HISTORICAL PICTURES FOR ACTING ON THE VIEW OF MATHEMATICS

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The article illustrates the underlying philosophy of an in progress book in which pictures taken from historical books are used to hint some fundamental ideas of the history of mathematics. Both epistemological and disciplinary issues are taken into account. The aim of the book is to let its potential readers know different aspects of mathematics as a science operating inside the socio-cultural context.

Keywords: historical pictures, original sources, mathematics view.

INTRODUCTION

This paper deals with the problem of the view of mathematics held by students and the means suitable to act on it. In previous works we have studied students' view of mathematics as a socio-cultural process with particular reference to the historical development, see (Demattè & Furinghetti, 1999). Our main conclusion was that this view was very narrow focused and based on common myths on mathematics. To answer the question "How to act on the image of mathematics held by students?" a book has been designed by one of the authors (A. D.) addressed to students of the final years of secondary school (16 years old onward) or readers who are interested in the popularisation of mathematics. The book is based on pictures taken from historical sources. Pictures have been largely used in history for communicating mathematical ideas, see (Mazzolini, 1993), and thus it is not difficult to collect materials for composing such a book. Words accompany pictures in order to create a unitary discourse and to focus on some aspects. Pictures strengthen what the verbal part say, like in a natural history museum where things and words, verbal and non-verbal communication coexist. Knowledge required for using the book in classroom (or elsewhere) is confined to elementary mathematics. As we will see in sections 3 and 4 some chapters are more suitable to develop mathematical topics *stricto sensu*, other are more oriented to raise reflections on historical-epistemological questions.

THE ROLE OF PICTURES

The idea of this book does not come out of the blue. We have already described in (Demattè, 2005; 2006a; 2006b) our work with pictures in the classroom. In particular, in the latter two papers we have discussed how students in front of a historical figure are able to mobilize some kind of narratives and to produce conjectures. This is due to the particular nature of the information provided by figures. Often images show supplemental details, which are not pertinent to the specificity of discourse. Readers can interpret these images in different ways. A discourse follows a logical track (sometimes very rigorous), a picture often permits freedom to the interpreter.

Therefore it is ‘friendly’ i.e. rich in possibility of reflections and personal reasoning. Our claim may be illustrated by some examples taken from the book.

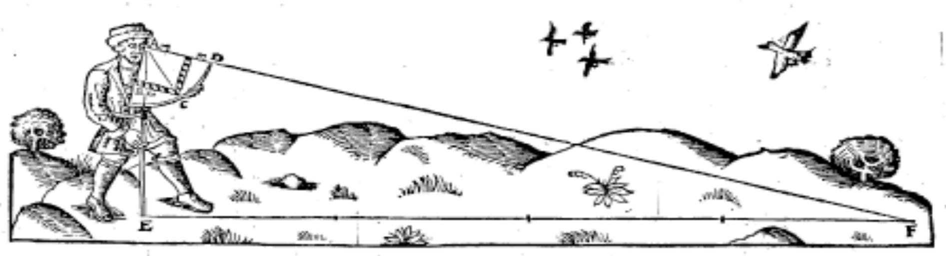


Fig. 1. Oronce Finé, *Protomathesis*, 1532

Pictures like Fig. 1 are aimed at showing how an instrument can be used, but the painter has added many details (hills, grass, trees, birds, elegant dress of the man) which make the scene realistic. The draw of the right-angled triangle and of the instrument (a “quadrant in a fourth part of a circle”) focuses on mathematical aspects.

To reflect on the use of the picture in Fig. 1 in classroom raises the following questions for the researcher: Can students appreciate these kinds of images? Do pictures like Fig. 1 make them want to use the facilities offered by mathematics? Do students see the relationship between the concepts and procedures shown in historical pictures and what they learn in school today? Maybe the answer is no, for each question. In any case the mathematics view suggested by this kind of pictures appears potentially positive in the fact that they address the attention to geometrical details and, in the same time, stimulate guessing the finalities of the action illustrated in the picture. A scene like the one in Fig. 1 suggests a simple story, a narration with a precise structure (some events happen before, some after, a goal of the action – including the implicit use of mathematics - is noticeable). (Demattè, 2006a; 2006b) report on an experiment where students were asked to write how they interpret Fig. 2.

