RECOMMENDED KNOWLEDGE OF PROBABILITY FOR SECONDARY MATHEMATICS TEACHERS

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Changes in school mathematics curricula in the last few decades have brought along an increase on the importance placed on probability (National Commission for Excellence in Education, 1983; National Council of Teachers of Mathematics, 2000). Since teachers' knowledge can have an impact on students' learning (Fennema & Franke, 1992), it is important that teachers have sufficient probability content and teaching knowledge. This paper identifies the suggested probability knowledge for secondary mathematics teachers through an examination of the recommendations from four professional organizations, namely the American Mathematical Society (AMS), the American Statistical Association (ASA), the Mathematical Association of America (MAA), and the National Council of Teachers of Mathematics (NCTM).

Keywords: teachers' knowledge, probability, professional recommendations

PROBABILITY CONTENT IN THE SECONDARY SCHOOL MATHEMATICS CURRICULUM

Since the late 1950s, there have been strong calls for an increase in the inclusion of probability in the US K-12 mathematics curriculum (NCSM, 1977; NCEE, 1983; NCTM, 2000). Probability has come to gain importance as a content area that students need to have experience with in order to be well-informed citizens since its study "can raise the level of sophistication at which a person interprets what he sees in ordinary life, in which theorems are scarce and uncertainty is everywhere" (Cambridge Conference on School Mathematics, 1963, p.70; as cited in Jones, 2004).

In 1963 a group of mathematicians and National Science Foundation (NSF) representatives published *Goals for School Mathematics* in which the importance of "some 'feeling' for probability" for all students was indicated (Jones, 1970, p. 291; as cited in Sorto, 2004). Following, the National Council of Supervisors of Mathematics (NCSM) defined probability as one of the basic skills that students should acquire (1977). In 1983, the National Commission for Excellence in Education (NCEE) published *A Nation at Risk*, a report aimed at pointing out the immediate need for reform in education, with the suggestion that high school graduates understand elementary probability and be able to apply it in everyday life.

More recently, the National Council for Teachers of Mathematics (NCTM) published the *Curriculum and Evaluation Standards for School Mathematics* (1989) in which it was recommended that in grades 5-8 students "explore situations by experimenting and simulating probability models", construct sample spaces in the attempt to determine probabilities of "realistic situations", and appreciate the use of probability in the real world (1989, p. 109). Particular to grades 9-12, recommendations included the understanding of the difference between experimental and theoretical probabilities, theoretical and simulation techniques for computing probabilities, and interpreting discrete probability distributions (p. 171). In the mid to late 1990s the NCTM standards were revised resulting in the publication of *Principles and Standards for School Mathematics* (2000). Here, recommendations stated that

> "middle-grades students should learn and use appropriate terminology and should be able to compute probabilities for simple compound events … In high school, students should compute probabilities of compound events and understand conditional and independent events." (NCTM, 2000, p. 51).

This increased attention on probability in school curricula is an indicator of how important it is that "teachers, mathematics educators, parents, and administrators, must provide their children and their students with alternative ways of approaching data and chance" (Shaughnessy, 2003, p. 223). Since "[T]here is perhaps no other branch of the mathematical sciences that is as important for *all* students, college bound or not, as probability and statistics" (Shaughnessy, 1992, p. 466, emphasis in original) and since misconceptions about probability are common among children, it is important that instruction allows students to confront their misconceptions and develop a deeper understanding of probability concepts (Garfield & Ahlgren, 1988; Konold, 1989; Shaughnessy, 2003). Since teachers' knowledge can have an impact on students' learning (Fennema & Franke, 1992), it is important that teachers be able to tackle these student difficulties and misconceptions on probability as they arise in mathematics classrooms. In order to be able to do so, teachers need to have sufficient probability content and teaching knowledge.

Teachers' Knowledge of Probability

Although there have been calls for an increased attention on probability in the school curriculum, one of the problems encountered is the inadequate preparation of teachers in probability (Penas, 1987; CBMS, 2001). Many teachers have not encountered probability in their own K-12 mathematics courses and sometimes need convincing as to why they need to learn and teach probability topics (CBMS, 2001). Batanero et al. (2004) suggest that educators need to provide better initial training for teachers by offering courses at the college level specific to the didactics of probability. Such a course should include an introduction to the history of probability; information on statistics journals, associations, and conferences; the study of fundamental probability concepts; readings of literature on heuristics and biases in probability, as well as students' difficulties and misconceptions in probability; identification of the educational theories and teaching approaches, assessment, teaching resources, and the use of technology; and examples of projects that can be used when teaching probability.

