THE IMPLEMENTATION OF THE HISTORY OF MATHEMATICS IN THE NEW CURRICULUM AND TEXTBOOKS IN GREEK SECONDARY EDUCATION

Yannis Thomaidis, Constantinos Tzanakis

Experimental High School, University of Macedonia, Thessaloniki, Greece

Department of Education, University of Crete. 74100 Rethymnon, Crete, Greece

The official textbooks for the teaching of mathematics in the Greek high school (7th-9th grades) include a lot of historical material, following the guidelines of the new curriculum. However, their use is questionable because of serious historical errors, obscurities, or omissions. We support this conclusion by some examples, suggest alternative ways to use this material, and outline a deeper and more demanding implementation of the history of mathematics in the context of cross-curricular teaching activities.

Keywords: historical snippet, mathematics curriculum, cross-curricular, original sources, junior high school.

1. INTRODUCTION

In the last two decades, there is an internationally increasing interest in introducing a historical dimension in mathematics education (ME), both in didactical research and in educational policy, curriculum design and textbook content. This is reflected in the appearance of several publications, the organization of conferences, especially in the context of the HPM Study Group (e.g. Fauvel & van Maanen 2000, Siu & Tzanakis 2004, Katz & Michalowicz 2005, Schubring 2006, Furinghetti *et al* 2006, 2007, Barbin *et al* 2008). In Greece, there has always been an active interest in this area, as early as the late ‘80s, mainly in didactical research (Fauvel & van Maanen 2000 §11.8, Kastanis & Kritikos 1991, Thomaidis *et al* 2006, Chasapis 2002, 2006) and occasionally in the inclusion of short historical comments in school textbooks. Possibly, the influence of active researchers and educators’ work in this area, made officials of the Ministry of Education more attentive to what international research and practice suggests on the role of the History of Mathematics (HM) in ME. Thus, for the first time in Greece the (new 2002) mathematics curriculum for compulsory education (Pedagogical Institute 2002) includes so extensive references to a historical dimension in ME, varying from the specific teaching objectives, to the didactical methodology and the textbook content, e.g. (Pedagogical Institute 2002 pp.311, 367-369; our translation):

Special objectives: *“….. to reveal the virtue of mathematics (historical evolution of mathematical tools, symbols and notions).”*
Didactical methodology: *“… It is important to provide students with “safety valves” in the pursuit of knowledge; namely, students should be given the possibility to approach a notion in a variety of ways, i.e.: (a) By means of several different representations (using symbols, graphs, tables, geometrical figures); (b) In an interdisciplinary way; (c) With reference to the HM (the HM is a field rich of ideas to*

approach a notion didactically).”
Didactical material: “*... Moreover, reference to the great historical moments that step by step have determined the development of mathematics should be included in the mathematics textbooks, so that the student becomes aware of the genesis of the ideas, which is a prerequisite for grasping each subject. It is not necessary that the historical notes appear separately at the end of each §. (If required), they can also be (briefly) presented, at intermediate parts of the text.*”

Though these guidelines follow what didactical research suggests on the role HM can play in ME, their actual classroom implementation is not satisfactory: the authors[1] have tried to follow these guidelines, incorporating in the new mathematics textbooks a great deal of material from the HM in the form of historical notes and associated activities. These notes and activities (called *historical snippets*; Fauvel & van Maanen 2000, ch.7) have different format and colors from the main text and usually contain pictures. Here we examine critically the validity of this material and its relevance to the curriculum, by means of specific examples and suggest other ways to integrate the HM in teaching, taking into account modern trends in this direction.

2. THE HISTORICAL TEXTBOOK MATERIAL & ITS RELEVANCE TO THE CURRICULUM

The quotations from the mathematics curriculum in §1 directly connect the use of the HM with a central issue of teaching and learning: how to pursue and grasp knowledge. Thus historical snippets in the textbooks should not be limited to factual information, but contribute to understanding the notions to be taught (Fauvel & van Maanen 2000, §7.4.1); they should provide ideas and material to organize teaching and motivate students to learn. Therefore, they should meet two reasonable requirements: (a) to be mathematically and historically correct; (b) to serve the objectives of the teaching units in which they are incorporated.

