiently various forms of representation of the same mathematical concept and move flexibly from one system of representation of the concept to another.

A representation is defined as any configuration of characters, images, concrete objects etc., that can symbolize or "represent" something else (Confrey & Smith, 1991, Goldin, 1998). Representations have been classified into two interrelated classes: external and internal (Goldin, 1998). Internal representations refer to mental images corresponding to internal formulations that we construct of reality. External representations concern the external symbolic organizations representing externally a certain mathematical reality. In this study the term "representations" is interpreted as the "external" tools used for representing statistical ideas such as tables and graphs (Confrey & Smith, 1991). The need for a variety of semiotic representations in the teaching and learning of probabilities is usually explained through reference to the cost of processing, the limited representation affordances for each domain of symbolism and the ability to transfer knowledge from one representation to another (Duval, 1987). By a translation process, we mean the psychological processes involving the moving from one mode of representation to another (Janvier, 1987). Several researchers in the last two decades addressed the critical problem of translation between and within representations, and emphasized the importance of moving among multiple representations and connecting them (Gagatsis & Elia, 2004; Goldin, 1998; Yerushalmy, 1997). Different representations refering to the same concept complement each ither and all these together contribute to a glibal understanding of it (Gagatsis & Siakalli, 2004). Duval

(2002) claimed that the conversion of a mathematical concept from one representation to another is a presupposition for successful problem solving. A person who cans easily transfer this knowledge from one structural system of the mind to another is more likely to be successful in problem solving by using a plurality of solution strategies and regulation processes of the system for handling cognitive difficulties. Kaput (1987) suggest that the concept of representation involves the following five components: a representational entity, the entity that it represents, particular aspects of the representation entity, the particular aspects of the entity that it represents that form the representation and finally the correspondence between the two entities. According to the above definition, the representation is considered a mental symbol or concept, which represents a concrete material symbol. It takes the place of another element and obtains more capabilities tan the object itself. Many studies identified the difficulties that arise in the conversion from one mode of representation of a mathematical concept to another. They revealed students inconsistencies when dealing with relative tasks that differ in a certain feature, i.e. mode of representation. This incoherent behavior was addressed as one of the basic features of the phenomenon of compartmentalization, which may affect mathematics learning in a negative way (Gagatsis & Elia & Mousoulidis, 2006). According to Duval (Duval, 2002), the phenomenon of compartmentalization reveals a cognitive difficulty that arises from the need to accomplish flexible and competent conversion back and forth between different kinds of mathematical representations.

In Greece, the introduction of Statistics in the mathematics textbook of primary schools took place at the end of nineties. The teaching of fundamental statistical concepts was assigned to primary school teachers who are responsible for teaching all the curriculum subjects in the primary level. (Anastasiadou, 2007). The emphasis on statistics and probability in curricula varies, often according to knowledge and feelings of the teacher.

Although that many researches have been done in relation to study of the of the representations role in mathematical understanding and learning, there only a few that explore students' performance in using multiple representations of statistical and probability concepts with emphasis on the effects exerted on performance and on the relations among the various conversion abilities from one representation to another.

The purpose in this study is to contribute to the statistics education research community understands of approach students build up and use in solving statistical tasks and to examine which approach is more associated with students' ability in solving statistical concepts. A main question of this study referred to the approach primary school students use in order to solve simple probability tasks. It is important to know whether students are flexible in using algebraic, graphical and verbal representations in probabilistic problems. Most of the students used an algebraic approach in order to solve the simple probabilistic tasks. This study intends to shed light on the role of different modes of representation on the understanding of some basic probabilistic concepts. This study investigated pre-service teachers' performance, in two aspects of probabilistic understanding: the flexibility in multiple representations and the problem solving ability.

METHOD

Participants- Data analysis-Tasks

The sample of the study involved 132 12th grade students from secondary schools in different regions of Thessaloniki (Western Thessalonki, Eastern Thessaloniki, Central Thessaloniki) in Greece. These regions were selected because of their diversity in size and population. In Greek secondary education only students of the 12th grade are taught basic concepts of probability theory.