Teachers' Knowledge of Mathematics

Several scholars in the past three decades have provided insight into the definition of teachers' knowledge. In his work, Shulman (1986) provided a framework of teachers' knowledge which includes the following three categories: i) *subject matter content knowledge* which refers to "the amount and organization of knowledge per se in the mind of the teacher" as well as not only understanding *that* something is so but also *why* it is so and why it is important to the discipline (p. 9); ii) *pedagogical content knowledge* which refers to

> "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others" (p. 9).

This category also includes knowledge of common conceptions/preconceptions that students have; and iii) *curricular knowledge* which includes knowledge about the

> "full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials …, and the set of characteristics that serve as both the indications and contraindications for the use of a particular curriculum or program materials in particular circumstances" (p. 10).

The difficulty faced by educators is how to blend the components of teacher knowledge so as to effectively prepare teachers to help all students to learn meaningfully.

FOCUS OF THE PAPER AND QUESTION

With the above issues under consideration, a study was carried out by the author in which US state and national mathematics standards for grades 6-12, secondary mathematics textbooks, and recommendations from professional organizations were analyzed in order to identify the content and teaching knowledge that secondary mathematics teachers need to have relative to the domain of probability. A report of the results relating to the probability topics that secondary mathematics teachers should know and be able to teach was presented at a previous conference (Papaieronymou, 2008), whereas this paper focuses on the teaching aspects of these probability topics and more specifically on the following question:

> What are the aspects of teaching knowledge of probability that secondary mathematics teachers need to have as suggested by professional organizations?

For the purposes of addressing this question, only the recommendations from professional organizations were analyzed. The data sources specific to students (i.e. national and state standards for grades 6-12 and secondary mathematics textbooks) were not very informative since they did not directly address teachers' knowledge.

METHODS

Data Sources

In particular, *A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics* (1991) published by the MAA, the *Professional Standards for Teaching Mathematics* (1991) published by the NCTM, *The Mathematical Education for Teachers* (CBMS, 2001) published by the AMS, and the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Report* (Aliaga et al., 2005) published by the ASA were analyzed. The ASA report presents a list of goals for college students – not specifically prospective mathematics teachers – and recommendations for the teaching of introductory statistics courses.

Data Analysis

The number of recommendations from each professional organization was as follows:

Data Source	Number of Recommendations before multi-coding
AMS (2001)	27
ASA (2005)	9
MAA(1991)	17
NCTM(1991)	6
Total	59

Table 1: Number of recommendations from each organization before multi-coding

These 59 recommendations were categorized according to Shulman's (1986) framework of teacher knowledge with 8 recommendations being placed under more than one category. In deciding under which knowledge category to place each recommendation, the verbs appearing in the recommendation and their use in association with the probability concepts mentioned in the respective recommendation were considered. Some examples of recommendations that were placed under each of Shulman's categories are:

Recommendation	Knowledge Category
Mathematics teachers should be able to use permutation and combinatorial computations in problems arising from several areas, including geometry, algebra, and graph theory. They should also understand how counting techniques apply in the calculation of the probability of events. (MAA report, p. 36)	Subject-matter content knowledge
The fact that, under random sampling, the empirical probabilities	Pedagogical

actually converge to the theoretical (the law of large numbers) can be illustrated by technology (computer or graphing calculator) so that an understanding of probability as a long-run relative frequency is clearly established. (AMS report, p.116)	content knowledge
Precede computer simulations with physical explorations (e.g. die rolling, card shuffling) (ASA report)	Curricular knowledge
Other topics that should be introduced include fair games and expected value, odds, elementary counting techniques, conditional probability, and the use of an area model to represent probability geometrically (NCTM, 1991, p. 138)	Subject-matter and pedagogical content knowledge

Table 2: Examples of recommendations under Shulman's (1986) knowledge categories

In the last recommendation provided in Table 2 above, the use of the area model to represent probability implies pedagogical content knowledge since this type of knowledge includes the ways of *representing the subject*. The reference to topics of probability that should be introduced implies subject matter content knowledge; the topics refer to the *amount* of knowledge that teachers should have with respect to probability so as to be able to introduce these topics in their mathematics classrooms.