Unfortunately, in many cases the historical snippets in the new high school textbooks violate these requirements; the authors’ insistence on restricting the historical material to (often inaccurate and contradictory) biographical information, is a typical case. In general this material is presented in an informal style, inserted in separate boxes in the text, usually emphasizing historical facts, rather than the mathematical exposition. In some cases it also includes related activities (cf. Fauvel & van Maanen 2000, §7.4.1). Table 1 gives a summary of the historical material in the new textbooks:

Table 1

Grade	Number of historical snippets	Percentage of textbook pages covered	Percentage of snippets which include activities	Comments in the teachers book
7	21	11/220 = 5%	5/11 = 45,5%	Some comments on the HM
8	9	6/230 = 2.6%	0/6 = 0%	2 additional activities are recommended
9	5	5/240 = 2.1%	2/5 = 40%	10 additional comments covering 12 of the 100 pages (1 activity recommended as an interdisciplinary activity.

We illustrate this material and its weaknesses by means of indicative examples, mainly from the 7th grade textbook (Vandoulakis et al 2007, Vlamos et al 2007)[2].

Example 1: factual information; no mathematics involved

In the 7th grade textbook, the authors cite 3 contradictory lifetimes of Euclid giving contradictory results: p.26: 330-275BC; p.147: 300-275BC; p.182: 330-270BC, ignoring that the only existing valid historical source on this point, is an extract from *Proclus' Commentary* on Book I of Euclid's *Elements* with no possibility to specify exact dates. In addition to historical confusion, this note does not serve any of the purposes of introducing HM in teaching as detailed in the new curriculum (cf. §4 below).

Example 2: factual information; reference to mathematical & scientific results

In a separate box of the same textbook (p.29), brief information is given on Eratosthenes' life and some of his scientific achievements (e.g. the measurement of the earth's circumference), claiming that: Eratosthenes lived from 276BC to 197BC; from 235BC and for 40 years he was director of Alexandria's famous library; at the age of 82, he committed suicide because he became blind. These data are contradictory, however: Since 276-197=79 and 235-40=195, he lived 3 years less than the age at which he died, and directed Alexandria's library for two years after his death! This note could include interesting activities in accordance to the regulations of the new curriculum (e.g. the simplicity of the measurement method of the earth's circumference), but being restricted to simply assert the results, it is mystifying, rather than enlightening!

Example 3: fiction, mathematical results and a related mathematical activity

Occasionally, the historical narrative is fictitious. In the 7th grade textbook, historical accuracy is sacrificed in favor of a controversial story, aiming to dramatize an episode from Gauss' childhood (p.75, our translation):

"Sometimes a simple thought of a man is more worthwhile than the whole world's gold. With some clever ideas battles are gained, monumental pieces of work are done, people become famous and at the same time, science is developed, technology evolves, history is shaped and life changes. Just an example is the "smart addition" that Gauss (Karl Friedrich Gauss 1777-1850) had thought of in a small German village, around 1789, when he started learning about numbers and arithmetical operations in his first year at school. When the teacher asked his students to calculate the sum 1+2+3+...+98+99+100*, little Gauss had found it before the others even started. Then, he wrote on the blackboard:*

(1+100)+(2+99)+(3+98)+...+(48+50)+(50+51)= 101+101+101+...+101+101=101·50=5,050

Try to calculate in Gauss' way the sum 1+2+3+...+998+999+1000 *and measure the time needed. How long would it have taken if calculated it in the normal way?"*

However, (a) Braunscheweig, Gauss' native place, was a political and cultural center, capital of a ducat, with about 20.000 residents in the late 18th century, not a village; (b) given that Gauss had been characterized as a mathematics "child-prodigy" from the age of 3, how is it possible that he began learning arithmetical operations in 1789, at the age of 12? Gauss entered the Volksschule (elementary school) in 1784, the Gymnasium in 1788 and the Collegium in 1792 (Wussing & Arnold 1978, p.318); (c) Gauss died in 1855, not 1850!