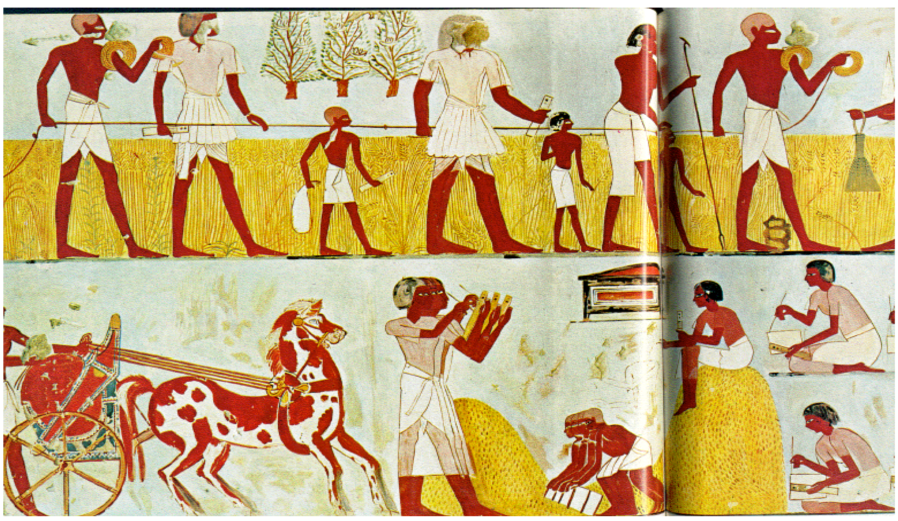


Fig. 2. A mural painted at Abd-el-Qurna, Egypt, around 1400 B.C

Some protocols show that they followed the pattern of a narrative. Because of the need to complete the story, students formulated also conjectures (e.g. the kings' servants on the cart have the task of rewriting the data and, as the student write, "the aim of giving an account of them to the king").

Students are rather naturally brought to formulate conjectures, which are coherent with context and with elements present in the scene, if they have adequate knowledge. To interpret mathematical aspects in the previous image from Finé's *Protomathesis* or in the following Fig. 3 the concept of similarity among triangles is required. But many other aspects require more knowledge: e.g. Why the square? Which is the purpose of the action of the man in the picture? etc.

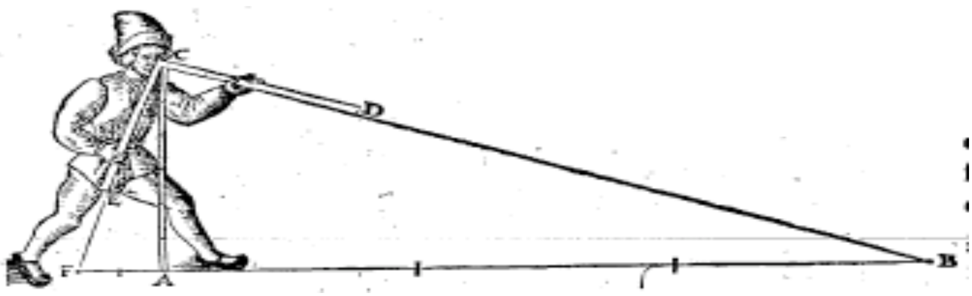


Fig. 3. Oronce Finé, *Protomathesis,* 1532

PICTURES AND MATHEMATICAL TOPICS

In the book the focus is on some grounding mathematical ideas that may be elaborated through the history of mathematics. These ideas regard the main chapters of mathematics (numeration, algebra, probability, etc., see Appendix). Some ideas are inherent to procedures and concepts: images suggest first of all the *incipit* of mathematical reasoning and its global structure. For example, the reader may reflect on the different ways of approaching the same theorem by considering the Chinese theorem of Pythagoras (Fig. 4) and what is done using Cartesian graphs.

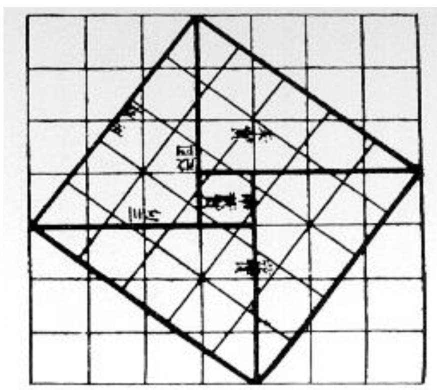


Fig. 4. 'Pythagorean' theorem from *Chou Pei Suan Ching*, about 500-200 b.C.