RESULTS

Once the 59 recommendations were categorized under Shulman's framework for teacher knowledge, with 8 recommendations being placed under two of the knowledge categories, the results were:

Data Source	Subject-matter content knowledge	Pedagogical content knowledge	Curricular knowledge	Total
AMS (2001)	22	5	1	28
ASA (2005)	7	1	1	9
MAA (1991)	13	6	2	21
NCTM (1991)	2	4	3	9
Total	44	16	7	67

Table 3: Number of recommendations under each of Shulman's (1986) categories

As can be seen from Table 3, about 66% (44 out of 67) of the recommendations from the four professional organizations relate to subject matter content knowledge, 24% (16 out of 67) of the recommendations refer to aspects of pedagogical content knowledge and 10% of the recommendations specify aspects of curricular knowledge that should be included in the preparation of secondary mathematics teachers.

The analysis also showed that the following topics were recommended by at least two of the organizations:

Common Topic	Professional Organizations in agreement
Combinatorics	AMS, MAA, NCTM
Experimental and Theoretical Probability	AMS, MAA, NCTM
Simulations	ASA, MAA, NCTM
Probability Distributions	AMS, MAA, NCTM
Hypothesis Testing	AMS, ASA, MAA
Conditional Probability	AMS, NCTM
Expected Value	AMS, NCTM
Probabilistic Misconceptions	AMS, NCTM
Uses/Misuses of Probability	AMS, MAA

Table 5: Probability topics recommended by at least two of the organizations

DISCUSSION

Given the small number (59) of recommendations overall across all four organizations specific to the area of probability and that 66% of the recommendations relate to subject matter content knowledge whereas 24% refer to pedagogical content knowledge and only 10% refer to curricular knowledge, the results imply that it is still unclear what exactly the pedagogical content knowledge and curricular content knowledge that secondary mathematics teachers need to have in the area of probability is.

A closer examination of the recommendations indicates that with respect to pedagogical content knowledge specific to probability, teachers need to acquire an awareness and ability to confront common probabilistic misconceptions and student difficulties relative to probability concepts (as suggested by the ASA, the MAA, and the NCTM). In addition, teachers need to be able to use technology to carry out simulations in order to illustrate probabilistic concepts (as recommended by all four of the professional organizations) and should also be able to use concrete objects such as dice, cards, and spinners to demonstrate probability concepts to students in the mathematics classroom (as suggested by the ASA and the NCTM). Furthermore, secondary mathematics teachers should be able to represent probabilities using various models such as the area model (as suggested by the NCTM).

Specific to curricular knowledge, secondary mathematics teachers should be aware of the various materials and programs that they can use to help students understand probability concepts. That is, they should be aware that they can use various computer programs such as Fathom and DataScope in their mathematics classrooms when working with probability concepts (as suggested by the AMS) and they should know the power of simulation as a technique that can be used to solve probability problems (as recommended by the MAA and the NCTM).

As can be seen from Table 5, the four professional organizations place considerable emphasis on experimental versus theoretical probability and simulations. Secondary mathematics teachers need to be able to plan and conduct experiments and simulations (Aliaga et al., 2005; CBMS, 2001; Committee of the Mathematical Education of Teachers, 1991; NCTM, 1991), distinguish between experimental and theoretical probability (Committee of the Mathematical Education of Teachers, 1991), determine experimental probabilities (CBMS, 2001; Committee of the Mathematical Education of Teachers, 1991), use experimental and theoretical probabilities to formulate and solve probability problems (Committee of the Mathematical Education of Teachers, 1991), and use simulations to estimate the solution to problems of chance (Committee of the Mathematical Education of Teachers, 1991; NCTM, 1991). Secondary mathematics teachers should be able to provide a model which gives a theoretical probability that can be compared to experimental results, which in turn is essential when studying the concept of relative frequency (CBMS, 2001). In order to help students develop an understanding of probability as a long-run relative frequency, secondary mathematics teachers need to understand the law of large numbers and be able to illustrate it using simulations (CBMS, 2001).

With regards to theoretical probability, teachers should know about and be able to use both discrete and continuous probability distributions (NCTM, 1991), understand probability distributions (CBMS, 2001) and especially the normal distribution (CBMS, 2001; Committee of the Mathematical Education of Teachers, 1991), as well as the binomial, poisson, and chi-square distributions (Committee of the Mathematical Education of Teachers, 1991). They should also be able to use simulations to study probability distributions (CBMS, 2001; Committee of the Mathematical Education of Teachers, 1991) and demonstrate their properties (CBMS, 2001). Moreover, they should be introduced to fair games (NCTM, 1991) and understand expected value (CBMS, 2001).