More importantly, this note makes an extreme statement, suggesting that mathematical progress is due to a few geniuses, not a collaborative enterprise in which personal skill is harmoniously combined with preceding achievements of the scientific community at the right moment. Thus, it implicitly gives a distorted view of history, which, considered didactically, is expected to discourage rather than engage students in mathematical activities in the classroom. Hence, this example shows lack of relevance of the textbook's historical material with the curriculum objective "*to provide students with 'safety valves' in the pursuit of knowledge*".

Example 4: historical snippets with historically motivated mathematical activity

In the same textbook there is the following activity (p.75, our translation):

ACTIVITY: On a gravestone the following problem is inscribed, whose solution gives the age of the great ancient Greek mathematician Diophantus:

"This tomb holds Diophantus. Ah, how great a marvel! The tomb tells scientifically the measure of his life. God granted him to be a boy for the sixth part of his life, and adding a twelfth part to this, he clothed his cheeks with down; He lit him the light of wedlock after a seventh part, and five years after his marriage He granted him a son. Alas! Late-born wretched child; after attaining the measure of half his father's life, chill Fate took him. After consoling his grief by this science of numbers for four years he ended his life."[3]

But where lies this gravestone? We do know that this story appears in the *Palatine Anthology,* of the Byzantine era, with no other reliable evidence for it. This activity, included in the chapter on "Equations and Problems", is not accompanied by any query, except mentioning in the teacher's book that (p.53, our translation):

"*A.* 4.2. Problem Solving: *Indicative design of the material of this unit. 1 teaching hour*. The suggested activity aims to understand: The notions used in problems, their solutions, as well as, the solution process followed [*Answer*: Diophantus lived for 74 years]".

If this requires the formulation of an equation for Diophantus' age x, then the epigram implies: $\frac{x}{6}+\frac{x}{12}+\frac{x}{7}+5+\frac{x}{2}+4=x \Leftrightarrow x=84$

However, 7[th] graders are not able to formulate and solve this equation, since solving such equations is taught in the 8[th] grade! Hence, this historical note is related neither to the mathematics of the textbook unit in which it is included, nor to the cognitive level of the students to whom it is addressed.

This epigram appears in an introductory note in the 8[th] grade textbook's chapter on "*Equations and inequalities*" with the following comments (Vlamos et al 2007, p.120, our translation):

"*…From his* [Diophantus'] *13 pieces of work only 10 had been found (6 in Greek manuscripts and 4 in Arabic translation). The most famous of his works is the "**Arithmetika**" (6 books). It is the most ancient Greek work in which for the first time a variable is used in problem solving…When he died, …his students composed a riddle and wrote it on his grave, upon his request. Here is Diophantus Epigram…*"

According to Diophantus' own statement, *Arithmetika* were divided into 13 "books"; 6 have been preserved in the Greek original and 4 in Arabic translation of the 9[th] century discovered in the 1960's. We also know another of Diophantus' works - "*On polygonal numbers"* – only fragments of which survive. Hence, the textbook confuses the 13 books of "*Arithmetika*" and the total number of his works.

3. SOME CONCLUSIONS

All examples in §2 concern historical errors (there are still more, reinforcing the bad flavor got from the textbooks' historical snippets) that nevertheless, could easily be corrected in a new textbook edition, though it is strange that they have not been avoided. It seems as if they were hurriedly written, mainly aiming to satisfy the relevant term of the announcement of the textbook writing competition and not to introduce a historical dimension in teaching.