Moreover pictures, suggest at a glance some metacognitive information e.g. the level of complexity and the need of a detailed mathematical reasoning, as exemplified by the Leibnizian graphs shown in Fig. 5 from *Nova methodus pro maximis et minimis, itemque tangentibus, quae nec fractas, nec irrationales quantitates moratur, et singulare pro illis calculi genus* (A new method for maxima and minima as well as tangents, which is impeded neither by fractional nor by irrational quantities, and a remarkable type of calculus for this), see (Dupont & Roero, 1991).

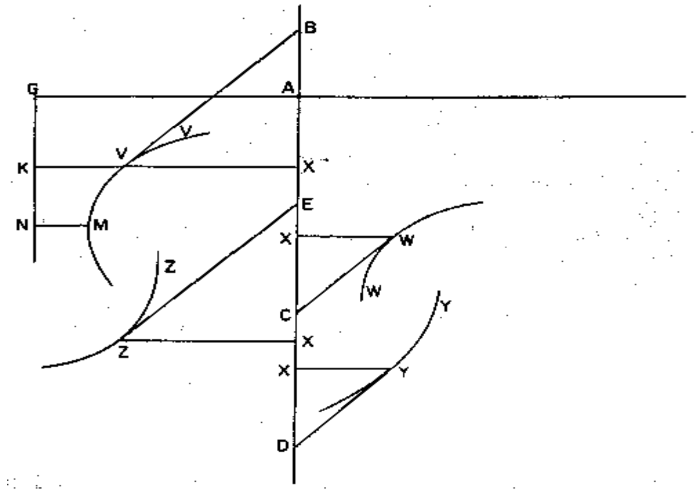


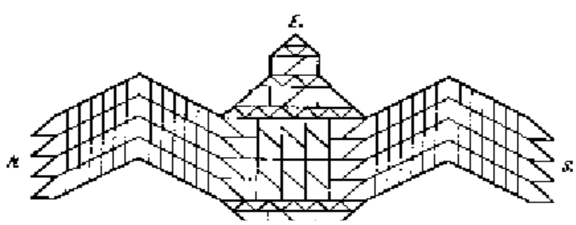
Fig. 5. Gottfried Wilhelm Leibniz, *Nova Methodus* …, 1684

1. PICTURES AND HISTORICAL-EPISTEMOLOGICAL IDEAS

Some chapters address historical and socio-cultural aspects such as: reckoning and measuring as answers to problems of human activities. The students may perceive the hypothetical-deductive structure of mathematics as a model for other branches of the human knowledge such as philosophy and economy, or for every day life. Through these chapters some myths about mathematics may be discussed: the development of mathematics seen as a linear progress from ancient to contemporary times, euro centrism, independence from external factors.

In our previous papers, see (Demattè & Furinghetti, 1999; Furinghetti, 2007) we discussed how students and teachers may conceive the development of mathematics just as an evolutionary process. In doing that they loose the richness of the path of mathematical ideas that are lateral to the main stream of the development of mathematical concepts. Moreover we know that the intertwining and the reciprocal influence of internalist and esternalist factors is a powerful perspective for studying mathematical concepts and its development, as shown in the paper (Radford, 2006). Mathematics has changed during the time but has become also different in different countries and cultural contexts.

Ethnomathematics (see a product in Fig. 6) is a fruitful branch of research in education. It is about learning mathematics connected to other areas, to social and



environmental problems (Joseph, 2003; Katsap, 2006). It lead to reflect on the fact that not only the European mathematics is the 'true mathematics'

Fig. 6. The most elaborate altar from the Indian *Sulbasutras* (the first part probably was written in the 6th century B.C.). Many of the triangular and trapezoidal altars described in the *Sulbasutras* use then theorem of Pythagoras

Some external factors influence the daily work of researchers: relations among colleagues (well known 'spy stories' regarded 16th century Italian algebraists, see Fig. 7), salary (not ethically impeccable 'involvements' come from the fact that ancient and modern war requires a wide apparatus of mathematical knowledge), national policy pushed by the dominating class, see (Barnett, 2006; Swetz, 1987), etc. This is enough to confirm that context influences advancement of science.

