INTRODUCTION
ON "STOCHASTIC THINKING"

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OVERVIEW

The Working Group 3 discussed 8 three aspects that reflect the diversity of the research approaches on stochastic thinking:

- theoretical issues of stochastic thinking,
- teachers' professional development, and
- students' learning in respect to their success in solving stochastical tasks.

The connective aim of all approaches was the students' learning of stochastical concepts, and the students' awareness that it is possible to use stochastics to cope with specific real situations. These aspects of the students' stochastical literacy (for the term statistical literacy see Gal, 2004), however, were discussed using three different perspectives, i.e. the stochastical content (C), the teaching of stochastics (T), and the students' learning about stochastics (S), that shape a didactical triangle referring to stochastics instruction.

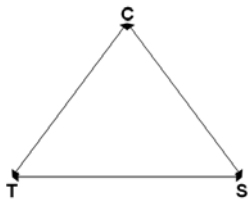

Figure 1: Didactical triangle involving three different perspectives on stochastics instruction, i.e. the content, the teachers, and the students

In the following we will introduce the papers that match one of the three perspectives, and we will sketch some results of our discussion.

STOCHASTICAL CONTENT

Stochastics is a cocktail of statistical ideas and probabilistic ideas. Although the latter thesis seems to be trivial, there is a lot of evidence that the emphasis on statistics and probability in curricula varies, often according to knowledge and feelings of the teachers. In the same way, the topics of interest to researchers vary over time.

Currently the focus of research concerning statistics is, for instance, on distributions, averages, variability (including informal inference, and co-variation and correlation), and graphs (Shaughnessy, 2007). Concerning probability the research focus is on random, sample space, and probability measurement (Jones, Langrall, & Mooney, 2007).

The research referring to these subjects has two aims:

- to clarify the notions, meanings or definitions of stochastical concepts. In our group, for instance, the talk of Hasan Akyuzulu deals with the undefined concept of risk highlighting the connection between risk and defined stochastical concepts.
- to develop and to evaluate teaching approaches that facilitate students' learning in respect to the different stochastical concepts. Matching this aspect, Herman Callaert discusses in his paper obstacles of the students' learning that emerge through ambiguous notations and explanations of stochastical concepts in widely-used text books and software.

Concerning the aspect of stochastical content, we, finally, discussed the recommendation of professional organisations regarding stochastics instruction. To this aspect, Irini Papaieronimou identifies in her paper many recommendations about the teaching of probability from four US professional organisations. We are concerned that there is insufficient support to effect a didactical transposition. Further, we noted an omission: such recommendations do not include the need for teachers to understand what it is that students know (as opposed to misconceptions).

TEACHING OF STOCHASTICS

A repeated claim towards the research on stochastic thinking is to increase the effort of investigating the teachers' knowledge and the teachers' beliefs concerning stochastical concept, and the learning and teaching of stochastics (Shaughnessy, 2007). According to this claim, we discussed two research approaches that concern both, the stochastics teachers' knowledge, and the stochastics teachers' beliefs.

- Carmen Batanero, Pedro Artega, and Blanca Ruiz discuss in their paper the knowledge of prospective primary Spanish teachers referring to statistical graphs based on the theoretical Framework of Curcio (1989). They found that some of the teachers were unable to use even basic statistical graphs, and that, in fact, only one third were able to draw a reasonable conclusion.
- the paper of Andreas Eichler refers to an analysis of "ordinary" upper secondary teachers' planning of stochastics instruction, the teachers' classroom practice and their students' learning. His report focus on teachers having differing orientations across two dimensions: seeing mathematics as: (i) emphasising applications or a formal subject; (ii) being dynamic or static.

The report gives some evidence about different modes of students' learning concerning their awareness of the benefit of stochastics in the real life.

We concluded on the one hand, that the teaching of stochastics needs to offer students experiences of statistics and probability before theoretical perspectives are introduced. On the other hand, we stated that there is much research to do to understand the teachers' knowledge and the teachers' beliefs about stochastics that both in some sense determine the students' learning of stochastics.

LEARNING ABOUT STOCHASTICS

Finally, we discussed three considerably different research approaches focusing students' learning in respect to their success in solving stochastical tasks.

- The paper of Zoi Nikiforidou and Jenny Page provides a psychological experiment on children (age 5 or 6 years), in which the children made decisions based on posterior information. The results of this research give some evidence that even such young children have some understanding of ideas that may be the roots of probability or inference. This result argues against the Piagetian framework.
- The paper of Theodosia Prodromou and Dave Pratt concerns students (15 years of age) using a computer simulation. This research yielded that it was possible to design a computer simulation such that students were able to make use of ideas about causality to make sense of distribution. In this sense, the deterministic and the stochastic worlds are not disconnected but connected through levels of complex causality.
- Finally, Sofia Anastasiadou provides in her paper a study referring to children's meaning-making with respect to set theory. She found that the students were not able to recognise the mathematical concept across differing representations. Perhaps the lack of transfer could be attributed to the students lack of preparation: time to discuss, interact and work on related tasks.

Although the papers focusing on the students' learning match some of the claims to the research into stochastics education, the three research approaches mentioned above showed the diversity of possible research questions in this field.

CONCLUSIONS

The papers of Working Group 3 highlighted the diversity of research approaches focusing on stochastic thinking. However, we concluded with thre claims for future research that often combine several perspectives on the teaching and learning of stochastics that shape a didactical triangle (fig. 1):

- We need empirical results that give evidence, how we can support the implementation of recommendations from professional organisations.

- We need empirical based strategies we support teachers to be more connectionist in their approach.
- We need to research how students can transfer ideas from one domain to another. Reference could be made to connectionist theoretical frameworks.

One of the problems to achieve these claims is that it is sometimes not possible to transfer results yielded into mathematics education on stochastics education due to the fundamental difference of stochastics in contrast to other mathematical disciplines. For instance, the role of context is very different in statistics from in mathematics. Mathematics as a discipline aims to be decontextualised whereas statistics may draw on context. However, in both mathematics and stochastics learning, the students must experience the underlying ideas in meaningful contexts.

Another problem seems to be that stochastics instruction in Europe still emphasise probability, and, for this reason, studies in the field of stochastics education often focus on probability. Hence, we hope to see more research in statistics in future conferences of the ERME. Otherwise, we are afraid that statistics will be lost from CERME. But also, we as educationalists fear this might parallel a loss of statistics to mathematics education.

However, stochastics and, in particular, statistics are certainly useful to many subjects and to citizens in general but it is also important to mathematics education.

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