Another topic among the recommendations from three of the four professional organizations is that of hypothesis testing. Secondary mathematics teachers should understand the concept of statistical significance including significance level and p-values, and that of confidence interval (Aliaga et al., 2005; Committee of the Mathematical Education of Teachers, 1991) including confidence level and margin of error (Aliaga et al., 2005).

Returning to the idea of theoretical probability, secondary mathematics teachers should be able to use counting techniques (NCTM, 1991) such as permutations and combinations to determine such (theoretical) probabilities (Committee of the Mathematical Education of Teachers, 1991). In addition, they should be exposed to the applications of combinatorics (CBMS, 2001) including their use in calculating the probability of events (Committee of the Mathematical Education of Teachers, 1991). Secondary mathematics teachers should also understand and be able to calculate the probabilities of independent and dependent events (CBMS, 2001), compound events made up of independent and dependent events (CBMS, 2001) and also understand conditional probability (CBMS, 2001; NCTM, 1991). Various representations such as area models and tree diagrams should be used by teachers to aid students in better understanding compound events (CBMS, 2001; NCTM, 1991).

In addition, teachers should know about the uses of probability in many fields and its misuses in such sources as newspapers and magazines (CBMS, 2001; Committee of the Mathematical Education of Teachers, 1991). Once experiments have been performed, teachers should be able to use probability to make decisions and predictions (CBMS, 2001; Committee of the Mathematical Education of Teachers, 1991).

An issue that arose as recommendations were being coded concerned the exact definition of the verbs that appeared in the documents. In many cases it was unclear as to what action or type of knowledge was expected of teachers based on the verb used since the meaning of the verb appearing in the report was unclear. Within the four documents of recommendations from the professional organizations, verbs appeared in different forms e.g. use, using, used or apply, applying, applied. Counting the different forms of a verb as one verb family gave rise to a total of 53 verb families being identified in the four reports. For example, consider the last recommendation on Table 2 which lists a set of probability topics that need to be ‘introduced’ in a mathematics classroom. The mere list of topics in this recommendation implies subject matter content knowledge. However, if the recommendation had established more clearly *how*, *in what order*, *what types of problems* should accompany these topics, and *how much emphasis* should be placed on each, the categorization might have been different. Let us also consider the verb family *understand* which had the highest frequency (29) in the four documents overall. In the mathematics education literature much has been written about the definition of this verb family. For example, Skemp (1976) makes a distinction between *relational understanding* (“knowing both what to do and why” (p.20)) and *instrumental understanding* (“rules without reasons” (p.20)). On the other hand, the National Research Council (2001) refers to *procedural* understanding and *conceptual* understanding. However, in the four reports examined in this study, it is not clearly indicated by the professional organizations which of these meanings the verb family *understand* carries when used in a recommendation. Such precise meanings are needed so as to accurately code the recommendations.

CONCLUSION

In recent decades, probability has come to gain importance as one of the content areas of school curricula in the United States. However, research on teachers’ knowledge in this content area is scarce. The identification of the knowledge of probability that secondary mathematics teachers need to have in the form of content topics and their aspects of teaching is an essential tool that can be used in future research in this area. The analysis of recommendations on probability provided by professional organizations has revealed the importance of language in attempting to communicate to mathematics educators and teachers what is expected that they know and teach. As mentioned, 53 verb families were identified in the data sources. However, no clear definitions of these verbs, as related to the probability topics they accompanied, were provided by any of the sources leaving much to the interpretation of the researcher. Precise definitions of action verbs are needed in such documents to avoid possible errors in the coding of the recommendations and to help educators as they plan courses for prospective mathematics teachers.

Last, the analysis of the reports on recommendations for the preparation of secondary mathematics teachers by the AMS, ASA, MAA, and NCTM, revealed the inadequate number of such recommendations especially with regards to pedagogical content knowledge and curricular knowledge requirements specific to the area of probability at the secondary level. Given the increased attention of probability in school curricula, it is essential that professional organizations provide more extensive and detailed reports regarding the recommended skills in probability for future mathematics teachers. It would perhaps be most beneficial if professional organizations provide such a report collaboratively so that there is common agreement about the expectations of probabilistic knowledge of secondary mathematics teachers.

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