The main characteristic of this historical material is the large amount of information and the rich illustrations, without however some methodological hints of how to benefit didactically from it. Though, the corresponding suggestions and instructions in the teacher's book in general emphasize the positive contribution of the HM, the way this could be realized is left to the initiative and ideas of the teacher, with reference to the relevant bibliography. E.g., the teacher's book for the 7th grade mentions that:

"*In some sections, there are historical notes, which intend to stimulate the student interest and love for Mathematics and to inform them on the historical development of mathematical thinking. Their use in teaching depends on the initiative and the ideas developed by the teachers*" (Vandoulakis et al 2007, p.31, our translation)

In the teacher's book for the 9th grade this issue is detailed more:

"*In some units there are topics from the HM intended to give the description of the problem that has been posed and the presentation of the conceptual tools applied to solve them. These topics, with the accompanying questions, aim to exploit the HM in the best possible way. Integrating the HM in teaching has become the subject of systematic studies at an international level. The positive contribution of the HM is corroborated in three groups of arguments: (a) It stimulates students' interest and contributes to the development of a positive attitude towards mathematics. (b) It reveals and stresses the human nature of the mathematical activity throughout history. (c) It contributes to the understanding of mathematical concepts and problems, revealing not only the context and circumstances in which they originated, but also the conditions of their development.*

These topics [from the HM and the accompanying questions]*, together with those points raised in the teacher's book, should not be considered as complete studies; it is for this reason that references to the literature are given for those teachers and students who will have a special interest.*" (Argyrakis et al 2007, pp.10-11, our translation)

Remark: Points (a)-(c) form part of the arguments for integrating HM in ME, put forward more systematically in Fauvel & van Maanen 2000, §7.2 (particularly §§(a1), (c1), (d1).

Introducing a historical dimension in the teaching of mathematics, based on teachers' interest, initiative and ideas, needs extra teaching time. But, apart from the usual obligation to cover the school material (a very difficult problem in itself!), teachers have also to cope with the innovations of the new curriculum, like group-cooperative teaching based on learning activities, or an interdisciplinary approach to mathematics. Hence, introducing a historical dimension in ME to the benefit of both teachers and students, requires additional support in the form of detailed guidelines (e.g. examples serving to illustrate how history could be integrated into teaching), extensive references for further reading and availability of relevant resources. Unfortunately, existing resources are limited (Fauvel & van Maanen 2000, p.212). In addition, from the evidence here, it is clear that the material of the new textbooks is not the most appropriate and valid guide in this direction. Therefore, high school mathematics teachers are not given any real motivation to take up the initiative to benefit from the new textbooks' historical material. In the next section, we examine whether the available historical snippets (after being corrected) can contribute positively to the teaching of high school mathematics.

4. USING HISTORICAL SNIPPETS IN CROSS-CURRICULAR ACTIVITIES

The errors in the historical notes of §2 indicate that integrating the HM in ME is a demanding activity, presuming, not only mathematical knowledge and the ability to approach, read and interpret the historical sources, but also to cross-check facts, to conclude and narrate. This seems to suggest cross-curricular activities as a privileged framework in this connection. Fortunately, such activities form an integral part of the new curricula and high school textbooks in Greece, an example being the determination of Euclid's lifetime: As mentioned in §2, the only valid historical source on this point comes from Proclus, who lived in the 5th century A.D. In his *Commentary* on the 1st Book of Euclid's Elements, he writes:

"[Euclid] *lived in the time of Ptolemy the First, for Archimedes, who lived after the time of the first Ptolemy mentions Euclid. It is also reported that Ptolemy once asked Euclid if there was not a shorter road to geometry than through the Elements, and Euclid answered that there was no royal road to geometry. He was therefore later than Plato's group, but earlier than Eratosthenes and Archimedes, for these two men were contemporaries, as Eratosthenes somewhere says.*" (Morrow 1970, pp.56-57)

This is a nice extract for an activity, combining mathematics, history and language (for Greek students). Translating the ancient text into modern Greek, collecting information for the persons involved, studying more the historical period in which they lived, could be a student activity to provide material for further discussion in the classroom, leading to the following conclusion:

We know that Ptolemy the 1st, a general of Alexander the Great had been the satrap of Egypt from 323 to 305 B.C., and its king from 304 to 283, and Archimedes lived from 287 to 212 BC. Proclus cites the dialogue of Euclid with Ptolemy the 1st and says that he was older than Archimedes. Therefore, Euclid's period of activity is very close to 300 BC.