For the analysis of the collected data the similarity statistical method (Lerman, 1981) was conducted using a computer software called C.H.I.C. (Classification Hiérarchique, Implicative et Cohésitive) (Bodin, Coutourier & Gras, 2000). This method of analysis determines the similarity connections of the variables. In particular, the similarity analysis is a classification method which aims to identify in a set V of variables, thicker and thicker partitions of V, established in an ascending manner. These partitions, when fit together, are represented in a hierarchically constructed diagram (tree) using a similarity statistical criterion among the variables. The similarity is defined by the cross-comparison between a group V of the variables and a group E of the individuals (or objects). This kind of analysis allows for the researcher to study and interpret in terms of typology and decreasing similarity, clusters of variables which are established at particular levels of the diagram and can be opposed to others, in the same levels. It should be noted that statistical similarities do not necessarily imply logical or cognitive similarities. The red horizontal lines represent significant relations of similarity.

The test consisted of 12 tasks of two "equivalent" problems in difficulty from the mathematical point of view. In particular, the tasks concerned concepts of the probability theory such as probability, Venn's diagrams, events and probability problems.

Right and wrong or no answers were scored as 1 and 0, respectively. Students' responses to the tasks comprise the variables of the study which were codified by an uppercase V (variable concerns Venn's diagrams) or P (probability problem), ή R (concept definition, e.g.event), followed by the number indicating the exercise number. Following is the letter that signifies the type of initial representation (e.g. r=representation, t=table, g=graphic, v=verbal) and, lastly, comes the letter that signifies the type of final representation.

For example the first and second tasks are the following ones: Task 1. Given two events A and B of a chance experiment and with the help of set theory we have the following event $A' \cap B'$. Present with a Venn diagram this event (V1sg). Task 2. Given two events A and B of a chance experiment and with the help of set theory we have the following event $(A'\cap B)\cup(A\cap B')$. Express the verbal representation of this event (V2sv).

RESULTS

Descriptive results

Table 1 presents the success rates of third, fifth and sixth grade indigenous students and immigrants in all types of conversions.

Tasks	Type of translation	12th grade success rate of students (%)	Tasks	Type of translation	12th grade success rate of students (%)
V1sg	Symbolic - Graphic	52.8%	P7va	Verbal - Algebraic	32.6%
V2sv	Symbolic - Verbal	51.6%	P8vg	Verbal - Graphic	28.3%
V3gs	Graphic - Symbolic	34.5%	P9vs	Verbal - Symbolic	22.6%
V4gv	Graphic - Verbal	30.7%	P10vv	Verbal - Verbal	27.5%
V5vg	Verbal - Graphic	46.2%	R11vv	Verbal - Verbal	23.1%
V6vs	Verbal - Symbolic	48.6%	R12vs	Verbal - Symbolic	22.9%

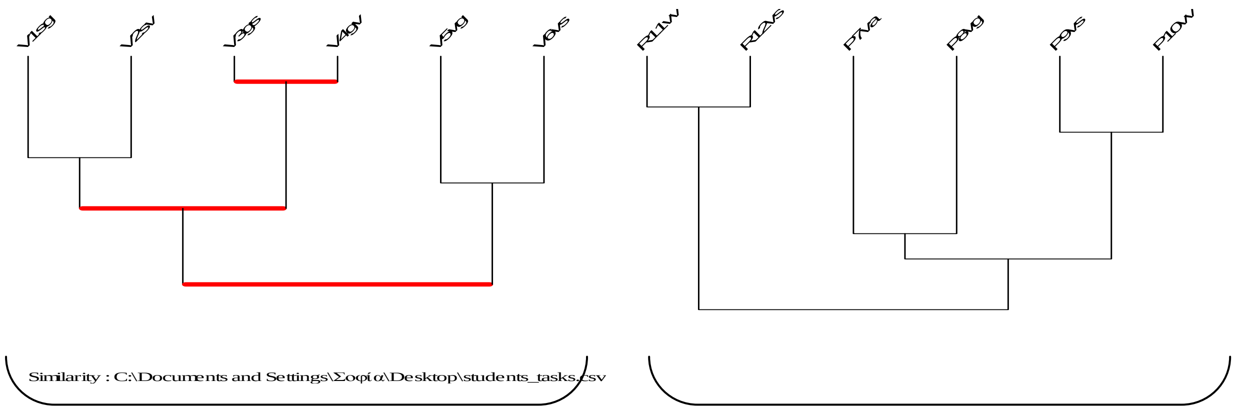
Table 1: Success rates of indigenous students and immigrants in the tasks