Fig. 7. Italian mathematicians Niccolò Fontana ("Tartaglia"; 1499-1557) and Gerolamo Cardano (1501-1576)

MATHEMATICS VIEW

The ultimate aim of the book is to suggest a different mathematics view. Every chapter ends with a discussion about beliefs on the nature of mathematics, which are connected with the aspect treated in it. This part of the book regards factors that are not always made explicit in the classroom, but influence the personal relation with mathematics. We deem it is important to stimulate students' awareness on these factors. In the book the pictures and the related comments show unusual, but in our opinion more realistic, aspects of mathematics. As discussed above, mathematics:

- is an historical construction which is socially and culturally bounded, therefore different cultural context have produced different forms of mathematics;
- is used in many professions and jobs; is present in the everyday life; has epistemological and also psychological aspects which are intertwined (such as the role of error and its acceptance by individuals);
- has relationships with other disciplines; requires debate, communication and involvement and may also originate wish to investigate.

We briefly recall some beliefs widespread among students and ordinary people that were detected in our study (Demattè & Furinghetti, 1999). These are some of the beliefs considered in the book with respect to the content of the chapters:

- it is better if I remember rules by heart and I don't attempt to reason with my brain;

- when I solve a mathematical problem I know that there is only one exact solution;
- mathematics learnt in school has not a practical use; not everybody has a 'mathematical mind';
- creativity is not necessary in mathematical reasoning; different topics, such as arithmetic, geometry, algebra, must be taught and learnt separately because they don't have any connections; in mathematics approximated results are incorrect and do not give useful information;
- in mathematics errors are absolutely negative experiences;
- mathematics doesn't depend on culture; I think that men have began to use the signs +, -, *x*, : before Christ;
- if I study alone (not with mates) I'll have better results in mathematics.

FINAL REMARKS

In a previous paper, see (Furinghetti, 1997) it is pointed out that there are two main streams in the use of history in the classroom: - to promote the image of mathematics, - to introduce mathematical contents. From our presentation it follows that our work is set in the first stream. Only a few parts of the chapters have been administered in the classroom. After completing the work it is planned to use it and to study students' reactions. We expect to carry out empirical research that allows to answer questions such as the following:

- How will readers consider the kind of mathematics presented in the book? Will they establish connections with mathematics they learned at school or will they consider it an 'extraneous entity'?
- What beliefs could change through learning the history of mathematics? What activities could be more useful?
- Learning history (in a broad sense) is also to remember facts and dates. What historical information could mathematics teacher require the students to remember? Could pictures create an opportunity to remember significant aspects of the history of mathematics?
- In our opinion, the citizen mathematics education requires new didactical choices. Could historical-epistemological analysis of mathematics replace some parts of traditional curriculum?

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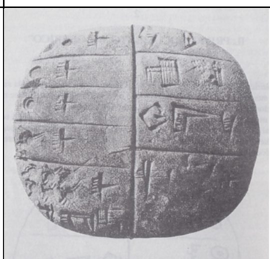
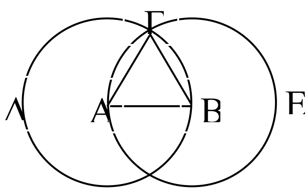
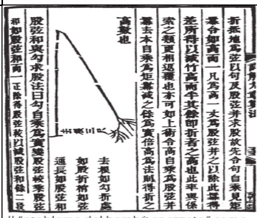
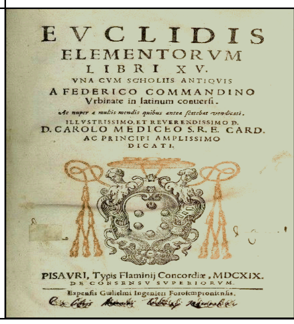
APPENDIX. The structure of the book

In the book there is a preface explaining the aim and the rationale of the work and 30 chapters whose titles and some representative figures are shown below.