This activity has interesting didactical extensions and could lead to insightful discussions on the concept of mathematical proof: The method and logical arguments leading, from historical sources to the above conclusion, can be paralleled to those used to justify a general mathematical result from definitions, axioms and others previously proven. Hints can also be given for those characteristics of theoretical geometry that led Ptolemy to ask Euclid for a "short" learning path to it. Similarly, ancient texts on Eratosthenes' life and work could be used, with emphasis on the measurement of the earth's circumference (Thomaidis & Poulos 2006, p.110).

Cross-curricular activities could be also disconnected from conventional teaching and be realized more efficiently in parallel school events, like the formation of a group of students, who, under the teachers' supervision and help, read mathematical works. E.g., studying Tent's book (2006) could be pedagogically and didactically more efficient results than the note on Gauss in § 2.

5. ANCIENT GREEK MATHEMATICAL TEXTS IN THE TEACHING OF EUCLIDEAN GEOMETRY IN HIGH SCHOOL: A CROSS-CURRICULAR APPROACH

We present some elements of a deeper and more demanding approach to integrate the HM in teaching mathematics, than the use of historical snippets; namely the use of original texts in carefully designed worksheets, implemented in cross-curricular activities (Fauvel & van Maanen 2000, ch.9).

We developed a cross-curricular activity in 4 classes of 10th-graders (15-16 year old students; 25 girls and 25 boys in total), for 2-hour sessions in which the teachers of mathematics, ancient Greek language and history were involved with alternating interventions. To this end excerpts from Euclid's *Elements* and Proclus' *Commentary*, have been used to construct 4 worksheets, each one of which was used in a 2-hour classroom session. They concern: (a) Euclid, Proclus and Pappus' different proofs of the equality of an isosceles triangle's angles; (b) the construction of an angle's bisector; (c) the triangle inequality for the sides of a triangle; (d) the sum of the angles of a triangle.

This activity aimed to (i) integrate original texts in a cross-curricular teaching of Euclidean Geometry in the 10th grade; (ii) to create a new didactical environment and accordingly explore the realization of specific teaching aims; "initiation in mathematical proof", and "development of critical thinking". More specifically, by the chosen excerpts and the questions addressed to the students, we sought to examine whether the students (i) share the criticism of the ancient philosophers against Euclid, (ii) understand the expediency of giving different proofs for the same geometrical proposition, particularly for obvious properties of geometric figures (as Proclus did while defending Euclid) and (iii) understand the expediency of mathematical proof in general. Under the teachers' supervision, students analyzed ancient texts mathematically, linguistically and historically, with focus on formulating corresponding questions emerging from this analysis and the classroom discussion of students' point of view on them.

The worksheets' structure was: (a) Ancient Greek mathematical text; (b) Request to read and translate the text; (c) Questions on the text: 2 to 3; (d) Homework: 1 or 2 assignments.

Remarks: (1) Three of the worksheets contained 2 excerpts, with this structure for each excerpt; the fourth included 4 excerpts. We outline this approach for worksheet No1. (2) The discussions in the classroom were videotaped. Students' answers below refer to questions raised in the classroom (Q1-Q3 below) and come from the analysis of videotapes and the teachers' hand-notes.

Worksheet No1

Excerpts: (i) Euclid "*Elements*" *Book I*, *prop.V*: equality of the basis angles of an isosceles triangle (Heath 1956, pp.251-252). (ii) Proclus' "*Commentary*", §§248, 250: Alternative proofs of this proposition by Proclus and Pappus (Morrow 1970, pp.193-195).

Questions: Find: (1) the corresponding theorem in the geometry textbookq
(2) similarities & differences between Euclid's and the textbook's proofs.