Similarity diagram of students' responses to the two tests

The similarity diagram in this study concern the data 11th grade and allow for the arrangement of students' responses ((V1sg), (V2sv), (V3gs), (V4gv), (V5vg), (V6vs), (P7va), (P8vg), (P9vs), (P10vv), (R11vv), (R12vs), to the tasks into groups according to their homogeneity.

Two clusters (Cluster A and B) of variables are identified in the similarity diagram of 11th grade students' responses as shown in Figure 1. Cluster A involves three pairs of variables V1sg-V2sv, V3gs-V4gv, V5vg-V6vs in Cluster A and concerns events representations with the aid of Venn diagrams. Cluster B involves three pairs of variables R11vv- R12vs, P7va-P8vg, P9vs-P10vv and involves variables relating to probability problem solving. This grouping suggests that students dealt similarly with the conversions involving probability problems.

The structure of the diagram reveals a cognitive difficulty that arises from the need to accomplish flexible and competent conversion back and forth between different kinds of probabilistic representations. Thus, this particular structure of the diagram indicates a compartmentalization of the tasks of the tests. Students approached in a completely distinct way the tasks which involved the use of Venn's diagrams and the probability problems. Therefore, possible instructive activities would focus on the identification of the two different groups. The strongest similarity (almost 1) occurs between variables (V3gs-V4gv) (Figure 1) that were the most difficult for the students of 12th grade (Table 1). Furthermore the similarity (V1sg-V2sv, V3gs-V4gv) is also important (0.923).



Cluster A Cluster B

Figure 1: **Similarity Diagram**

CONCLUSIONS RESULTS

Representations enable students to interpret situations and to comprehend the relations embedded in probabilistic problems. Thus, we consider representations to be extremely important with respect to cognitive processes in developing probabilistic concepts. The main contribution of the present study is the identification of secondary students' abilities to handle various representations and to translate among representations related to the same probabilistic relationship. Our findings provide a strong case for the role of different modes of representation on 12th grade students' performance to tasks on basic statistical concepts such as frequency. At the same time they enable a developmental interpretation of students' difficulties in relation to representations of Venn diagrams. Lack of connections among different modes of representations in the similarity diagram indicates the difficulty in handling two or more representations in probabilistic tasks. This incompetence is the main feature of the phenomenon of compartmentalization in representations, which was detected in students if both grades. This inconsistent behavior can be seen as an indication of students' conception that different representations of the same concept are completely distinct and autonomous mathematical objects and not just different ways of expressing the meaning of a particular notion. An alternative explanation for the difficulty in transferring knowledge could be the emphasis on stating with representations and defining transfer as connecting those representations. Perhaps links that were more powerful and meaningful for the students would have led to a space of the utility of the statistical and probability construct (Ainley and Pratt, 2002). Transfer might then be achieved by recognizing new situations which are consistent with the same meaning. In addition the lack of transfer may be attributed to the students' lack of preparation: time to discuss, interact and work on related tasks.

Probability instruction needs to encourage pupils' involvement in activities including translations between different modes of representation. Even more educators should focus on reasons that we use a specific representation or another of the same probability concept. As a result, students will be able to overcome the compartmentalization difficulties and develop their flexibility in understanding and using a concept within various contexts or modes of representation and in moving from one mode of representation to another. Moreover there is a strong need for teachers to understand what it is that students know about stochastic and offer them experiences of probability before theoretical perspectives are introduced.

It seems that there is a need for further investigation into the subject with the inclusion of a more extended qualitative and quantitative analysis. In the future, it is interesting to compare the strategies and modes of representations students used in order to solve the problems. Besides, longitudinal performance investigation in the multiple representation flexibility tasks for secondary students should be carried out.

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