<table>
<tr><td colspan="2">Legenda
E: *Chapters mainly concerning historical or* ***E****pistemological ideas.*
M: *Chapters over mainly concerning relevant* ***M****athematical topics.*</td><td>1. The first files of data (*M**)</td><td></td></tr>
<tr><td>2. Mathematics for administering a Nation (*E*)</td><td colspan="3">
Egyptian peasants seized for non-payment of taxes… (Pyramid Age)</td></tr>
<tr><td>3. Is mathematics we learn at school ancient? (*E*)</td><td></td><td>4. How to write a number (*M*)</td><td></td></tr>
<tr><td>5. Algebra begins (*M*)</td><td></td><td>6. Mathematics is full of errors (*E*)</td><td></td></tr>
<tr><td>7. Pythagoras in China (*M*)</td><td>
Il "problema del bambù spezzato" come viene illustrato nello *Hsiang Chieh Chiu Chang Suan Fa*.</td><td>8. A model to be imitated (*E*)</td><td></td></tr>
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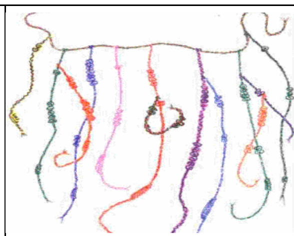
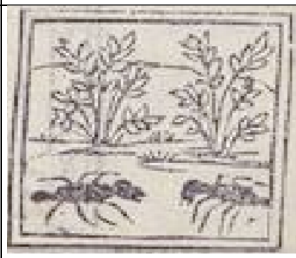
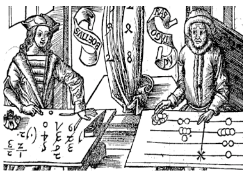
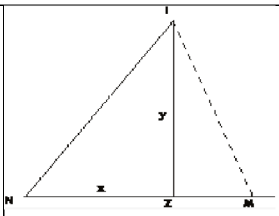
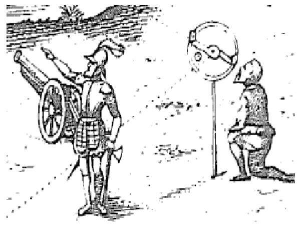
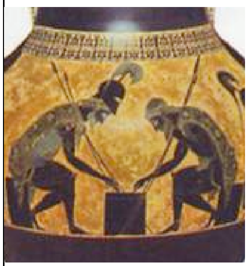
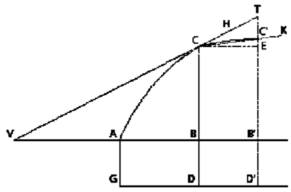
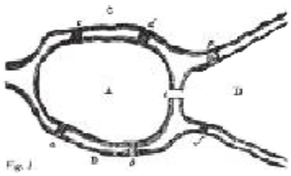
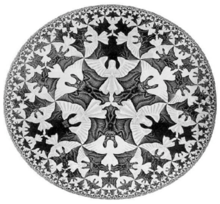
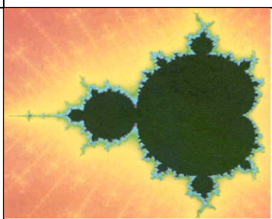
9. What is geniality? (*E*)		10. Does it depend on material we have? (*E*)	
11. Mathematical knowledge doesn't "accumulate in layers" (*E*)		12. Recreational problems (*M*)	
13. Does an authority hold knowledge? (*E*)		14. Mathematics is culture (*E*)	
15. Masters of abacus (*E*)		16.Mathematics and trade (*E*)	
17. Geometry for builders (*M*)		18. Mathematics and politics (*E*)	 ALLO ILLVSTRISSIMO ET ECCELLENTISSIMO S. IL S. COSIMO DE MEDICI, DVCA DI FIRENZE ET DI SIENA, SIG. ET PATRONE MIO OSSERVANDISSIMO.
19. More recent than we think (*E*)		20. Is mathematics the same everywhere? (*E*)	

21. Problems of paternity (*E*)		22. Mathematics and war (*E*)	
23. Let's bet everything (*M*)		24. Calculus (*M*)	
25. Mathematics and other sciences (*E*)		26. Geometry of position (*M*)	
27. Beyond infinity (*M*)		28. Etnomathematics (*E*)	
29. Past, present and future (*E*)		30. Imagine a mathematician (*E*)