Homework: (1) Translate the ancient text keeping to Euclid's spirit as close as possible (e.g. avoid terminology and notation not used by Euclid).
(2) Get information on Euclid and his *Elements* from encyclopedias or other resources.
(3) Translate Proclus' text to modern Greek.
(4) Find similarities and differences among Euclid, Proclus and Pappus' proofs.
(5) Try to explain why all ancient proofs are different from that in the textbook[4].

Classroom discussion on the following questions:
Q1. In your opinion, why did Euclid give a complicated proof?
Q2. Why did the ancients avoid using the bisector of the angle at the top vertex? How it can be ensured that the usual construction (by ruler and compass) of the bisector of an angle, does indeed bisect the angle?
Q3. Comment on Proclus' and Pappus' proofs.

Some of students' responses
*On **Q1, Q2***:
(i) Euclid wanted to impress his readers, because when scientists do complicated things, their authority increases.
(ii) Euclid wanted to show how to use the triangles' equality criteria.
(iii) Euclid wants a theoretical, not a practical proof. Bisecting an angle is a practical issue and is not accurate. This construction is naïve, possible for all people, because it is like folding in two a piece of paper.
(iv) Euclid could not draw the bisector accurately; he could not prove that the two angles are equal. The bisector concept had not been discovered yet.
(v) Euclid wanted to exploit that particular proof in order to prove other properties that exist in that particular figure.

*On **Q3** (for Pappus' proof)*:
(i) It looks like proofs that we gave at the elementary school.
(ii) It is a proof appropriate for babies(!)[5]
(iii) It is more difficult; it requires more thinking (more probable to make a mistake).
(iv) It is adapted to practice, whereas, Proclus' and Euclid's proofs have elements of logic and scientific reasoning.

Remarks on methodological issues concerning cross-curricular activities:
(1) This cross-curricular approach helped to face important issues concerning translation & interpretation and placed original texts in the appropriate historical context.
(2) The original texts and the translation process led to etymological comments on the origin, meaning and accurateness of mathematical terminology.
(3) The clarity and conciseness of ancient Greek mathematical language was revealed by connecting two apparently disjoint disciplines; ancient Greek language and mathematics.

Some results: The remarks, and the analysis of the classroom discussion stimulated

by the study of the other three worksheets suggests:
(a) Studying original texts created a new didactical environment, in which students actively participated in the classroom discourse and exhibited a positive attitude towards the subject, which never happens in conventional geometry teaching (this was particularly clear in the critical discussions on worksheet No3 on the triangle inequality and Stoics' objections reported by Proclus, that tried to ridicule Euclid).
(b) Students' commented that this activity led them to a more global understanding of what Euclidean geometry really is (e.g. see answers (ii) and (v) to Q2).
(c) The variety and mutual incompatibility of students' answers produced by studying original texts, reveal factors that influence the understanding of metamathematical concepts, like the concept of proof (e.g. compare answers to Q3; (i) & (ii) to (iii)).
(d) Critical thinking requires both the technical ability to formulate particular proofs, and more general abilities to globally conceive notions, to formulate correct assertions etc (e.g. see answers (iii) to Q3 and (iv) to Q2).
(e) The requirements for studying original texts, link the didactical aims of learning specific pieces of mathematics, with wider pedagogical aims of ME: raising metamathematical issues, access to philosophical & epistemological concepts, links to the historical & cultural tradition etc (e.g. see answers (i), (iii) and (iv) to Q2).

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[1]In Greece, there is only one textbook per subject in each grade of primary or secondary education, imposed by state regulation as a result of a public competition for writing these textbooks.
[2]In Greece, grades 1 to 9 constitute compulsory education: the elementary school (grades 1-6; students 6-12 year-old) and the "gymnasium" (junior high-school, grades 7-9, students 13-15 year-old). There are essentially no historical aspects in the elementary school textbooks; hence we restrict the discussion to junior high school.
[3] See Cuomo 200, p.245.
[4] In the textbook, the angle at the top vertex is bisected and the two resulting triangles are shown to be equal.
[5] In Pappus' proof an isosceles triangle is turned and the resulting triangle is shown to be equal to the initial one.

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